

## Synchronized hysteresis regulator with reference adaptation and hysteresis correction

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**Abstract:** In this paper are presented the results of a study of the dynamic modes and bifurcations in a hysteresis regulator with clocked commutation. Regulator dynamics is examined in case of reference adaptation use and differential correction use. The behavior of the hysteresis regulator with clocked commutation is investigated in the dc/dc and dc/ac mode.

**Keywords:** Dynamics, stability analysis, hysteresis, current regulators, clocked commutation.

### INTRODUCTION

Pulse-relay regulators (PRR) have the best dynamic behavior among all pulse regulators. A fast response, simplicity and constant switching frequency ( $f_{sw}$ ) are advantages of these regulators. The operation principle of the simple PRR with one-stand relay border is shown in Fig. 1.

Such kinds of PRR with the one-stand relay border have two important disadvantages among the evidential advantages. The principle instability in the whole range of a duty cycle variation is the first one (PRR with the top-stand relay border and with the low-stand relay border are unstable in the range of duty cycle ( $d$ ) from 0,5 to 1 and from 0,5 to 0 respectively) (Kolokolov, 1975). The static control error is the second one (Fig. 1).

At the end of 80-th some scientists almost simultaneously proposed the structure of the two-stand relay current regulator that is synchronized with a double switching sequences - so-called the hysteresis controlled regulator with clocked commutation (HRCC) (Kolokolov 1988, Anunciada and Silva 1989). It implements automatic changing from the algorithm with the top-stand relay border to the algorithm with the low-stand relay border and guaranties the stability of the synchronized periodic mode within the whole range of duty cycle variation in case of the correct hysteresis value ( $H$ ) without supplementary device using (Fig. 2). It has been using since 1995 in the dc-motor current control loop of the high-speed trains ER-200.

The disadvantage of the all enumerated PRR is a static control error ( $\varepsilon_{st}$ ) Fig. 1. Algorithms of hysteresis adaptation for the hysteresis regulators have been proposed since middle of 90-th (Malesani et al., 1996; Malesani et al., 2000; Tae-Won 1996; Bode and Holmes, 2001). Hysteresis adaptation

algorithm essence consists in the continuous real time hysteresis recalculation with respect to the current reference. Load parameters, desirable  $f_{sw}$ , voltage of dc source, commutation moment, current derivative against to the algorithm are the hysteresis calculation initial data (Malesani et al., 1996; Malesani et al., 2000; Tae-Won 1996; Bode and Holmes, 2001). It is evident that regulator becomes much more complex and naturally loses a fast response. HRCC with the reference current adaptation (HRCCRA) was proposed in (Kolokolov and Koschinsky, 2004). It avoids  $\varepsilon_{st}$  and saves simplicity and high quick-action of HRCC. Its principle of operation consists in changing of the reference current in order to eliminate  $\varepsilon_{st}$  (Fig. 2). The main property of this regulator is a simplicity – regulator can be realized by simple analog and logic elements.

HRCC dynamics was completely investigated in (Kolokolov and Koschinsky, 2004). Periodic processes (PP) with period  $T_s$  lose stability, if  $H$  is less than the current pulsations maximum value ( $\Delta x_{1max}$ ) only. In this case, C-bifurcation appears and PP, which have the switching period less than  $T_s$ , arise. Reference current adaptation (RCA) reduces  $\varepsilon_{st}$  to the zero value. At the same time, it provides overshoot increasing. The RCA time constant ( $\tau$ ) must be reduced to enhance the regulator dynamic performance. The time constant decrease may result in the regulator stability loss (Kolokolov and Koschinsky, 2006). Also, the combination of the  $\tau$  and  $H$  values influences on  $\varepsilon_{st}$  magnitude in the dc/ac conversion system.

The purpose of the paper consists in perform an analysis of the mechanisms and reasons of stability and regulation quality loss in HRCCRA and HRCCRA with deferential correction in the dc/dc and dc/ac energy conversion system. The numerical investigations are proved experimentally.

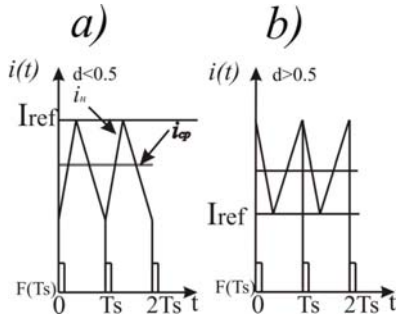


Figure 1 – Time diagrams of PRR with a) “top-stand” relay border b) “low-stand” relay border

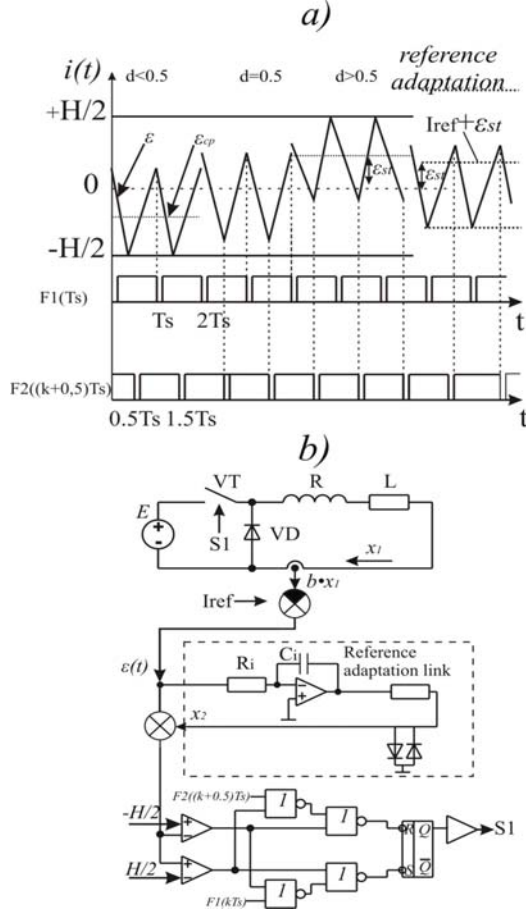


Figure 2 – Time diagrams a) HRCRA b) Function scheme of HRCRA

## 1. DYNAMICS OF HYSTERESIS REGULATOR WITH REFERENCE ADAPTATION

The equivalent circuit of the considered hysteresis regulator is shown in the Fig. 2 (b). The mathematical model of such system is described by:

$$\begin{cases} \frac{dx_1}{dt} = -\frac{x_1}{\mu} + B(\xi(t)) \\ \frac{dx_2}{dt} = \frac{1}{\tau} \cdot (I_{ref} - b \cdot x_1) \end{cases}, \quad (1)$$

where  $x_1$  - a load current;  $x_2$  - an output value of the reference adaptation link;  $I_{ref}$  - a reference current;

$\xi(t)$  - a HRCRA commutation function;  $\mu=L/R$  - a load time constant;  $B(1)=E/L$ ,  $B(0)=0$ ;  $\tau=R_i \cdot C_i$  - a RCA time constant;  $R=2.8$  Ohm - a load resistance;  $L=0.0025$  H - a load inductance;  $E=10$  V - a voltage source;  $b=0.05$  - a current sensor transfer ratio. The commutation function states are described by the system:

$$\xi(t) = \begin{cases} 1, \text{if } I_{ref} - b \cdot x_1 + x_2 > H/2 \\ 0, \text{if } I_{ref} - b \cdot x_1 + x_2 < -H/2 \\ 0 \rightarrow 1, \text{if } t = k \cdot T_s \\ \text{or } I_{ref} - b \cdot x_1 + x_2 = H/2 \\ 1 \rightarrow 0, \text{if } t = (k+0.5) \cdot T_s \\ \text{or } I_{ref} - b \cdot x_1 + x_2 = -H/2 \end{cases}, \quad (2)$$

where  $H$  - a regulator hysteresis,  $k=1,2,3 \dots$

The system (1) partial solution can be obtained by the individual solution cross-linking on the all intervals of the constancy (2). The cross-linking surfaces (CS) are surfaces of relay commutation (two first equations of (2)) and stroboscopic surfaces  $t=k \cdot T_s$ ,  $t=(k+0.5) \cdot T_s$ ,  $k=1,2,3 \dots$  (Kolokolov and Koschinsky, 2004). Let's numerate CS as:  $H/2 \rightarrow 1$ ,  $-H/2 \rightarrow 3$ ,  $t=k \cdot T_s \rightarrow 2$ ,  $t=(k+0.5) \cdot T_s \rightarrow 4$ . In this case, PP can be marked by the series of CS numbers, which indicate CS crossing by the phase trajectory of the system ( $P_{2342}$  is shown on Fig. 1 (a),  $P_{2412}$  - on Fig 1 (b)). PP, which have period  $T_s$  and have two different states of  $\xi(t)$  at this period, are “desirable” ones in HRCRA. In this case,  $f_{sw}$  is equal to the synchronization frequency and the current pulsations have a minimum value.  $P_{2342}$ ,  $P_{2412}$  and “degenerative”  $P_{242}$  satisfy just listed conditions.

The local stability of the system (1) solution is determined by the variation equation, if  $H \geq \Delta x_{1max}$ :

$$\varepsilon_{K+1} = J \cdot \varepsilon_K, \quad (3)$$

where  $\varepsilon_K$  is a deviation of state vector  $X_K$  from the fixed point  $X_C$ ;  $J$  is a Jacobian of the periodic solution (3).

The state vector  $X=\{x_1; x_2\}$  of the system (1) for  $P_{2342}$  is determined by means of the trajectory 23, 34, 42 solution sewing and leads to:

$$X_k = \begin{cases} C0 \cdot x_{1,k-1} - \mu \cdot \left( \frac{B(1) \cdot (C0 - C2) +}{+B(0) \cdot (C2 - 1)} \right) \\ x_{2,k-1} + \frac{b \cdot \mu}{\tau} \cdot x_{1,k-1} \cdot (C0 - 1) + \\ + \frac{T_s \cdot I_{ref}}{\tau} - \frac{b \cdot \mu^2}{\tau} \cdot (B(1) \cdot (C0 - C2 + \frac{t_k}{\mu}) + \\ + B(0) \cdot (C2 - 1 + \frac{(T_s - t_k)}{\mu})) \end{cases}, \quad (4)$$

where  $C0=e^{-T_s/\mu}$ ,  $C1=e^{-tk/\mu}$ ,  $C2=e^{-(T_s-tk)/\mu}$ ,  $tk$  - smallest root of the equation:

$$|I_{ref} - b \cdot x_1 + x_2| = H/2. \quad (5)$$

The fixed point coordinates of  $P_{2342}$  can be evaluated from (3) and (4) as:

$$x_{1c} = \left[ \begin{array}{l} \mu \cdot (B(1) \cdot ((C2 - C0) \cdot \\ \cdot (1 - \frac{b \cdot \mu}{\tau}) - \frac{b}{\tau} \cdot t_k) + \\ + B(0) \cdot ((1 - C2) \cdot (1 - \frac{b \cdot \mu}{\tau}) - \\ - \frac{b}{\tau} \cdot (T_s - t_k))) - \frac{T_s \cdot I_{ref}}{\tau} \end{array} \right] \cdot \left[ (C0 - 1) \cdot (1 - \frac{b \cdot \mu}{\tau}) \right]^{-1}; \quad (6)$$

$$x_{2c} = b \cdot X_{1c} \cdot (C1 - \frac{\mu}{\tau} \cdot (C1 - 1)) + \\ + b \cdot \mu \cdot B(1) \cdot (\frac{t_k}{\tau} - (C1 - 1) \cdot (1 - \frac{\mu}{\tau})) - \\ - I_{ref} \cdot (\frac{t_k}{\tau} + 1) + \frac{H}{2}. \quad (7)$$

Jacobian for  $P_{2342}$  is equal:

$$J|_{X=X_c} = \frac{\partial F}{\partial X} + \frac{\partial F}{\partial t} \cdot \left( -\frac{\partial \xi}{\partial t} \right)^{-1} \cdot \frac{\partial \xi}{\partial X}, \quad (8)$$

where  $\frac{\partial F}{\partial X} = \begin{pmatrix} C0 & 0 \\ \frac{b \cdot \mu}{\tau} (C0 - 1) & 1 \end{pmatrix};$

$$\frac{\partial F}{\partial t} = \begin{pmatrix} C2 \cdot (B(1) - B(0)) \\ \frac{b \cdot \mu}{\tau} (C2 - 1) \cdot (B(0) - B(1)) \end{pmatrix};$$

$$\frac{\partial \xi}{\partial t} = \left( \frac{I_{ref}}{\tau} - X_c \cdot C1 \cdot (T + \frac{b}{\tau}) - B(1) \cdot (C1 - \frac{b \cdot \mu}{\tau} \cdot (C1 - 1)) \right);$$

$$\frac{\partial \xi}{\partial X} = \left( -C1 + \frac{b \cdot \mu}{\tau} \cdot (C1 - 1) \quad 1 \right).$$

By the same way was obtained the solutions for the periodical processes  $P_{2412}$  and  $P_{242}$ .

Let's make an analysis of HRCCRA typical evolution scenario. Periodical processes evolution will be considered within the quasistatic change of the reference current. HRCCRA "desirable" situation is an existence of the "desirable" PP at  $0 < d < 1$ , i.e.  $P_{2342}$  is realized at the interval  $0 < d < 0.5$ ,  $P_{242}$  at the point  $d = 0.5$  and  $P_{2342}$  - at the interval  $0.5 < d < 1$ . The periodical processes with the period different from  $T_s$  appears if  $\tau$  is less then some critical value  $\tau_{crit}$  (Fig 3 (c)). The monodromy matrix eigen value module ( $|\rho|$ ) of  $P_{2342}$  crosses unit level in the point  $d_1$  (Fig. 4 (b)). It means, that PP losses stability and a bifurcation situation appears. Complex PP  $P_{234242}$  arises at this point (Fig. 3(c)). It realizes one by one the algorithms with the "top-stand" relay border and the "degenerative"  $P_{242}$ .  $P_{234242}$  appearance leads to the load current pulsation increase, however  $f_{sw}$  stays equal to the synchronization frequency. The C-bifurcation arises in the point  $d_2$  and  $P_{241242}$  appears. This process has resembling to  $P_{234242}$  algorithm, which realizes in turns  $P_{2412}$  and  $P_{242}$ . "Desirable"  $P_{2412}$  arises in the point  $d_3$  after the C-bifurcation. It is stable at  $d_3 < d < 1$ . The  $\tau_{crit}$  value is equal  $4.55 \cdot T_s$  for system (1) parameters and does not depend on the hysteresis value, if  $H > \Delta x_{1max}$ .

The complex PP, which have  $f_{sw}$  bigger than the synchronization frequency, appear in HRCCRA, if  $H < \Delta x_{1max}$ .  $P_{234132}$  arises, if  $d < 0.5$ , the  $P_{234132}$  arises, if  $d > 0.5$  (Fig 3 (d)). The amount of the switchings increases to 4 during  $T_s$  when these PP appear. It is equivalent to two times increase of  $f_{sw}$  with respect to the synchronization frequency. PP with the 6 switching during  $T_s$  arise if  $\Delta x_{1max}/H > 1.5$  ( $P_{2314132}$  arises if  $d < 0.5$ ,  $P_{2314132}$  - if  $d > 0.5$ ). It means, free times  $f_{sw}$  increase happen in against of the synchronization frequency.

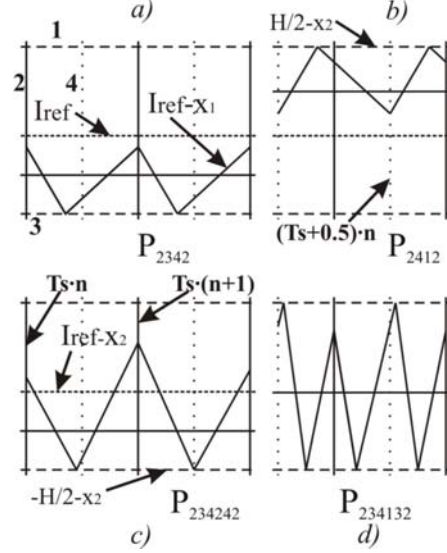


Figure 3 – Time diagrams of HRCCRA periodic process

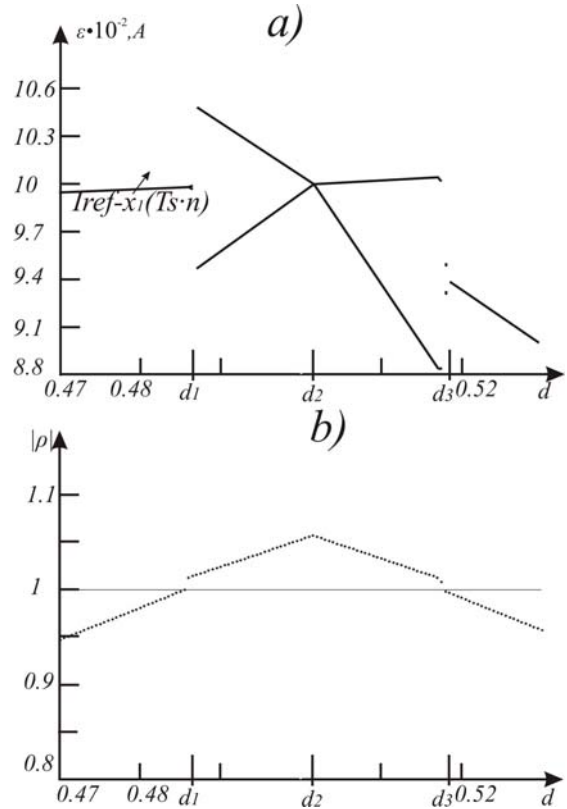


Figure 4 – a) Bifurcation diagram of HRCCRA b) multiplier evolution diagram for  $m=1$ ,  $\tau=3 \cdot T_s$

## 2. DYNAMICS OF HYSTERESIS REGULATOR WITH DIFFERENTIAL CORRECTION LINK

The local  $\varepsilon_{st}$  increase arises (Fig 5 (a),  $(t_1; t_2)$ ) in case of the variable reference use (Kolokolov and Koschinsky, 2006). It is a consequence of “degenerative”  $P_{242}$ , when the  $x_2$  polarity changing process happens. The nature of this PP is that the switchings occur by means of synchronization pulses  $F_1$  and  $F_2$  only (Fig 5 (a)).

The  $\varepsilon_{st}$  value of HRCC is a non-positive value if  $d < 0.5$  and is non-negative if  $d > 0.5$  (Kolokolov and Koschinsky, 2004). Let's assume, that  $\varepsilon_{st} = 0$ , then  $x_2$  can be evaluated as:

$$|x_2| = \frac{H - b \cdot \Delta x_1(d)}{2}, \quad (9)$$

where  $\Delta x_1(d)$  - load current pulsations.

“Degenerative”  $P_{242}$  exists until  $|I_{ref} - x_1 + x_2| < H/2$  (Fig. 5 (a)). Let's assume that  $\varepsilon_{st} = 0$  in the point  $t_1$ , than duration of  $P_{242}$  ( $t_{trans}$ ) can be evaluated with respect to (9):

$$\left| I \cdot \sin(\omega \cdot t_{trans}) - \frac{b}{\tau} I \cdot \cos(\omega \cdot t_{trans}) - \frac{H - b \cdot \Delta x_1(0.5)}{2} \right| = \frac{H}{2}, \quad (10)$$

where  $I$  - reference current amplitude;  $\omega$  - angular frequency.

The dependence of current pulsation on  $d$  is determined (Kolokolov and Koschinsky, 2004):

$$\Delta x_1 = \frac{Vg \cdot T_s}{L} d \cdot (1 - d). \quad (11)$$

It is evident, that  $t_{trans}$  is equal to 0, if the current pulsation is equal to the hysteresis value (10) (Fig. 6). The additional differential correction link is used to decrease  $t_{trans}$  and consequently area of  $\varepsilon_{st}$  increase. The effect of the differential link inclusion can be interpreted as decreasing of the hysteresis for the differential correction value (Fig.5 (b)). The  $t_{trans}$  and  $\varepsilon_{st}$  values are equal to 0 at  $d = 0.5$ , if  $x_1$  is equal to the corrected hysteresis value.

In case of the differential correction, systems (1,2) have form:

$$\begin{cases} \frac{dx_1}{dt} = -\frac{1}{\mu} \cdot x_1 + B(\xi(t)) \\ \frac{dx_2}{dt} = \frac{1}{\tau} \cdot (I_{REF} - b \cdot x_1) \\ \frac{dx_3}{dt} = k_d \cdot \frac{d(I_{REF} - b \cdot x_1)}{dt^2}, \end{cases} \quad (12)$$

where  $k_d$  - a differential link time constant. Commutation function values are determined by:

$$\xi(t) = \begin{cases} 1, \text{ if } I_{ref} - b \cdot x_1 + x_2 + x_3 > H/2 \\ 0, \text{ if } I_{ref} - b \cdot x_1 + x_2 + x_3 < -H/2 \\ 0 \rightarrow 1, \text{ if } t = k \cdot T_s \\ \text{or } I_{ref} - b \cdot x_1 + x_2 + x_3 = H/2 \\ 1 \rightarrow 0, \text{ if } t = (k + 0.5) \cdot T_s \\ \text{or } I_{ref} - b \cdot x_1 + x_2 + x_3 = -H/2. \end{cases} \quad (13)$$

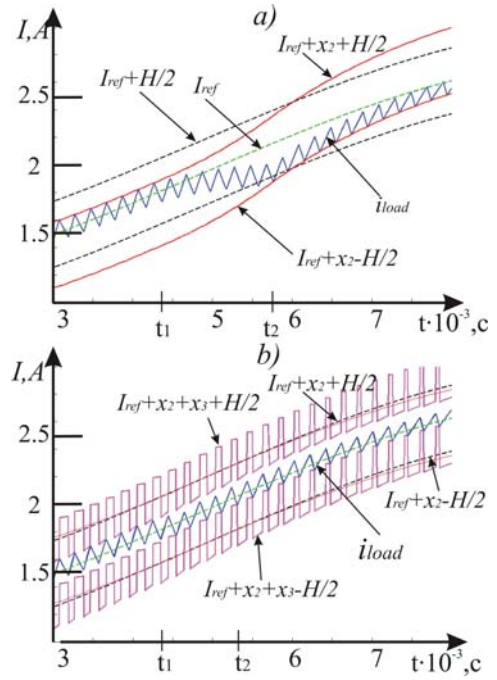


Figure 5 – Time diagrams of a) HRCCRA b) HRCCRA with differential correction,  $\tau = 1/(5T_s)$

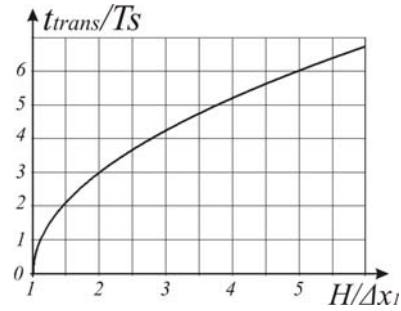


Figure 6 – Plot of duration of  $P_{242}$  existence vs. hysteresis value,  $\tau_1 = 5 \cdot T_s$ ,  $\omega = 314$  radian/s.

The  $\tau_{crit}$  value increases if differential correction is used (Fig. 7,  $\tau_{crit} = 5 \cdot T_s$ ). Also evolution of the regulator dynamics differs in case of  $\tau \leq \tau_{crit}$ . The  $|p|$  value of the  $P_{2342}$  m-cycle crosses unit line at the point  $d_1$  and doubling period bifurcation appears. C-bifurcation situations arise in the points  $d_2, d_3, d_4$ , at which processes  $P_{234242}, P_{241242}, P_{2342}$  appear. The “desirable”  $P_{2412}$  is stable at  $d > d_5$ . Stability loss of HRCCRA with the differential correction leads to current pulsation increase, however  $f_{sw}$  stays equal to the synchronization frequency similar to HRCCRA.

The complex periodical processes  $P_{234242}$  and  $P_{241242}$  arise, if the corrected hysteresis value is less than  $\Delta x_{1max}$ . Main feature of these PP is a double  $f_{sw}$  against the synchronization frequency. Let's assume that the reference current is a permanent during  $T_s$ , then the maximum value of the differential link time constant, with respect to (12,13), is described by equation:

$$k_{d\max} = \frac{\mu(H - b \cdot \Delta x_{1\max})}{2 \cdot e^{(-T_s/(2 \cdot \mu))} \cdot \left( \frac{b \cdot \Delta x_{1\max}}{2} + \mu \cdot B(1) \right)} \quad (14)$$

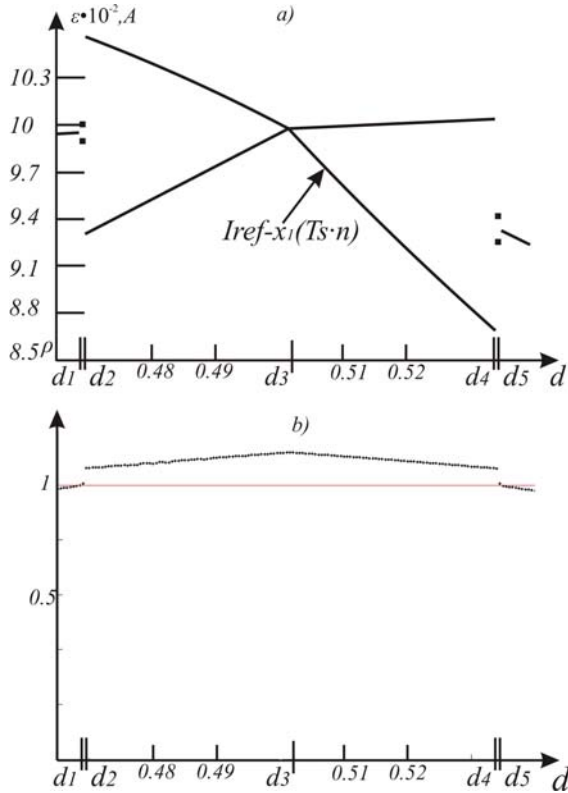


Figure 7 – a) Bifurcation diagram of HRCCRA with differential correction b) multiplier evolution diagram for  $m=1$ ,  $\tau=3 \cdot T_s$

### 3. EXPERIMENTAL RESEARCHES OF HYSTERESIS REGULATORS

The experimental bifurcation diagrams, which corresponds to the diagrams (Fig. 4,7) of HRCCRA and HRCCRA with differential correction are shown in the Fig. 8. The experimental time diagrams of just listed regulators in case of sine wave reference current use are shown in the Fig. 9. Disadvantage of the differential correction is small noise immunity, which can provide deterioration of the regulation quality and switching frequency increase if the differential link time constant is close to  $\tau_{d\max}$ .

### CONCLUSION

The dynamics of the astatic synchronized hysteresis regulator was investigated in this paper. The main advantage of this regulator is simplicity, that it can be realized by means of the ordinary logic and analog elements. At the same time, reference adaptation use leads to HRCC dynamics change, which was examined in detail in the paper (Kolokolov and Koschinsky, 2004). Regulator investigation was shown, that “desirable” periodical processes lose stability if RCA link time constant is less than critical value. In this case current pulsations increase.

Stability of HRCCRA is guaranteed if hysteresis value is bigger than maximum current pulsations. Otherwise, two times switching frequency increase arises, if  $P_{234132}$  and  $P_{234132}$  arise and increases more if  $\Delta x_{1\max}/H > 1.5$ .

The situation when a hysteresis value is less than maximum current pulsation is another way of “undesirable” periodical processes appearance. In this case switching frequency increases in two times if  $P_{234132}$  and  $P_{234132}$  arise. The regulator dynamics research was made for dc/dc and dc/ac energy conversion systems. The local increase of static control error is a disadvantage of HRCCRA in case of variable reference use. Existence of the natural RCA link polarity change transient process is a reason of it. Differential correction link use was proposed to avoid the regulator disadvantage in dc/ac application. It decreases the transient process duration and, consequently, static control error in this area. On the other hand, differential correction link use leads to increase of the RCA link time constant and reduces the regulator noise immunity.

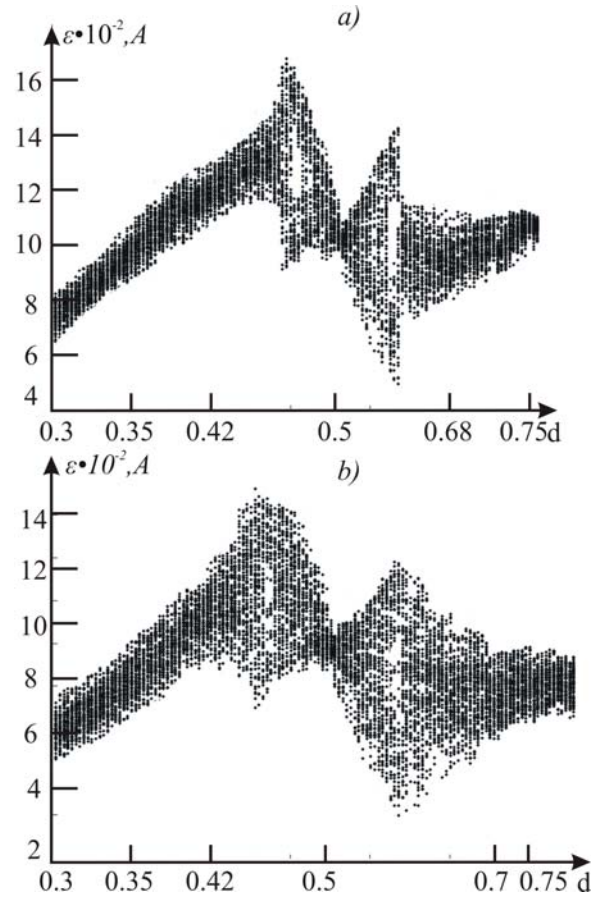


Figure 8 – Experimental bifurcation diagram of the a) HRCCRA b) HRCCRA with differential correction,  $\tau=2 \cdot T_s$ ,  $k_d=0.3 \cdot T_s$



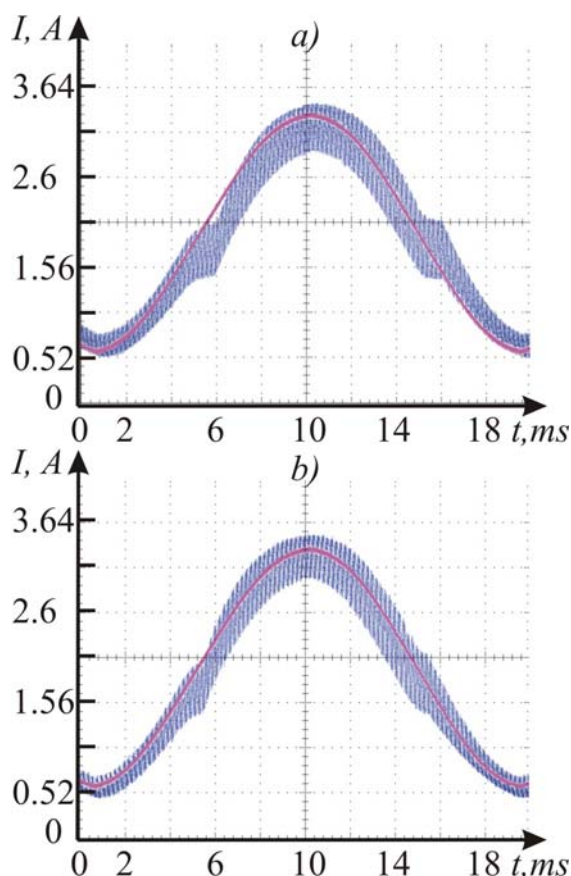


Figure 9 - Time diagrams of a) HRCRA b) HRCRA with differential correction,  $\tau=6 \cdot T_s$ ,  $k_d=0.3 \cdot T_s$

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