SLOW OSCILLATIONS OF THE UNBALANCED VIBROEXCITERS' ROTORS WHEN PERTURBING THE SELF-SYNCHRONIZATION REGIMES

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Abstract

The results, presented in article [Indeitsev, Blekhman and Fradkov, 2007] are extended on the case of the system with self-synchronizing mechanical vibroexciters.It is shown, that so-called ”inner pendulum” is obtained when perturb a steady mode of self-synchronization of several inertial vibroexciters. In the case of two activators the equation of semi slow oscillations of ”inner pendulum” is obtained. Frequency of these oscillations is much above the change of anqular speed rate of rotors in start-up mode frequency.

Key words

Self-synchronization, mechanical vibroexciters, inner pendulum, oscillations.

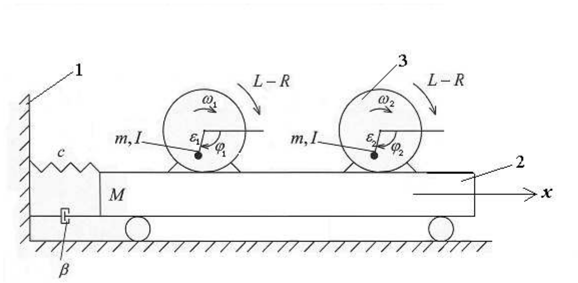

Figure 1. The scheme of system

1 Introduction

Oscillations of the angular speed of rotation when starting and stopping the inertial vibroexciter, the unbalanced rotor that is established on resiliently supported rigid platform and set in rotation by the engine of asynchronous type, are considered in article [Indeitsev, Blekhman and Fradkov, 2007].By use of a method direct separation of motions the equation of ”semi-slow” oscillations, so-called ”inner pendulum”, was obtained for a second approximation.The existence of such pendulum is important for formation of an algorithm of optimum control of start-up mode of the device which is characterized by the so called Sommerfeld effect.Roughly speaking, Sommerfeld effect is regarded to be ”capturing” of rotor angular velocity near system igene frequency.In [Indeitsev, Blekhman and Fradkov, 2007] there are also reviews of researches of this effect. In this work for the case of two activators the equation of semi slow oscillations of inner pendulum is obtained. It is remarkable, that the obtained formula for frequency of indicated semi slow oscillations differs from corresponding frequency found in work [Indeitsev, Blekhman and Fradkov, 2007], only by simply interpreted multiplier.By another much more complex method - with use of asymptotic methods and by decomposition of a small parameter on fractional powers for another system semi slow rotor motions are investigated in [Pechenev, 1992].

2 The Scheme and the Description of System

The considered system (Fig.1) represents the rigid platform, connected with the motionless basis by means of an elastic element with rigidity c and linear dampening element with decrement factor β. Here 1 - The motionless basis; 2- A platform; 3 - Vibroexciter. Two unbalanced vibroexciters, unbalanced rotors, which are set in rotation by asynchronous type electric engines; rotors' axes are perpendicular to the oscillation direction of a platform, are set on the platform.The described system represents the elementary system considered in the theory of self-synchronization of vibroexciters.[Blekhman,2000]

It is described system of three equations:

$$I_s\ddot{\varphi}_s = L_s(\dot{\varphi}_s) - R_s(\dot{\varphi}_s) + m_s\varepsilon_s(\ddot{x}\sin\varphi_s + g\cos\varphi_s)$$
$$M\ddot{x} + \beta\dot{x} + Cx = \sum_{j=1}^{2} m_j\varepsilon_j(\ddot{\varphi}_j\sin\varphi_j + \dot{\varphi}_j^2\cos\varphi_j) \quad (1)$$

where $s = 1, 2; m_s, I_s$ are weight and the inertia moment of a rotor of s-th vibroexciter with respect to axis passing through its centre of gravity; L_s - the rotating moment of the asynchronous electric motor;R_s - the force moment of resistance to the rotation, caused by the resistance in bearings; M - weight of the whole system;ε- eccentricity of a rotor; φ_s- angle of rotation of the rotors, counted from an axis OX on a clockwise direction; x - displacement of a platform from the position corresponding to the unstressed elastic element.

3 The Equation of Movement of the Inner Pendulum (Semi Slow Oscillations)

As the theoretical researches confirmed by numerous experiments [Blekhman,2000] show, system (1) under certain conditions admits stable stationary solutions

$$\varphi_s = \omega t + \alpha_s + \psi_s(\omega t) \quad (2)$$

where α_s=const; $\psi_s, x - \ 2\pi$ - periodical functions with zero averages on this arqument; these solutions correspond to the synchronous rotation of activators' rotors with average frequency ω.

Near to stationary value α_s=const phase shifts α_s represent slowly changing time functions t, satisfying to the equations:

$$\begin{aligned} I_s\ddot{\alpha}_s + k_s\dot{\alpha}_s &= \sigma_s L_s(\sigma_s\omega) - R_s^0(\omega) + \\ &+ V_s(\alpha_1 - \alpha_k, ..., \alpha_{k-1} - \alpha_k, \omega), \qquad s = 1, 2 \end{aligned} \quad (3)$$

where the vibrating moments

$$V_s = -\frac{m_s\varepsilon_s\omega^2}{2M\Delta}\sum_{j=1}^{k} m_j\varepsilon_j\sin(\alpha_s - \alpha_j - \gamma) \quad (4)$$

And each of sizes σ_s can be either 1 or -1; rotor rotation of the $s-$ th activator in positive direction corresponds to the first case, rotor rotation of the $s-$ th activator in negative direction corresponds to the second case.Here:

$$\begin{aligned} &\Delta = \sqrt{(1-\lambda^2)^2 + 4n^2}, \quad \sin\gamma = -\frac{2n}{\Delta}, \\ &p = \sqrt{\frac{c}{M}}, \ n = \frac{\beta}{2M\omega}, \quad \lambda = \frac{p}{\omega} \end{aligned} \quad (5)$$

It is also supposed, that $\dot{\alpha}_s << \omega$. For two identical vibrators $L_s - R_s = 0$,and the system transforms into:

$$\begin{aligned} &I\ddot{\alpha}_1 + k\dot{\alpha}_1 = \\ &= -\frac{(m\varepsilon\omega)^2}{2M\Delta}(\sin(\alpha_1 - \alpha_1 - \gamma) + \sin(\alpha_1 - \alpha_2 - \gamma)) \\ &I\ddot{\alpha}_2 + k\dot{\alpha}_2 = \\ &= -\frac{(m\varepsilon\omega)^2}{2M\Delta}(\sin(\alpha_2 - \alpha_1 - \gamma) + \sin(\alpha_2 - \alpha_2 - \gamma)) \end{aligned} \quad (6)$$

It is obvious the stationary values of slow variables $\alpha_s = const$ correspond to synchronous motion.

Indicate $\alpha = \alpha_1 - \alpha_2$ and subtracting the second system equation from the first one (6), we will obtain

$$I\ddot{\alpha} + k\dot{\alpha} + \frac{(m\varepsilon\omega)^2}{M}\frac{p^2-\omega^2}{((p^2-\omega^2)^2 + 4n^2\omega^2)^2}\sin\alpha = 0 \quad (7)$$

Here $k-$ total decrement coefficient. This equation has two stationary solutions $\alpha = \alpha_1 = 0$ and $\alpha = \alpha_2 = \pi$. The first solution $\alpha = \alpha_1 = 0$ is steady in preresonant area $\omega < p$, and the second $\alpha = \alpha_2 = \pi$- in postresonant area $\omega > p$.

4 Semislow Oscillations Frequency; Comparison with Results of Work [Indeitsev, Blekhman and Fradkov, 2007]

Frequency of relative free rotors' oscillations in both cases is determined from the equation:

$$q_2 = \frac{m\varepsilon\omega^2}{\sqrt{MI}}\frac{\sqrt{|p^2-\omega^2|}}{\sqrt{(p^2-\omega^2)^2 + 4n^2\omega^2}} \quad (8)$$

It is remarkable, that this frequency in $\sqrt{2}$ times more than semi slow rotor oscillation frequency

$$q_1 = \frac{m\varepsilon\omega^2}{\sqrt{2MI}}\frac{\sqrt{|p^2-\omega^2|}}{\sqrt{(p^2-\omega^2)^2 + 4n^2\omega^2}} \quad (9)$$

obtained in work [Indeitsev, Blekhman and Fradkov, 2007].

Note, that such difference completely corresponds to difference of frequencies of free oscillations of a cargo weight m on a spring with rigidity c and two free identical weights m connected by a spring with the same rigidity.

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References

Blekhman, I. I.(2000).*Vibrational Mechanics*.World Scientific. Singapore-New Jersey-London-Hong Kong

Indeitsev, D., Blekhman, I. and Fradkov, A. (2007) Slow motions in systems with inertially excited vibrations. IFAC ALCOSP 2007, http://lib.physcon.ru

Pechenev, A. V.(1992) Hierarchical separation of motions in problems about self-synchronization mechanical vibroexciters with an asynchronous drive.In *Problems of Mechanical Engineering and reliability of machines*, Moscow, 1992, 2,pp.15-22.