Control of UAV Formation Over a Finite Data Rate Packet Erasure Channel with Adaptive Coder

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***Abstract*—The data transmission techniques minimizing information flow for tracking the moving object over the limited capacity packet erasure communication channel of finite data rate with adaptive coding/decoding procedure is studied by the example of tracking the maneuvering UAV. Dependence of tracking accuracy on the probability of erasure p is numerically found based on the simulations.**

***Index Terms*—data transmission, communication channel, adaptive coding**

I. Introduction

At the present time, the distributed control of complex industrial, transportation, power and agricultural sytems becomes increasingly important. The relevance of the intensive research in designing multi-agent systems with decentralized control over the communication network is widely demonstrated in scientific and technical literature, see [1]–[10] for mentioning a few. The *Networked Control System* is a real-time system in which the sensor data and control signals are transmitted over the common or switched communication network. Design of such a kind of systems requires a simultaneous consideration of control, computation and information aspects. The networked control includes the cooperative control of a group of moving agents, such as transport robots, aircrafts, water vehicles, etc. Due to the digital nature of the communication channel, each signal, transmitted over the digital network, is represented by the symbol of a finite set [1]. Thus, the finiteness of the data set should be explicitly taken into account when the networked cooperative flight control systems are developed.

During the recent years the limitations in estimation and control, imposed by boundedness of the communication channel capacity, were widely studied in control theoretic literature, see [11]–[14] and references therein. In particular, it has been shown that stabilization of linear systems with limited data rate over a communication channel is available, if and only if the channel capacity exceeds the rate of entropy production in the system vicinity of the equilibrium – the so-called '*data rate theorem*' [11], [15].

In several engineering applications (for example, in distributed sensor networks or remote monitoring systems), there is no possibility to mount complicated devices for measuring or estimation at the transmitter side and only the scalar output may be transmitted over the communication channel. For nonlinear systems such a task is considered in [16], where the results on the observer-based synchronization are obtained and optimality of binary encoding using one-step memory coder is proven. In [17]–[20] a scheme for minimization of the channel load by means of encoding and transmission only the *update signal*, produced by an observer on the side of the transmitter and using an adaptive tuning procedure is demonstrated. In [21] application of the adaptive coding procedure for maneuvering UAV tracking is given, and dependence of the estimation accuracy on data transmission rate R is numerically found, fitting well the general theoretical statements.

In the mentioned works [11], [15]–[18], [21] the communication channel is assumed of limited capacity otherwise ideal. The cases of packet erasure channel and 'blinking' channel are widely appear in different real-world applications and are intensively studied in Information theory, Computer and Physical sciences and Control theoretic literature, see, e.g. [4], [22]–[31].

In the present paper the results of [21] are used to the flight formation control with the information exchange based on the proposed algorithm and are extended to studying accuracy of state estimation over the finite data rate packet erasure channel.

The rest of the paper is organized as follows. The adaptive coding procedure of [17], [19], [21] is outlined in Section II. Method for coding the UAV coordinates of [21] is briefly represented in Section III. An example of formation control for three UAVs, following in string with data exchange over the limited capacity communication network is presented in Section IV. Case of the data erasure channel is considered in Section V. Concluding remarks and the future work intentions are given in Section VI.

II. Coding procedure

Let $\sigma[k]$ be a scalar signal to be transmitted over the digital communication channel at discrete instants $t_k = kT$, where $k = 0, 1, \ldots$ is a sequence of natural numbers, $T > 0$ is the *sampling interval*. Introduce the following binary static quantizer

$$q(\sigma, M) = M\,\mathrm{sign}(\sigma), \qquad (1)$$

where $\text{sign}(\cdot)$ is the *signum* function: $\text{sign}(\sigma) = 1$, if $\sigma \geq 0$, $\text{sign}(\sigma) = -1$, if $\sigma < 0$. Parameter M is referred to as a *quantizer range*. The output signal of the quantizer

$$\bar{\sigma}[k] = q(\sigma[k], M[k]) \tag{2}$$

is transmitted over the communication channel to the decoder.

Range M is updated with time by the following event-based *adaptive zooming* strategy:

$$\begin{aligned} &\lambda[k] = (\bar{\sigma}[k] + \bar{\sigma}[k-1])/2, \\ &M[k+1] = m + \begin{cases} \rho M[k], & \text{as } |\lambda[k]| \leq 0.5, \\ M[k]/\rho, & \text{otherwise,} \end{cases} \\ &\lambda[0] = 0, \quad M[0] = M_0, \end{aligned} \tag{3}$$

where M_0 stands for the chosen initial value of $M[k]$.

The full-order coding/decoding procedure employs the embedded observer. Let us use the following drive process model:

$$\dot{x}(t) = Ax(t) + B\varphi(y), \; y(t) = Cx(t), \quad x(0) = x_0 \tag{4}$$

where $x(t) \in \mathbb{R}^n$ is the process state space vector; $y(t)$ is the scalar measured signal; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$ are real matrices; $\varphi(y)$ a Lipschitz continuous function along all the trajectories of the drive system (4).

The *quantized observation error* $\bar{\sigma}[k]$ is defined as a deviation between measured signal $y(t)$ and its estimate $\hat{y}(t)$ quantized with given $M[k]$ as follows:

$$\bar{\sigma}[k] = q\big(y(t_k) - \hat{y}(t_k), M[k]\big), \quad t_k = kT. \tag{5}$$

where the estimate $\hat{y}(t)$ is generated by the observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\varphi\big(\hat{y}(t)\big) + L\bar{\sigma}(t), \; \hat{y}(t) = C\hat{x}(t), \tag{6}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state estimation vector; $\hat{y}(t)$ is the drive process estimate; L is $(n\times 1)$-matrix (the column vector) of the observer parameters; continuous-time observation error $\bar{\sigma}(t)$ is found as an expansion of $\bar{\sigma}[k]$ over the sampling interval. In the case of the *zero-order extrapolation*, $\bar{\sigma}(t)$ has a form $\bar{\sigma}(t) = \bar{\sigma}[k]$ as $t_k \leq t < t_{k+1}$.

III. METHOD OF CODING THE UAVS COORDINATES

The leader-follower flight formation configuration of the group of identical UAVs is considered. Several UAVs should follow the leader, keeping the specified relative position.

The leading UAV coordinates are measured in the normal Earth's coordinate frame $X_gY_gZ_g$ and are transmitted over a communication channel to the follower.

The measurements of the leading UAV at instants kT ($k = 0, 1, \ldots$) are transformed by means of algorithm (3), (5), (6) to be used by the autopilot of the following UAV for maintaining the prescribed flight formation. It is assumed that the leader performs motion with constant velocity in each coordinate. Under this assumption the 'generalized' coordinate $x(t)$ (which may be reffered to as the UAV center of gravity coordinates x, y, z) is governed by the equation $\ddot{x}(t) = 0$ or a difference equation with respect to output $x(t_k)$ of the following state-space form:

$$z[k+1] = Az[k], \quad x[k] = Cz[k], \quad k = 0, 1, \ldots, \tag{7}$$

where $z[k] \in \mathbb{R}^2$ is a state space vector, $A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $C = [1, 0]$.

Discrete-time representation of observer (6) for system (7) reads as

$$\begin{cases} \hat{x}[k+1] = \hat{x}[k] + T\hat{V}[k] + l_1\bar{\sigma}[k], \\ \hat{V}[k+1] = \hat{V}[k] + l_2\bar{\sigma}[k], \end{cases} \tag{8}$$

where $\bar{\sigma}[k]$ is defined by (5) as $\bar{\sigma}[k] = q\big(x(t_k) - \hat{x}(t_k), M[k]\big)$ is the observation error signal with respect to coordinate x; $\hat{V}[k]$ is the the estimate of velocity V along OX axis at instant $t_k = kT$, $k = 0, 1, \ldots$; l_1, l_2 are the components of the observer gain vector L.

Finally, the coding algorithm with the binary adaptive coder and the observer is described by (3), (5), (6), (8) for each UAV's coordinate x, y, z in the normal Earth axis frame.

IV. EXAMPLE OF UAV NETWORKED FORMATION CONTROL

A. Formation structure

Let three UAVs should follow in a string, one after another, keeping the prescribed relative distance d_x between the neighbouring UAVs along axis OX of the Earth reference frame. Let UAV # 1 be a *leader*, its altitude be $h_1(t)$ and lateral position be $z_1(t)$. Coordinates $h_1(t)$, $z_1(t)$ should be maintained by the followers – the UAVs # 2, 3 with the given constant 'shift' for collision avoidance. The information flow graphs of the formation may be various, which impacts on the overall formation dynamics, cf. [32]. At the present paper the string network structure is taken under the assumption that UAV # 2 receives position data from UAV # 1, and UAV # 3 receives data from UAV # 2.

B. Algorithms for control of following UAVs

For control of altitude h_i and lateral position z_i of follower # i the reference signals $h_i^*(t)$, $z_i^*(t)$ are used. $h_i^*(t)$ and $z_i^*(t)$ are the estimates of the current position $\hat{h}_{i-1}(t_k)$, $\hat{z}_{i-1}(t_k)$ ($t_k = kT_s$, $k = 0, 1, \ldots$) of the leader (UAV # $i-1$), which are transferred over the communication channel. It shoud be mentioned that the motion along the longitudinal axis X_g differs from vertical and lateral motions due to the high longitudinal speed V_k (we assume that the Earth reference frame $X_gY_gZ_g$ is chosen in the way that plane X_kY_k of the trajectory coordinate frame is close to plane X_gY_g). Then, for control of the follower in the longitudinal direction, the reference signal in the speed control loop V_x^* is governed by the following PD-control law:

$$\begin{aligned} &e_x(t) = \hat{x}_{i-1}(t) - x_i(t) - d_x, \\ &V_{x,i}^*(t) = \hat{V}_{x,i-1}(t) + k_x e_x(t) + k_{dx}\dot{e}_x(t). \end{aligned} \tag{9}$$

where d_x is the prescribed distance between following (ith) UAV from leading one ($(i-1)$th).

C. Simulation results

Consider the following numerical example. Let the prescribed distance be as $d_x = 30$ m, the UAV cruise speed be as $V_k = 250$ m/s, cruise altitude h be as 6.0 km. The gains in (9) are taken as $k_x = 0.3$ 1/s, $k_x = 6$.

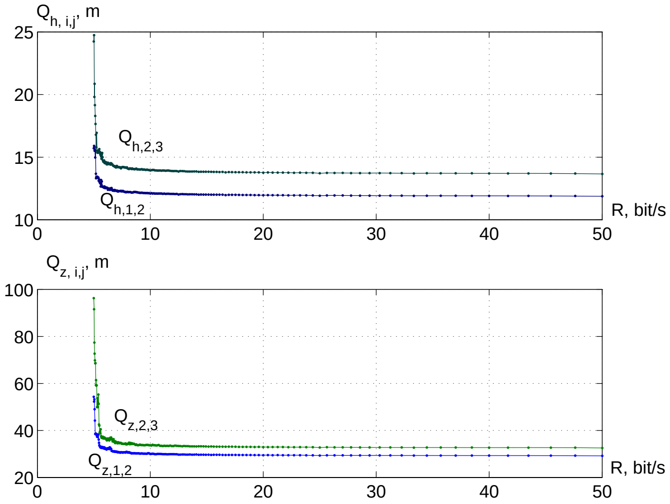

Fig. 1. Magnitudes of tracking errors on altitude h and lateral coordinate z vs transmission rate R.

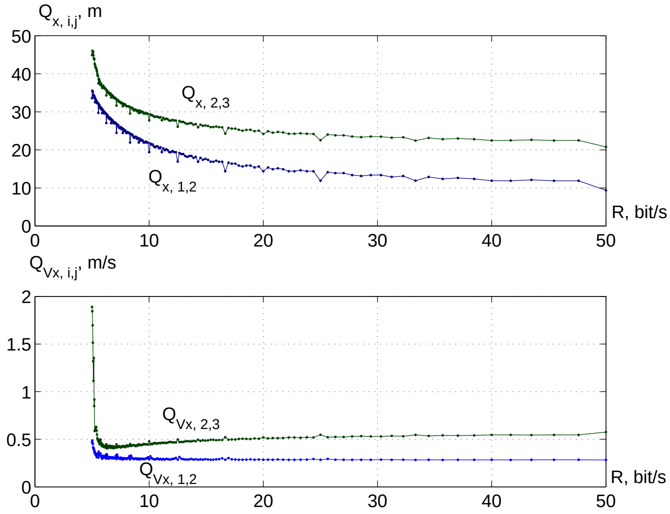

Fig. 2. Magnitudes of tracking errors on longitude coordinate x and speed V_x vs transmission rate R.

Assume that the leader tracks the harmonic reference signal with lateral magnitude as 200 m, and period 100 s. The altitude reference signal has a magnitude of 100 m and a period of 150 s. Let us find numerically an accuracy of tracking the followers by a leader under the condition of data quantization.

Accuracy estimates are shown in Figs. 1, 2, where the magnitudes of tracking errors vs data transmission rate R for vertical h, lateral z coordinates, longitudinal position x and speed V_x are plotted. As is seen from the plots, if the data transmission rate is sufficiently high (exceeds 30 bit/s), the UAV control loop dynamics introduce a dominating contribution to the tracking error, rather than data quantization. Based on the results obtained, the data transmission rate as $R^* = 25$ bit/s for each channel may be considered as an acceptable one for the considered example. It is worth mentioning that if R is lower some threshold value (5 bit/s in our example) then the estimation error drastically increases, which fits well the existing theoretical statements, cf. [33]–[38].

The similar results for coder with observer of the third order, based on model (7) with matrices

$$A = \begin{bmatrix} 1 & T_s & T_s^2/2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix}, \quad C = [1, 0, 0]$$

are plotted in Figs. 3–4, showing the better accuracy in the case of the higher order coder.

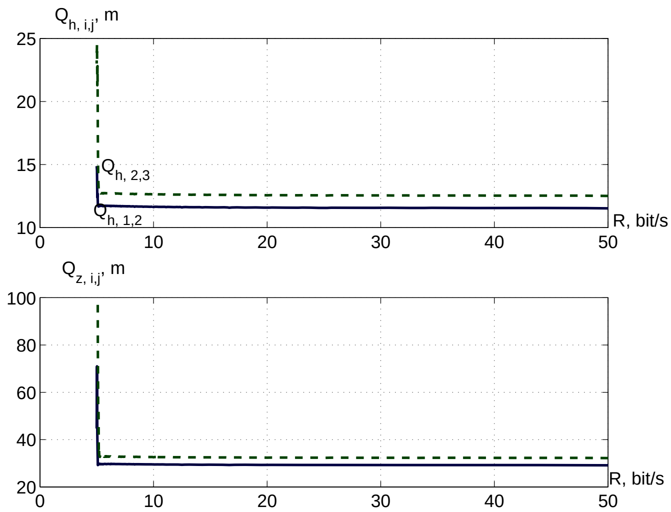

Fig. 3. Magnitudes of tracking errors on altitude h and lateral coordinate z vs transmission rate R. The 3rd order coder.

V. Case of the Data Erasure Channel

A. Erasure channel description

By analogy with [31] we assume that output measurement is encoded by an encoder and transmitted to a decoder through packet erasure channel with erasure probability p. Assume that the decoder will feed back to the encoder an acknowledgment whether the packet is erased or not. Therefore the encoder knows what information has been delivered to the decoder (i.e. the so-called *equi-memory condition* [39] is fulfilled). Denote the acknowledgment signal at time k which is sent by the decoder and received by the encoder by $\sigma[k] \in \{0, 1\}$ as follows:

$$\sigma[k] = \begin{cases} 0, & \text{if no erasure occurs at } k, \\ 1, & \text{otherwise.} \end{cases} \quad (10)$$

The random variables $\sigma[k], k = 0, 1, \ldots$ are assumed to be independent and identically distributed with common distribution: $P_r(\sigma[k] = 0) = 1 - p$ and $P_r(\sigma[k] = 1) = p$.

B. Numerical results

Let $R = 20$ bit/s ($T = 0.05$ s) be taken. For the example, described in Section IV, let us made simulations of the system with erasure communication channel. The estimates of the 3rd UAV speed along with its true time history for various $p \in \{0, 0.4, 0.5\}$ are plotted in Figs. 5–7.

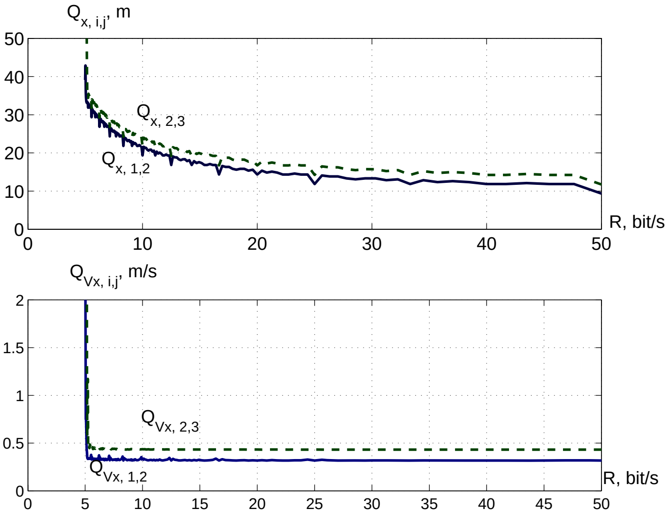

Fig. 4. Magnitudes of tracking errors on longitude coordinate x and speed V_x vs transmission rate R. The 3rd order coder.

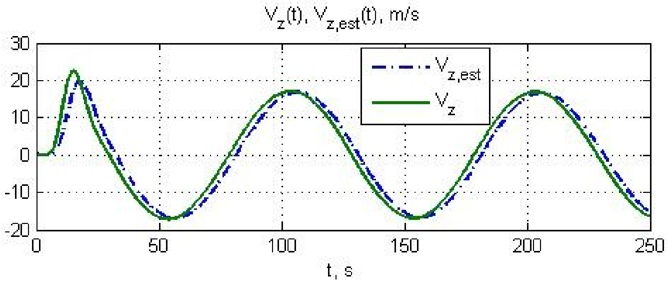

Fig. 5. Time histories of the 3rd UAV speed and its estimate. An ideal channel, $p = 0$.

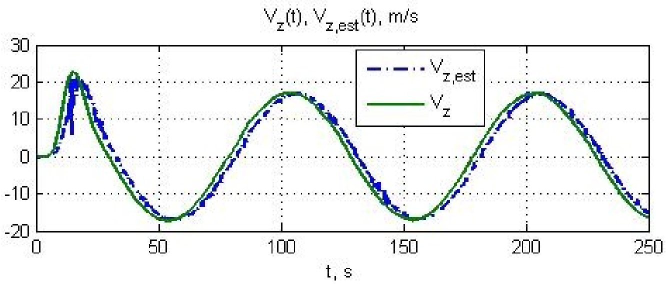

Fig. 6. Time histories of the 3rd UAV speed and its estimate. Erasure channel, $p = 0.4$.

The latest case ($p = 0.5$) may be referred to as a critical one for $R = 20$ bit/s: the process diverges as $p = 0.6$.

Influence of packets erasure to follower's position in the formation may be evaluated by Fig. 8, where the magnitude of tracking error on lateral coordinate z vs transmission rate R for the case of erasure channel with $p = 0.4$ is pictured, cf. Fig. 1.

VI. Conclusions

In the paper the data transmission techniques for tracking the moving object over the limited capacity packet erasure communication channel of finite data rate with adaptive coding/decoding procedure is studied by the example of tracking the maneuvering UAV. Control of UAV formation over the communication network is studied and the accuracy characteristics are obtained by simulations. Dependence of tracking accuracy on the probability of erasure p is numerically found based on the simulations.

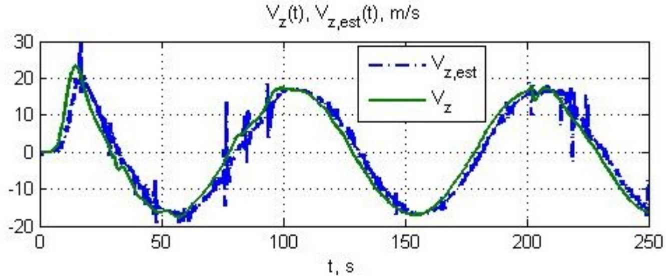

Fig. 7. Time histories of the 3rd UAV speed and its estimate. Erasure channel, $p = 0.5$.

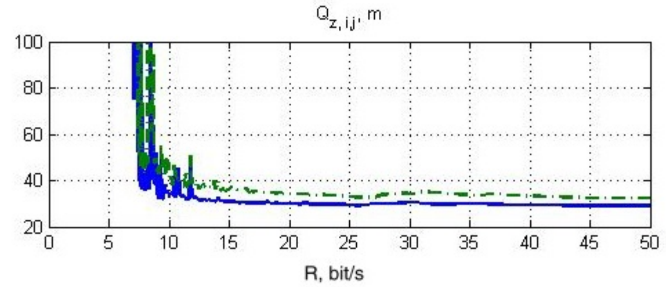

Fig. 8. Magnitude of tracking error on lateral coordinate z vs transmission rate R. Erasure channel, $p = 0.4$.

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