Indirect control of the asymptotic states of a dynamical semigroup

Raffaele Romano*
Department of Theoretical Physics, University of Trieste,
and The Abdus Salam International Centre for Theoretical Physics

The largest obstacle to the implementation of quantum technologies is the unavoidable interaction of quantum systems with the surrounding environment. Because of this interaction, the system dynamics is subject to loss of coherence, irreversibility and dissipation, and the appealing properties of quantum systems are lost or compromised [1].

In many situations the environmental action can be accounted for by describing the dynamics of a system s by a quantum dynamical semigroup, that is a Markovian reduced dynamics whose generator L, defined by $\dot{\rho}_s = L[\rho_s]$, has the standard form

$$L[\rho_s] = -i[H_s, \rho_s] + \sum_{i,j} c_{ij} \Big(F_i \rho_s F_j^\dagger - \frac{1}{2}\{F_j^\dagger F_i, \rho_s\}\Big), \tag{1}$$

where ρ_s is the statistical operator associated to the system s, H_s is an Hermitian operator and the set $\{F_i; i\}$ satisfies $\mathrm{Tr} F_i = 0$, $\mathrm{Tr}(F_i F_j^\dagger) = \delta_{ij}$. The Kossakowski matrix $C = [c_{ij}]$ must satisfy $C^\dagger = C \geqslant 0$ in order to guarantee a consistent physical interpretation of the formalism [2].

The second contribution in the right hand side of (1) is a non-coherent term responsible of the irreversible behavior of the system s. The introduction of this term leads to the appearance of attractors in the state space of s, producing relaxation to equilibrium of the states of the system, not realizable in the absence of the environment. A stationary state for the dynamics, ρ_s^∞, is defined by the condition $L[\rho_s^\infty] = 0$. Necessary conditions for the existence of stationary states and for the convergence of $\rho_s(t)$ to them have been derived in terms of the operators $\{V_i; i\}$ appearing the diagonal form of (1),

$$L[\rho_s] = -i[H_s, \rho_s] + \sum_{i} \Big(V_i \rho_s V_i^\dagger - \frac{1}{2}\{V_i^\dagger V_i, \rho_s\}\Big), \tag{2}$$

The conditions that are relevant to our purposes are summarized by the following theorem.

Theorem 1 *Given the quantum dynamical semigroup (2), assume that it admits a stationary state ρ_0 of maximal rank. Defining $\mathcal{M} = \{H_s, V_i, V_i^\dagger; i\}$ the commutant of the Hamiltonian plus the dissipative generators, the following conditions hold true:*

1. If $\mathcal{M} = span(\mathbb{I})$, then ρ_0 is the unique stationary state. Moreover, if $\{V_i; i\}$ is a self-adjoint set with $\{V_i; i\}' = span(\mathbb{I})$, then for all $\rho_s(0)$

$$\lim_{t \to +\infty} \rho_s(t) = \rho_0.$$

2. If $\mathcal{M} \neq span(\mathbb{I})$, then there exist a complete family $\{P_n; n\}$ of pairwise orthogonal projectors such that $\mathcal{Z} = \mathcal{M} \cap \mathcal{M}' = \{P_n; n\}''$. If $\{V_i; i\}' = \mathcal{M}$, two extreme cases together with their linear superpositions may occur. If $\mathcal{Z} = \mathcal{M}$, then for all $\rho_s(0)$

$$\lim_{t \to +\infty} \rho_s(t) = \sum_n Tr(P_n \rho_s(0) P_n) \frac{P_n \rho_0 P_n}{Tr(P_n \rho_0 P_n)}.$$

If $\mathcal{Z} = \mathcal{M}'$, then for all $\rho_s(0)$

$$\lim_{t \to +\infty} \rho_s(t) = \sum_n P_n \rho_s(0) P_n.$$

*Electronic address: rromano@ictp.trieste.it

In this work we address the following question: is it possible to modify the stationary states of a system s_1, that evolves under a quantum dynamical semigroup, by enlarging the system as $s = s_1 + s_2$, considering the evolution of the joint system, and then discarding s_2? In other words, we consider the joint evolution of two systems s_1 and s_2, assuming that the semigroup form is preserved, and study the stationary states of the system s_1 alone. The system s_2 has the role of an auxiliary system, and its impact on the stationary states of the system s_1 depends on the correlations between s_1 and s_2. There can be initial correlations, or rather correlations created during the joint evolution.

The use of an ancillary system has been proved useful in the field of quantum control, since it provides an alternative to the usual coherent control approach, while still using an open-loop procedure [5]. In a different approach, some information gain about the system (via an indirect measurement) is used to modify the controls in order to steer or stabilize the target system. This closed loop procedure is the so-called quantum feedback [3, 4].

In this work, for a large class of quantum dynamical semigroups, we describe to what extent it is possible to affect the stationary states of the relevant system through the ancilla. In particular, depending on the dynamical parameters characterizing the semigroup, we describe all the possible scenarios for these stationary states. We further provide examples and applications of our results.

---

[1] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information*, Cambridge, 2000

[2] R. Alicki and K. Lendi, *Quantum Dynamical Semigroups and Applications*, Lecture Notes in Physics, vol. 286, Springer-Verlag, 1987

[3] H.M. Wiseman and G. J. Milburn, Phys. Rev. Lett. 70, 548 (1993)

[4] S. Mancini, D. Vitali, and P. Tombesi , Phys. Rev. Lett. 80, 688 (1998)

[5] R. Romano and D. D'Alessandro, Phys. Rev. A 73, 022323 (2006)