THERMO-MECHANICAL MODELLING OF GROUND DEFORMATION IN VOLCANIC AREAS

Danila Scandura
Istituto Nazionale di Geofisica e Vulcanologia
Sezione di Catania
Italy
Dipartimento di Matematica e Informatica
Universita' degli studi di Catania
Italy
scandura@ct.ingv.it

Alessandro Bonaccorso
Istituto Nazionale di Geofisica e Vulcanologia
Sezione di Catania
Italy
bonaccorso@ct.ingv.it

Gilda Currenti
Istituto Nazionale di Geofisica e Vulcanologia
Sezione di Catania
Italy
currenti@ct.ingv.it

Ciro Del Negro
Istituto Nazionale di Geofisica e Vulcanologia
Sezione di Catania
Italy
delnegro@ct.ingv.it

Abstract

Numerical models are valuable tools to investigate the effect of mechanical heterogeneities when studying the deformation of structures subjected to complex loading conditions. A thermo-mechanical numerical model is performed in order to evaluate the temperature dependence of the rheological behaviour. Accounting for viscoelastic and elasto-plastic material around the volcanic source remarkably reduces the magmatic pressure necessary to trigger the observed magnitude of surface deformation with respect to solutions from elastic models. Model assumptions on the medium rheology strongly affect the resulting solutions for ground deformation which is important for estimating eruption hazards.

Key words

Numerical model, Finite Element Method, Etna volcano.

1 Introduction

The presence of heterogeneous materials and high temperatures in volcanic areas affects the rheological behaviour of the Earth's crust, resulting in anelastic deformational responses of the magmatic host rocks. Most volcano deformation models published to date assume that the Earth's crust behaves as a perfectly elastic solid. Over the last decades, elastic numerical models have contributed to assess how medium heterogeneity can influence ground deformation [Bonaccorso et al., 2005, Cayol et al., 1998, Currenti et al., 2008, Williams and Wadge, 2000, Trasatti et al., 2003]. The elastic approximation is generally appropriate for small scaled deformations of crustal materials with temperatures cooler than the brittle-ductile transition, which is between 600 and 800 K, mainly depending on composition and strain rate. Therefore, the elastic rheology assumption is oftentimes an overly simplification. The thermal state makes the elastic approximation inappropriate and can greatly influence the surface deformation field. We hence performed a thermo-mechanical numerical model in order to evaluate the temperature dependence of ground deformation in volcanic areas. This effects may be relevant for the interpretation and quantitative assessments of the pressure changes within magmatic sources inferred from geodetic data. We reviewed the ground deformation observed on Etna volcano during the 1993-1997 inflation period by setting up a fully 3-D axi-symmetric model. The main aim of this work is to study how the solutions vary if the description of a specific volcanic region becomes more realistic, focusing on the heterogeneous structure and the inelastic properties of Mount Etna volcano.

2 Methods

Simulation are based on 3-D axi-symmetric model of Mount Etna to assess the effect of rheological heterogeneity on the surface deformation field. Numerical computations are performed using the FE software COMSOL. Our approach assumes radial symmetry above the source centre. Source is located in an area that has been a preferential pathway of magma rising and a region of intermediate magma storage. Source parameters has been taken from Bonaccorso et al.(2006)l. The source has a spherical geometry with a radius of $0.7km$ and is centered at $4km$ depth. A step-like increase in overpressure of $P = 320MPa$ is applied at time t=0 onto the source wall as reported in Bonaccorso et al.(2006). In order to approximate a half-space, the axi-symmetric FE model is composed of $400,000$ triangular elements, covering a region that extends $50km$ horizontally from the source centre and $70km$ below the surface. We developed the model in two steps solving separately: (i) the heat conduction equation to compute the temperature profile, and (ii) the elasto-static mechanical equation in order to obtain solutions for the resulting deformation field. Comparison between different rheologies are performed considering: (i) elastic, (ii) viscoelastic and (iii) elasto-plastic rheology.

2.1 Thermal model

The mechanical properties of the medium depend on the temperatures of the magma chamber and the surrounding rocks. If we assume a magma chamber maintained at a constant temperature, the temperature distribution around the magmatic source can be computed by solving a heat conduction equation. To derive the temperature profile, we numerically solved the equations for heat conduction in an axially symmetric formulation [Ranalli, 1995,Turcotte and Schubert, 1982]:

$$\nabla \cdot (k \nabla T) = -A(z) \tag{1}$$

where $T = T(r, z)$ is the temperature field, r is the radial coordinate, z is the vertical coordinate, k is thermal conductivity, and $A(z) = A_s exp(-z/b)$ is the volumetric crustal heat production, where A_s is the volumetric rate of heat production, and b is a characteristic depth of the order of $105km$. We assume the surface temperature constant at atmospheric temperature, since the thermal conductivity of the air is much smaller than that of the ground. Geothermal temperature values are assigned at bottom and lateral boundaries because the boundaries are in a distance large enough to not be affected by the magmatic source. We use the steady-state geothermal profile given by [Ranalli, 1995,Turcotte and Schubert, 1982]:

$$T(z) = T_s + \frac{q_m z}{k} + \frac{A_s b^2}{k}(1 - e^{-z/b}) \tag{2}$$

where T_s is the surface temperature and q_m is the heat flow coming from the mantle. The temperature on the magma chamber wall is set to $T_0 = 1500K$. This condition is equivalent to the assumption that the magma chamber walls act as heat sources, simulating a continuous refill of the magma chamber [Civetta et al., 2004, Dragoni et al., 1997]. The parameters used in our calculations are reported in Table 1, the thermal profile obtained is shown in Figure 1.

Table 1. Thermal parameters

	Thermal parameters	
T_s	Surface temperature	$273K$
q_m	Heat flow	$0.03mWm^{-}2$
k	thermal conductivity	$4Wm^{-}1K^{-}1$
A_s	Vol. rate of heat production	$2.4710^{-}6Wm^{-}3$
b	Length radioactive decay	$14.170km$

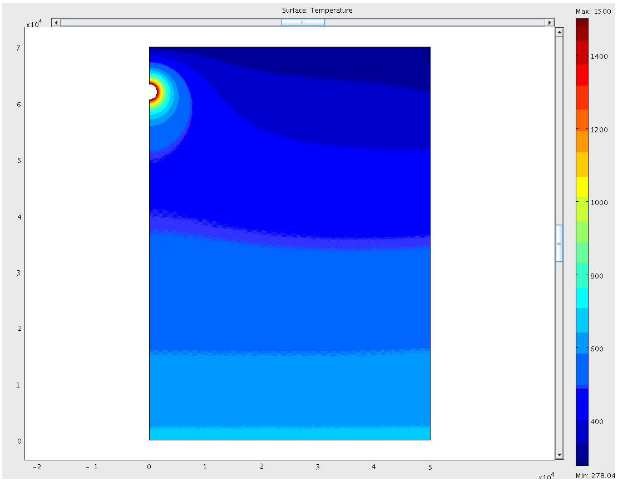

Figure 1. Temperature profile assuming a temperature of the magma chamber wall of 1500 K

2.2 Ground deformation models accounting for mechanical heterogeneity

We considered three different mechanical models considering: (i) elastic, (ii) viscoelastic and (iii) elasto-plastic rheology. First, we consider a fully elastic half-space with Poisson's ratio $\nu = 0.25$ and rigidity modulus $\mu = 30GPa$, a reasonable approximation to the values estimated in the upper crust on Mt Etna. The second and third models are obtained by implementing a spherical shell with a radius of $1.7km$ around the magmatic source. The shell is assigned with viscoelastic and elasto-plastic properties, respectively, while the host rock around the shell is considered to behave elastically. For the viscoelastic model we assume a generalized Maxwell rheology with Poisson's ratio $\mu = 0.25$, rigidity moduli $\mu_1 = \mu_2 = 30GPa$, and viscosity $\eta = 2 \cdot 10^{16} Pas$. For the elasto-plastic model we implement the yield stress/strain laws considering an ideal plastic behaviour obeying to von Mises criterion; the yield strength of surrounding rocks is assumed as $\sigma_y = 15MPa$, while the elastic parameters of the medium are those of previously described models. Model results are shown in Figure 2 comparing ground uplift due to a overpressure of $320MPa$ in the source.

The numerical models that include an anelastic rhe-

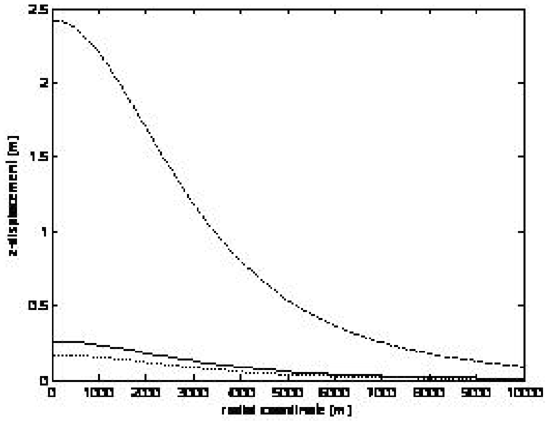

Figure 2. Comparison of ground uplift due to a pressure source of 320 MPa considering elastic (dotted line), viscoelastic (solid line) and elasto-plastic (dashed line) rheology . See text for details.

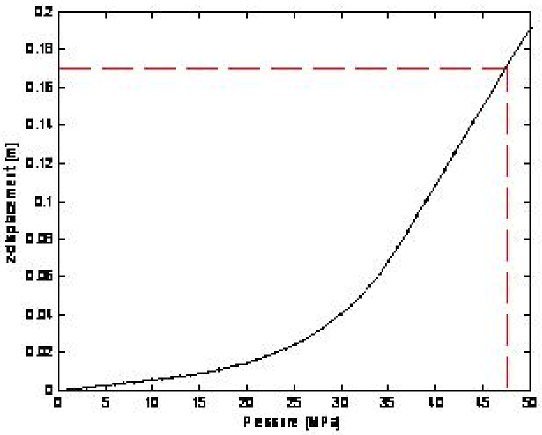

Figure 3. Non linear dependence between pressure source and ground uplift. Simulations were computed considering an ideal plastic medium with yield strength of $\sigma_y = 15MPa$.

ology give spatial deformation patterns equal to what is obtained from an elastic model. However, significantly lower values for overpressure are required in order to obtain the same magnitude of deformation. Viscoelastic response after 3 years and half are enhanced of about 1.6 in comparison with the deformation of the elastic solution. For the elasto-plastic model, a non-linear dependence between pressure source and magnitude of deformation can be found (Fig.3). Figure 3 shows that the maximum uplift of $0.17m$ (elastic deformation) in the case of elasto-plastic model requires a pressure changes of $47MPa$.

3 Thermo-mechanical model of ground deformation

We combined the mechanical and thermal model approaches presented above, as we modify the properties of the host rocks depending on computed thermal profile. In the viscoelastic model we assume a viscoelastic behaviour in areas where the temperature is higher than a fixed threshold and an elastic behaviour where it is lower. Therefore, we modify the properties of the medium through the constitutive equations, allowing the element of the domain to behave elastically or viscoelastically as a function of the temperature threshold. Consequently, the resulting solutions for ground deformation vary as a function of temperature. We estimated the medium viscosity surrounding the source region using the Arrhenius formula. Figure 4 shows the ground uplift as the temperature threshold increases from $600K$ to $1000K$. When the threshold decreases to $600K$, the thickness of the viscoelastic shell become about $1.2km$ and the deformation is enhanced. Conversely, when the threshold increases to $1000K$, the thickness of the viscoelastic shell become about $0.3km$ and the deformation is reduced (Fig. 5).

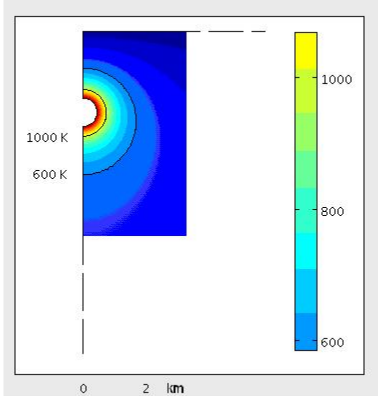

Figure 4. Temperature distribution around the magma source.

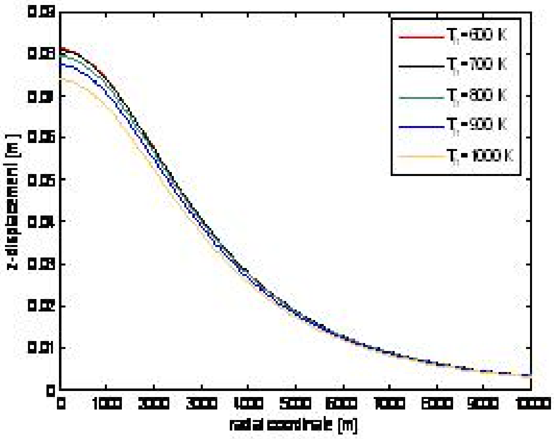

Figure 5. Ground uplift as predicted from the viscoelastic model with the shell thickness dependent on the threshold temperature. Simulations are performed for threshold temperature ranging from 600 K to 1000 K.

For the elasto-plastic model we suppose an elasto-

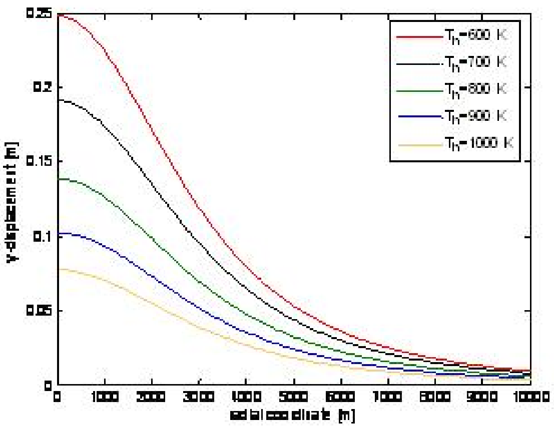

Figure 6. Ground uplift as predicted from the elasto-plastic model as the threshold temperature increases from 600 K to 1000 K.

plastic behaviour where the temperature is higher than the threshold value, an elastic behaviour where it is lower. Figure 6 shows the predicted ground uplift when changing the threshold from $600K$ to $1000K$. In the elasto-plastic model the transition temperature is found to be a very sensitive parameter. Indeed, as the threshold temperature decreases, the volume participating to the elasto-plastic flow increases and gives more contribution to the plastic deformation.

4 Conclusions

The numerical model, including anelastic rheology of the host rock surronding the magmatic source, enables to produce deformation comparable with those obtained from elastic model, requiring a significantly lower pressure. The thermo-viscoelastic model requires a lower pressure change ($200MPa$) that is near to the lithostatic load ($170MPa$), but still higher than the crustal strength ($45MPa$). Instead, the elasto-plastic model requires a pressure change comparable to the crustal strength. These evidences point to the elastoplastic rheology as the more reasonable behavior to perform more realistic numerical simulations. The low pressure change obtained using the elasto-plastic model is compatible with the low levels of both volcanic and seismic activity in the 93-97 inflation phase occurred on Etna volcano.

References

Bonaccorso, A., Cianetti, S., Giunchi, C., Trasatti, E., Bonafede, M., Boschi, E. (2005). *Analytical and 3D numerical modeling of Mt. Etna (Italy) volcano inflation*. Geophys. J. Int. 163, 852862.

Cayol, C., Cornet, F. H. (1998). *Effects of topography on the interpretation of the deformation field of prominent volcanoes, Application to Etna*. Geophys. Res. Lett. 25 , pp. 1979-1982.

Civetta, L., DAntonio, M., De Lorenzo, S., Di Renzo, V., Gasparini, P. (2004). *Thermal and geochemical constraints on the deep magmatic structure of Mt. Vesuvius*. J. Volcanol. Geotherm. Res. 133 , pp. 112

Currenti, G., Del Negro, C., Ganci G., Scandura D. (2008). *3D numerical deformation model of the intrusive event forerunning the 2001 Etna eruption*. Phys. Earth Planet. Interiors 168, pp. 8896 10.1016/j.pepi.2008.05.004.

Dragoni, M., Harabaglia, P., Mongelli, F., (1997). *Stress field at a transcurrent plate boundary in the presence of frictional heat production at depth*. Pure Appl. Geophys. 150, pp. 181201.

Ranalli, G., (1995). *Rheology of the Earth*. Chapman and Hall, London (1997) pp. 413.

Trasatti, E., Giunchi, C., Bonafede, M., (2003). *Effects of topography and rheological 705 layering on ground deformation in volcanic regions.*. J. Volcanol. Geotherm. Res. 706 122, 89110

Turcotte, D. L., Schubert, G., (1982). *GeodynamicsApplications of Continuum Physics to Geological Problems*. Wiley, New York.

Williams, C., Wadge, G. (2000). *An accurate and efficient method for including the effects of topography in three-dimensional elastic models of ground deformation with applications to radar interferometry*. J. Geophys. Res. 105 (2000), pp. 1038120.