# Identification of Programmed Control and Intensity of Perturbations for a Flying Vehicle

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**Abstract:** Necessary conditions to identify parameters and control of nonlinear variable stochastic systems being used to describe the operation of a flying vehicle and its subsystems are analyzed, the system parameters, control function, and states vector being bounded. The break moments as well as break points characteristics are related to a set of parameters being identified.

Keywords: identification, random disturbance, flying vehicle, control, parameter

## 1. INTRODUCTION

This paper considers the problem of identification of parameters and control of nonlinear variable stochastic systems by using an adjustable model of a diffusion Markov process class with system requirements limitations. Supporting these requirements narrows down the set of states of the system and allowed definitional domains for the parameters of the system and control in agreement with the prescribed set-up and being subject to new mathematical methods to research identification problems.

## 2. THE PROBLEM STATEMENT

Let consider the problem of identification of program control u = u(t), parameters a and time-points T of switching of the structure of a controlled nonlinear stochastic system

$$dX_{i} = \sum_{q=1}^{n} \left( C_{iq}^{j}(t) dt + dW_{iq}(t) \right) \varphi_{iq}^{j}(t, X, u, a) + \\ + \sum_{q=1}^{n} \sigma_{iq}^{j}(t, X) d\eta_{q}(t) , \qquad (1)$$
$$X_{i}(t_{0}) = X_{i0}, t \in [t_{j-1}, t_{j}], \quad (i = 1, ..., n; j = 1, ..., k) ,.$$

which describes the functioning a flying vehicle or its subsystems over the time-interval  $[t_0, t_k]$  in a successive manner over adjoining sections  $[t_{j-1}, t_j]$ , (j = 1, ..., k). Control objectives and different system requirements are described by mixed limitations of equalities and inequalities of system parameters, control functions, and phase coordinates.

$$I_{s}(v) = \sum_{j=1}^{k} \int_{\Omega} F_{s}^{j}(x) p(t_{j}, x \mid \overline{z}) dx = c_{s}, \qquad (2)$$

$$s \in J_{1} = \{1, \dots, q\},$$

$$I_{s}(v) = \sum_{j=1}^{k} \int_{\Omega} F_{s}^{j}(x, a, t_{j}) p(t_{j}, x \mid \overline{z}) dx \leq b_{s},$$

$$s \in J_{2} = \{1, \dots, q_{0}\},$$
(3)

$$\Psi_{s}(u) \leq 0 , \ s \in J_{3} = \{1, \dots, r_{0}\},$$
(4)

$$g_{*}(a) \le 0, \ s \in J_{4} = \{1, \dots, m_{0}\}.$$
 (5)

Identification efficiency by observations

$$Z_{l} = \sum_{\nu} c_{l\nu} X_{\nu} + \dot{w}_{l}(t) .$$
<sup>(6)</sup>

is estimated by the minimum of a functional

$$I_{0}(v) = \sum_{j=1}^{k} \int_{t_{j-1}}^{t_{j}} \int_{\Omega} \sum_{l=1}^{n} \alpha_{l} \left( z_{l} - \sum_{\nu=1}^{n} c_{l\nu} x_{\nu} \right)^{2} \times \\ \times p(t_{j}, x \mid \overline{z}) dx dt \to \min .$$

$$(7)$$

Here t - a time;  $t_0, t_k - initial$  and final points of the interval considered  $[t_0, t_k]$ ;  $T = (t_1, ..., t_k)$  are points of switching of the structure satisfying the condition  $t_1 < \cdots < t_i < \cdots < t_k$ ; X(t) - the *n*-dimensional random vector function of a system state being continuous of t over  $[t_{j-1}, t_j]$ , (j, ..., k),  $X_k = X(t_k); u(t) - a$  deterministic *r*-dimensional vector function of control being section continuous over timesegments  $[t_{j-1}, t_j]$ ; *a* – a deterministic *m*-dimensional vector of controlling parameters which defines constructive as well energy parameters of the system.  $dW_{ia}(t)$ , as  $d\eta_a(t)$  - stochastic Stratonovich differentials of the Wiener processes  $W_{iq}(t)$ ,  $\eta_{q}(t)$ . Right-hand members of (1) being considered over continuity segments meet known requirements that a solution exists and they have discontinuities of the first kind at time-points  $t_i$ . The upper index j at the right-hand members of (1) characterizes the system structure over time-segment  $[t_{i-1}, t_i]$ . The functionals  $I_s(v), s \in \{0\} \cup J_1 \cup J_2$  are limited and differentiable sets of variables v = (u, a, T).  $\psi_s(u)$ ,  $g_s(a)$  are continuous along with their derivatives of the function.  $\dot{w}_i(t)$  – the derivative of the Wiener process - an additive white noise component of a measuring instrument.

The equations (1) describe a diffusion markovian process along adjoining segments  $[t_{j-1}, t_j]$ , (j=1,...,k) in a consecutive manner over  $[t_0, t_k]$ ., its a posteriori probability density of states  $p(t, x | \overline{z})$  in accordance with Evlanov and Konstantinov (1976) satisfying the equation

$$\frac{\partial p(t, x | \overline{z})}{\partial t} = L^{j}(t, v, x) p(t, x | \overline{z}) + \left( F(x, z) - \int_{\Omega} F(x, z) p(t, x | \overline{z}) dx \right) p(t, x | \overline{z}),$$

$$p(t, x | \overline{z}) \Big|_{t=t_{0}} = p(t_{0}, x), \quad t \in [t_{j-1}, t_{j}], \quad (j = 1, ..., k),$$
(8)

and the conjugation conditions at break-points  $t_i$ 

(i = 1, ..., n; p = 1, ..., n; j = 1, ..., k - 1),

$$\begin{bmatrix} A_i^j(t_j,\cdot)p(t_j,x|\overline{z}) - \frac{1}{2}\sum_{p=1}^n \frac{\partial}{\partial z_p} (B_{ip}^j(t_j,\cdot)p(t_j,x|\overline{z})) \end{bmatrix} = 0,$$
  
$$[B_{ip}(t_j,\cdot)p(t_j,x|\overline{z})] = 0], \qquad (9)$$

Here

$$\begin{split} L^{j}(t,x,u,a) p(t,x|\overline{z}) &= -\sum_{i=1}^{n} \frac{\partial}{\partial x_{i}} \Big[ A_{i}^{j}(t,v,x) p(t,x|\overline{z}) \Big] + \\ &+ \frac{1}{2} \sum_{i,p=1}^{n} \frac{\partial^{2}}{\partial x_{i} \partial x_{p}} \Big[ B_{ip}(t,v,x) p(t,x|\overline{z}) \Big], \end{split}$$

 $\overline{z}$  means that the whole observed realization of the output signal of the measuring instrument is used over the time-interval  $[t_{j-1}, t_j]$ , (j, ..., k);  $A_i^j(t, v, x)$ -drift coefficients with discontinuities of the first kind at time-points  $t_j$ , (j = 1, ..., k-1)

$$\begin{split} A_i^j(t,v,x) &= \sum_{q=1}^n C_{iq}^j \varphi_{iq}^j + \frac{1}{2} \sum_{k,p,q=1}^n \left[ -\frac{\partial \varphi_{ik}^j}{\partial z_p} (\varphi_{pq}^j G_{ikpq}^W + \\ &+ \sigma_{pk}^j G_{ikq}^{W\eta}) + \frac{\partial \sigma_{ik}^j}{\partial z_p} (\varphi_{pq}^j G_{kpq}^{\eta W} + \sigma_{pq}^j G_{kq}^\eta) \right]; \end{split}$$

 $B_{ip}(t,v,x)$  is diffusion matrix coefficients with discontinuities of the first kind at time-points  $t_i$ , (j = 1,...,k-1)

$$\begin{split} B_{ip}(t,v,z) &= \sum_{k,q=1}^{n} \left( \varphi_{ik}^{j} \varphi_{pq}^{j} G_{ikpq}^{W} + \varphi_{ik}^{j} \sigma_{pq}^{j} G_{ikq}^{W\eta} + \right. \\ &+ \sigma_{ik}^{j} \varphi_{pq}^{j} G_{kpq}^{\eta W} + \sigma_{ik}^{j} \sigma_{pq}^{j} G_{kq}^{\eta} \right) \,. \end{split}$$

 $G_{ik}^{W}, G_{pq}^{W}, G_{k}^{\eta}, G_{q}^{\eta}$  are intensities of Wiener processes according to  $W_{ik}(t), W_{pq}(t), \eta_{k}(t), \eta_{q}(t); G_{ikpq}^{W}, G_{kq}^{\eta\eta}, G_{kq}^{\eta}$  are mutual intensities of Wiener processes. The expressions in square brackets (9)  $[F(\cdot)] = F(\cdot)_{-} - F(\cdot)_{+}$  designate a difference of the expressions contained in them left and right off breakpoints  $t_{j}$ , (j = 1, ..., k - 1). The scalar function F(x, z) is defined by the formula

$$F(x,z) = \sum_{l,p,q=1}^{n} \frac{1}{G_{pl}^{R}} c_{pl} x_{q} \left( z_{l} - \frac{1}{2} \sum_{\nu=1}^{n} c_{l\nu} x_{\nu} \right),$$

where  $G_p^R$ ,  $G_k^R$  are noise intensities of the measuring instrument;  $G_{pk}^R$  is mutual noise intensities of the measuring instrument.

Out of existence and uniqueness of the solution (1), and also out of continuous dependence of this solution on the acceptable control v = (u, a, T) (Gihman and Skorohod, 1977) follows the existence and uniqueness of the solution (8) by the distribution and continuous dependence of the probability density  $p(t, x | \overline{z})$  on the acceptable control v = (u, a, T) and the initial distribution  $p(t_0, x)$ . Thus, the initial problem of identification (1) - (7) is reduced to an equivalent problem with spread parameters relative to a posteriori distribution density  $p(t, x | \overline{z})$ , which is reduced to a terminal problem, its states vector extending to  $\overline{X} = (X, X_{n+1})$ 

$$I_{0}(v) = \sum_{j=1}^{k} \int_{\Omega} x_{n+1} p(t_{j}, \overline{x} | \overline{z}) d\overline{x} \to \min, \qquad (11)$$

$$\frac{\partial p(t, \overline{x} | \overline{z})}{\partial t} = \overline{L}^{j}(t, v, \overline{x}) p(t, \overline{x} | \overline{z}) + \left( F(x, z) - \int_{\Omega} F(x, z) p(t, x | \overline{z}) dx \right) p(t, \overline{x} | \overline{z}), \qquad (12)$$

$$p(t_{j}, \overline{x} | \overline{z})_{-} = p(t_{j}, \overline{x} | \overline{z})_{+}, \quad (j = 1, ..., k - 1),$$

with conjugation conditions (9) and limitations (3) – (5). Here  $x_{n+1}$  is a realization of the component  $X_{n+1}$  of the extended vector function of the system state  $\bar{x} = (x, x_{n+1})$  defined by the differential equation

$$dX_{n+1} = \sum_{l=1}^{n} \alpha_l \left( Z_l - \sum_{\nu=1}^{n} c_{l\nu} X_{\nu} \right)^2 dt , \ t \in [t_{j-1}, t_j]$$
$$X_{n+1}(t_0) = 0, \ (j = 1, \dots, k).$$

 $\overline{L}^{j}(t,v,\overline{x})p(t,\overline{x}|\overline{z})$  is defined by the expression:

$$\overline{L}^{j}(t,v,\overline{x}) p(t,\overline{x}|\overline{z}) = L^{j}(t,v,x) p(t,x|\overline{z}) - \frac{\partial}{\partial x_{n+1}} \left[ \sum_{l=1}^{n} \alpha_{l} \left( z_{l} - \sum_{\nu=1}^{n} c_{l\nu} x_{\nu} \right)^{2} p(t,x|\overline{z}) \right].$$

## 3. NECESSARY IDENTIFICATION CONDITIONS

Necessary identification of control v = (u, a, T) of problem (11) - (12) over uniformly close neighborhood of identifiable control is established by theorem 1.

**Theorem.** (weak necessary conditions). To identify the system (1) by observations (6) with limitations (2) – (5) at a point  $v^0 = (u^0, a^0, T^0)$  by criterion (7), it is necessary that there exist a simultaneously nonzero vector  $\gamma = (\gamma_1, ..., \gamma_q) \neq 0$ , nonnegative vectors  $\alpha = (\alpha_1, ..., \alpha_{q_0}) \ge 0$ ,  $\beta = (\beta_1, ..., \beta_{m_0}) \ge 0$  and a bounded function  $\lambda(t, x | \overline{z}) \in C^{1,2}$  defined by a solution to a boundary problem

$$\begin{aligned} \frac{\partial \lambda}{\partial t} &= -L^{*j}(t, v^0, x)\lambda + M \left[ F(X, Z) \middle| \overline{z} \right] \lambda - G(x) ,\\ t &\in [t^0_{j-1}, t^0_j], \ (j = 1, ..., k), \end{aligned}$$
(13)

$$\lambda(t_k, x | \overline{z})_{-} = x_{n+1} - \sum_{s=1}^{q} \gamma_s F_s(x) + \sum_{s=1}^{q_0} \alpha_s F_s(x) , \qquad (14)$$

$$\lambda(t_j, x | \overline{z})_{-} = \lambda(t_j, x | \overline{z})_{+}, \quad (j = 1, \dots, k-1), \quad (15)$$

in which:

a) for almost all  $t \in [t_{j-1}^0, t_j^0]$ , (j=1,...,k) and all u over uniformly close neighborhood of the point u, the optimum control  $u^0$  satisfies the inequality

$$\left(M\left(\frac{\partial R^{i}}{\partial u}\right)+\sum_{s=1}^{r_{0}}\mu_{s}\chi_{T_{s}}\frac{\partial\psi_{s}}{\partial u}\right)(u-u^{0})\geq0;$$
(16)

b) the parameters  $a^0$ ,  $T^0$  satisfy the conditions

$$\sum_{j=1}^{k} \left( \sum_{s=0}^{q_0} \alpha_s M\left(\frac{\partial F_s^j}{\partial a}\right) + \int_{t_{j-1}^0}^{t_j^0} M\left(\frac{\partial R^j}{\partial a}\right) dt \right) + \sum_{s=1}^{m_0} \beta_s \frac{\partial g_s}{\partial a} = 0$$

$$(17)$$

$$\sum_{s=0}^{q_0} \alpha_s M\left(\frac{\partial F_s^j}{\partial t_j}\right) + M(R^j(\cdot) - R^{j+1}(\cdot))|_{t=t_j^0} = 0.$$

$$(j = 1, \dots, k).$$
(18)

Expressions (13) - (18) demonstrate the following:

$$G(x) = \sum_{l=1}^{n} \alpha_l \left( z_l - \sum_{\nu=1}^{n} c_{l\nu} x_{\nu} \right)^2,$$
  

$$L^{*j}(t, \nu^0, x) \lambda = \sum_{i=1}^{n} \frac{\partial \lambda}{\partial x_i} A_i(t, \nu^0, z) +$$
  

$$+ \frac{1}{2} \sum_{i, p=1}^{n} \frac{\partial^2 \lambda}{\partial x_i \partial x_p} \partial B_{ip}(t, \nu^0, z),$$
  

$$R^j = L^{*j}(t, \nu^0, x) \lambda, \ R^0 = R^{k+1} = 0$$

The proof of the theorem is analogous to proof samples (Rodnishev, 2001a).

The solution to the identification problem (11) - (12) requires the solution to the equation (12) and to the parabolic equation (18) being conjugated to (12). However, as it is known, the solutions to these equations for higher order systems can only be obtained for linear stochastic systems (Krasovsky, 1974; Kazakov, 1977) and only for exceptional cases of nonlinear systems of no higher than third order. So, to solve (11) – (12), it is suggested using a method by Rodnishev (2001b) based on employing statistics – a posteriori semi-invariants of the process (1) having a distribution density (8).

## 4. IDENTIFICATION OF STOCHASTIC SYSTEMS BY STATISTICS OF A PHASE STATE OF THE SYSTEM

The identification problem (11) - (12) being relative to statistics is reduced to the problem

$$I_0(v) = \sum_{j=1}^k \int_{\Omega} x_{n+1} p(t_j, \overline{x} | \overline{z}) \, d\overline{x} \to \min \,, \tag{19}$$

$$\dot{\omega}_{1}^{i} = M[A_{i}^{j}(\cdot)] + M[x_{i}F(x,z)] - \omega_{1}^{i}M[F(x,z)];$$
  

$$\dot{\omega}_{11}^{ip} = M[\tilde{x}_{i}A_{p}^{j}(\cdot) + \tilde{x}_{p}A_{i}^{j}(\cdot)] + M[B_{ip}(\cdot)] +$$
  

$$+ M[\tilde{x}_{i}\tilde{x}_{p}F(x,z)] - \omega_{11}^{ip}M[F(x,z)];$$
(20)

$$\begin{split} \dot{\omega}_{N_{i}}^{i} &= N_{i} M[\tilde{x}_{i}^{N_{i}-1} A_{i}^{j}(\cdot)] + \frac{1}{2} N_{i} (N_{i}^{-1}) \times \\ &\times M[\tilde{x}_{i}^{N_{i}-2} B_{ii}^{i}(\cdot)] - \sum_{q_{i}^{-1}}^{N_{i}-2} C_{N_{i}}^{q_{i}} (\mathbf{m}_{q_{i}}^{i}) M[\tilde{x}_{i}^{N_{i}-q_{i}}] + \\ &+ M[(\tilde{x}_{i})^{N_{i}} F(x,z)] - \omega_{N_{i}}^{i} M[F(x,z)] ; \\ &(i = 1, \dots, n; p = 1, \dots, n; j = 1, \dots, k; N_{i} = 3, 4, \dots) \\ &t \in [t_{j-1}, t_{j}], (j, \dots, k), \\ &\omega_{i}^{i}(t_{0}) = c_{0}^{i}, \omega_{11}^{ip}(t_{0}) = c_{0}^{ip}, \omega_{N_{i}}^{j}(t_{0}) = b_{0}^{i}, \\ &\omega_{i}^{i}(t_{j})_{-} = \omega_{i}^{i}(t_{j})_{+}, \omega_{11}^{ip}(t_{j})_{-} = \omega_{11}^{ip}(t_{j})_{+}, \omega_{N_{i}}^{i}(t_{j})_{-} = \omega_{N_{i}}^{i}(t_{j})_{+}; \\ &\omega_{2N_{i}}^{i}(t_{j})/(\omega_{2}^{i}(t_{j}))^{N_{i}} \ge -K_{2N_{i}} \end{split}$$

with limitations (3) – (5). Being relative to a posteriori semiinvariants  $\omega_{1}^{i}$ ,  $\omega_{11}^{ip}$ ,  $\omega_{N_{i}}^{i}$  of the random process X(t), ordinary differential equations (20) are of degree 2n+n(n+1)/2+(N-2)Upper indices of the semi-invariants designate numbers of the state vector components, lower ones – orders of the semiinvariants. (N-2) – number of semi-invariants with upper than second order,  $\tilde{x}_{i} = x_{i} - \omega_{1}^{i}$ . A posteriori semi-invariants of the first order  $\omega_{1}^{i}$  have equal mathematical expectations of components  $X_{i}(t)$  of a phase states vector X(t) being optimum values of components  $X_{i}(t)$ . A posteriori semiinvariants of the second order  $\omega_{11}^{ip}$  have equal a posteriori covariance moments of components  $X_{i}(t)$  and  $X_{p}(t)$  of a phase states vector X(t).

Problem (19) - (21) belongs to the type of problems of the theory of optimum processes with limitations by equalities and inequalities. To solve this problem, numerical methods, particularly stated in Bodner et al. (1987), may be used.

## 5. IDENTIFICATION OF CHARACTERISTICS OF A FLYING VEHICLE WHILE ITS TRANSFERING TO A PRESELECTED ALTITUDE

Let consider the problem of identification of characteristics of a flying vehicle with accelerator while its transferring to a preselected altitude at constant disturbance  $n_1(t)$ characterizing the disturbance of jet acceleration with white noise of an angular traction force vector and parametric noise  $n_2(t)$  caused by erosive fuel burn in the combustion chamber. The functioning the flying vehicle over adjoining segments  $[0, t_1]$ ,  $[t_1, T]$  in a successive manner,  $t_1$  being an accelerator detachment moment, is described by the stochastic differential equations:

$$X_{1} = -c_{j}u + n_{1}(t),$$

$$\dot{X}_{2} = X_{3},$$

$$\dot{X}_{3} = c_{j}(v + n_{2}(t))u - g,$$

$$X_{1}(0) = 1, \quad X_{2}(0) = 0, \quad X_{3}(0) = 0;$$

$$X_{1}(t_{1})_{-} = X_{1}(t_{1})_{+}, \quad X_{2}(t_{1})_{-} = X_{2}(t_{1})_{+},$$

$$X_{3}(t_{1})_{-} = X_{3}(t_{1})_{+}, \quad j = 1, 2$$
(22)

Here  $X_1(t)$  – mass of the flying vehicle related to the initial mass,  $X_2(t)$  – altitude over the ground (km),  $X_3(t)$  – speed (km/sec); u(t) – mass consumption per time unit (1/sec) before and after the detachment of the accelerator accordingly; v=2 km/sec is a gas outflow rate from the nozzle, g=0.01 km/sec<sup>2</sup> – gravitational acceleration.  $G_{n_1}$  – intensity of the additive white noise  $n_1(t)$ ,  $G_{n_2}$  – intensity of the parametric white noise  $n_2(t)$  of the gas outflow rate.  $c_1=1$ ,  $c_2=0.5$  – set parameters of the system which characterize energy characteristics of the propulsion unit of the flying vehicle.

It is needed to determine the time-point  $t_1$  of the accelerator detachment, muss consumption u(t), disturbance intensities  $G_{n_1}, G_{n_2}, G_{n_3}, G_{n_4}$  and estimates of the flying vehicle mass  $X_1(t)$  and altitude  $X_2(t)$  at the final time-point T = 100c by altitude observations

$$Z_2 = X_2 + n_3(t)$$

at white noise intensity of a measuring instrument  $G_{n_3}$  and measuring the speed of the flying vehicle

$$Z_3 = X_3 + n_4(t)$$

with white noise intensity  $G_{n_4}$ . The estimate of these characteristics is carried out under the condition that when transferring the flying vehicle to the preselected altitude its weight must amount to 40 % of the initial mass, and the mass consumption u(t) must be in the range:  $0 \le u(t) \le 0.04$ . The effectiveness of identification of the characteristics mentioned is estimated by minimum of the functional

$$I_{0} = \sum_{i=1}^{2} \int_{t_{i-1}\Omega} \int_{\Omega} \left[ k_{2}(x_{2} - z_{2})^{2} + k_{3}(x_{3} - z_{3})^{2} p(t, x \mid \overline{z}) dx dt \to \min, \right]$$
(23)

where  $t_0 = 0$ ,  $t_2 = T$ ,  $k_2 = k_3 = 0.5$ . Equations (22) describe a diffusion markovian process with coefficients of drift and diffusion:

$$A_{1} = -c_{j}u, A_{2} = x_{3}, A_{3} = c_{j}vu - g,$$
$$B_{11} = G_{n_{1}}, B_{33} = (c_{j}u)^{2}G_{n_{2}},$$

A scalar function F(x,z) characterizing changes of altitudes and speed is defined by the expression:

$$F(x,z) = \frac{1}{G_{n_3}} x_2 \left( z_2 - \frac{1}{2} x_2 \right) + \frac{1}{G_{n_4}} x_3 \left( z_3 - \frac{1}{2} x_3 \right).$$

The diffusion coefficients and the scalar function F(x,z) depend on the intensity of noises. Being subject to a posteriori mathematical expectations  $m_i$ , (i = 1, 2, 3), variances  $D_i$ , (i = 1, 2, 3), and covariance moments  $D_{ip}$ , (i = 1, 2; p = 2, 3), the identification problem is reduced to

$$I_0 = y_4(T) \to \min \tag{24}$$

$$\begin{split} \dot{m}_{1} &= M[A_{1}] + M[X_{1}F] - m_{1}M[F], \\ \dot{m}_{2} &= M[A_{2}] + M[X_{2}F] - m_{2}M[F], \\ \dot{m}_{3} &= M[A_{3}] + M[X_{3}F] - m_{3}M[F], \\ \dot{D}_{1} &= 2M[(X_{1} - m_{1})A_{1}] + M[B_{11}] + \\ &+ M[(X_{1} - m_{1})^{2}F] - D_{1}M[F], \\ \dot{D}_{2} &= 2M[(X_{2} - m_{2})A_{2}] + M[B_{22}] + \\ &+ M[(X_{2} - m_{2})^{2}F] - D_{2}M[F], \\ \dot{D}_{3} &= 2M[(X_{3} - m_{3})A_{3}] + M[B_{33}] + \\ &+ M[(X_{3} - m_{3})^{2}F] - D_{3}M[F], \\ \dot{D}_{12} &= M[(X_{1} - m_{1})A_{2} + (X_{2} - m_{2})A_{1}] + \\ &+ M[(X_{1} - m_{1})(X_{2} - m_{2})F] - D_{12}M[F], \\ \dot{D}_{13} &= M[(X_{1} - m_{1})A_{3} + (X_{3} - m_{3})A_{1}] + \\ &+ M[(X_{1} - m_{1})(X_{3} - m_{3})F] - D_{13}M[F], \\ \dot{D}_{23} &= M[(X_{2} - m_{2})A_{3} + (X_{3} - m_{3})A_{2}] + \\ &+ M[(X_{2} - m_{2})(X_{3} - m_{3})F] - D_{23}M[F], \\ \dot{y}_{4} &= k_{2}((m_{2}^{2} + D_{2}) - 2m_{2}z_{2} + z_{2}^{2}) + \\ &+ k_{3}((m_{3}^{2} + D_{3}) - 2m_{3}z_{3} + z_{3}^{2}), \\ \\ \text{with initial conditions at time-point } t_{0} = 0 \end{split}$$

$$m_{1} = 1, m_{2} = 0, m_{3} = 0,$$
  

$$D_{1} = 0, D_{2} = 0, D_{3} = 0,$$
  

$$D_{12} = 0, D_{13} = 0, D_{23} = 0, y_{4}(0) = 0;$$
  
(26)

conjugation conditions at time-point  $t = t_1$ 

$$m_{1}(t_{1})_{-} = m_{1}(t_{1})_{+}, m_{2}(t_{1})_{-} = m_{2}(t_{1})_{+},$$
  

$$m_{3}(t_{1})_{-} = m_{3}(t_{1})_{+};$$
  

$$D_{1}(t_{1})_{-} = D_{1}(t_{1})_{+}, D_{2}(t_{1})_{-} = D_{2}(t_{1})_{+},$$
  

$$D_{3}(t_{1})_{-} = D_{3}(t_{1})_{+};$$
  

$$D_{3}(t_{1})_{-} = D_{3}(t_{1})_{+};$$

$$D_{12}(t_1)_{-} = D_{12}(t_1)_{+}, \quad D_{13}(t_1)_{-} = D_{13}(t_1)_{+},$$
$$D_{23}(t_1)_{-} = D_{23}(t_1)_{+}; \quad y_4(t_1)_{-} = y_4(t_1)_{+}$$

and limitations

$$I_1 = m_1(T) - 0, 4 = 0 ; (28)$$

$$\psi_1(u) = -u \le 0 ; \tag{29}$$

$$\psi_2(u) = u - 0, 04 \le 0 . \tag{30}$$

Identification problem (24) - (30) belongs to the known class of problems of the theory of optimum processes with limitations by equalities and inequalities (Bodner et al., 1987). According to the specified gradient procedure, the identification of parameters and the assessment of components of the states vector was carried out relative to the characteristics obtained over mass consumption  $u^{(0)} = u^{(0)}(t)$ , see Tab. 3.

Let  $t_1 = 10c$  - accelerator detachment time. The computing experiment was carried out relative to "measurements" of coordinates  $X_2 \ \mu \ X_3$ , which were calculated with relative intensity  $G_{n_1} = 0.54 \cdot 10^{-4}$ , parametric errors with intensity  $G_{n_2} = 0.24 \cdot 10^{-3}$  and the measuring errors with intensities  $G_{n_3} = 0.55 \cdot 10^{-4}$  and  $G_{n_4} = 0.56 \cdot 10^{-6}$ . To make the convergence-check of the identification gradient procedure, the initial values of disturbance intensities were assumed equal to:  $G_{n_1} = 0.56 \cdot 10^{-4}$ ,  $G_{n_2} = 0.26 \cdot 10^{-3}$ ,  $G_{n_3} = 0.57 \cdot 10^{-4}$ ,  $G_{n_4} = 0.58 \cdot 10^{-6}$ .

Assessments of the states vector components and the time point of the accelerator detachment  $t_1$  are given in Tab. 1, variances of components of the states vector are presented in Tab. 2, and identification of mass consumption per a unit of time  $u^{(n)}$  over n iterations (n = 1, 2, 3, 4) is given in Tab. 3.

Iteration number	$m_1(T)$	$m_2(T)$	$m_3(T)$	$t_1$
1	0,414	57.004	0.246	10.48
2	0,406	63.961	0.273	10.42
3	0,398	66.074	0.300	10.07
4	0,391	66.964	0.328	9.70

Table 1.

Iteration number	$D_1(T)$	$D_2(T)$	$D_3(T)$	$I_0$
1	0.005	6.528	0.001	9.960
2	0.005	8.897	0.002	3.039
3	0.005	11.138	0.002	0.926
4	0.005	11.288	0.002	0.036

Time	<i>u</i> <sup>(0)</sup>	Approximation of mass consumption				
t		<i>u</i> <sup>(1)</sup>	<i>u</i> <sup>(2)</sup>	<i>u</i> <sup>(3)</sup>	<i>u</i> <sup>(4)</sup>	
0	0.040	0.0405	0.0410	0.0415	0.0420	
5	0.040	0.0405	0.0400	0.0395	0.0390	
10	0.040	0.0155	0.0160	0.0165	0.0170	
20	0.015	0.0155	0.0160	0.0165	0.0170	
30	0.015	0.0155	0.0160	0.0165	0.0170	
40	0.015	0.0155	0.0160	0.0155	0.0169	
50	0.015	0.0005	0.0010	0.0015	0.0019	
60	0.000	0.0005	0.0001	0.0006	0.0011	
70	0.000	0.0005	0.0000	0.0005	0.0000	
80	0.000	0.0005	0.0000	0.0005	0.0000	
90	0.000	0.0005	0.0000	0.0005	0.0000	
100	0.000	0.0005	0.0000	0.0005	0.0000	

Obtained results for a flying vehicle and shown in the Tables, confirm the efficiency of suggested approach to identification of stochastic systems.

#### 6. CONCLUSION

The analyzed approach to identify nonlinear stochastic systems enables to identify control functions, constructive parameters and energy parameters for rather broad range of expected operating conditions of flying vehicles and their subsystems; taking into consideration parametric and additive disturbances as well as limitations describing various requirements for a flying vehicle and its subsystems.

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