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CONTROL OF DISTILLATION COLUMN UNDER PERTURBATIONS: A CASE STUDY

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Abstract

The control law of distillation column under unknown parameters and external disturbances in feed is obtained. The control law design is based on the robust suboptimal auxiliary loop algorithm for rejecting perturbations and ensuring the track of column output to reference signal. The simulations illustrate an efficiency of proposed scheme and comparison with some existing ones.

Key words

Distillation column, robust control, optimal control, unknown parameters.

1 Introduction

Currently a distillation column is a widely used technical system in many areas of industry such as chemical, petroleum refining, pharmacological, food, etc. Area of application of the distillation column is constantly expanding. This is facilitated by the introduction of new products and processes, increasing demands for environmental protection, etc.

In [Hsu, Yu, and Liou, 1990; Xianku and Yicheng, 2005] PI and PID controllers are proposed for distillation column regulation. In [Tyreus, 1979] the controller is based on inversion of the distillation column transfer function. In [Diggelen, Kiss, and Heemink, 2010] the LGQ, LGQ/LTR, DNA/INA, IMC methods are applied to controller design. The LQR optimal control problem of the distillation column is considered in [Musch and Steiner, 1995]. However, the methods [Hsu, Yu, and Liou, 1990; Xianku and Yicheng, 2005; Tyreus, 1979; Diggelen, Kiss, and Heemink, 2010; Musch and Steiner, 1995] are proposed under known model parameters. In [Bouyahiaoui et sl., 2005; Khelassi, 1991; Afanasyev, Kolmanovskii, and Nosov, 2003; Skogestad, Morari, and Doyle, 1988; Razzaghi and Shahraki, 2006; Yu, Poznyak, and Alvarez, 1999] an optimal fuzzy control law and H_{inf}-control are proposed under small change in parameters.

In [Skogestad, Morari, and Doyle, 1988] it is noted that the processes in the distillation column are sensitive to the change of external feeds and less sensitive to the change of internal processes in the column. Therefore, even an insignificant difference of model parameters from the original ones will lead to a failure of stated quality indicators or to a loss of stability, if the control system is based on algorithms [Hsu, Yu, and Liou, 1990; Xianku and Yicheng, 2005; Tyreus, 1979; Diggelen, Kiss, and Heemink, 2010; Musch and Steiner, 1995; Bouyahiaoui et sl., 2005; Khelassi, 1991; Afanasyev, Kolmanovskii, and Nosov, 2003; Skogestad, Morari, and Doyle, 1988; Razzaghi and Shahraki, 2006; Yu, Poznyak, and Alvarez, 1999].

In the present paper the robust algorithm for control of the distillation column is proposed. Column model is represented by the differential equation with unknown parameters and disturbances presented in feeds. The goal is a design of the control law that provides tracking of distillation column output to the reference signal with the desired accuracy, where the parameters of the reference model are chosen optimal. The control law design is based on the auxiliary loop algorithm [Furtat, 2014].

The efficiency of the proposed scheme is illustrated by the simulations for the model of the distillation column with parameters from [Bouyahiaoui et sl., 2005].

2 Model of Distillation Column and Problem Formulation

Consider the distillation column in Fig. 1.

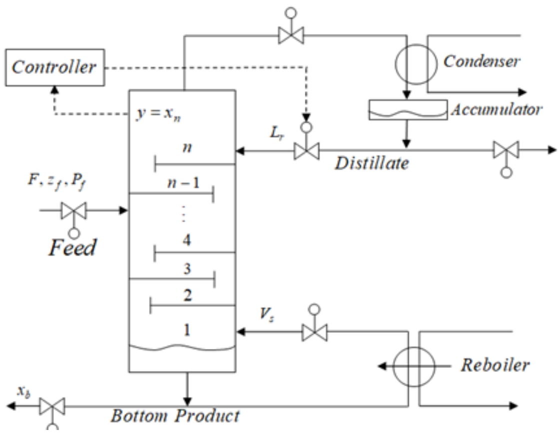

Figure 1. Simple scheme of the distillation column.

Let a model of the distillation column be described by the equations

$$\begin{aligned} \dot{x}(t) = Ax(t) + Bu(t) + Df(t), \\ y(t) = Cx(t), \end{aligned} \quad (1)$$

where

$$\begin{aligned} x(t) = [x_d(t), x_n(t), ..., x_f(t), ..., x_1(t), \\ x_b(t), P_c(t), V_s(t)]^{\mathrm{T}} \end{aligned}$$

is the state vector, $x_d(t)$ is a concentration of the light component in the distillate, $x_n(t)$ is a concentration of the light component in the condensor, $x_2(t), ..., x_{n-1}(t)$ is a concentration of the light component in trays $2, ..., n-1$, $x_1(t)$ is a concentration of the light component in the heater, $x_f(t)$ is a concentration of the light component in the part of the column, which receives the feed stream, $x_b(t)$ is a concentration of the light component in the bottoms product, $P_c(t)$ is a pressure in the upper tray of the column, $V_s(t)$ us a boilup flow rate, $u(t) = L_r(t)$ is a reflux flow rate, $f(t) = [P_f(t), F(t), z_f(t), P_{ss}(t), X_v(t)]^{\mathrm{T}}$ is a vector of uncontrolled perturbations, $P_f(t)$, $F(t)$ and $z_f(t)$ is a pressure, flow rate and concentration of the light component in the feed stream respectively, $P_{ss}(t)$, $X_v(t)$- pressure and the amount of light component in the boilup flow, $C = [0, 1, 0, ..., 0]$- matrix of corresponding dimension.

Let a reference model be defined as follows

$$\begin{aligned} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) + D_m f_m(t), \\ y_m(t) = C x_m(t). \end{aligned} \quad (2)$$

Here $x_m(t) \in R^{n+5}$ is the state vector of the reference model, $u_m(t) \in R$, $f_m(t) \in R^5$, $y_m(t) \in R$, $A_m \in R^{(n+5)\times(n+5)}$, $B_m \in R^{n+5}$, $D_m \in R^{(n+5)\times 5}$ are matrices and vector with known constant values. All signals and parameters in (2) have the same physical meaning as the corresponding signals and parameters in (1). Obviously, equation (2) represents the ideal case of the model (1), i.e. when the model (1) is not affected by parametric uncertainty and the disturbance $f(t)$. The reference control law $u_m(t)$ is calculated as $u_m(t) = -R^{-1}B_m^T P x_m$, $A_m^T P + PA_m - PBR^{-1}B^T P + Q = 0$, $R>0$ and $Q = Q^T \geq 0$.

Assumptions.

1. Unknown elements of the matrices$A$, B and D depend on a vector of unknown parameters $\vartheta \in \Xi$, where Ξ is a known set.
2. Ordered pair (A, B) is controllable and the pair (L, A) is observable.
3. Plant (1) is minimum-phase.
4. The relative degree of the plant and the reference model equals γ.
5. Signals $y(t)$, $u(t)$ and $y_m(t)$ are available for measurement.

It is required to design a control law that provides the following inequality

$$|y(t) - y_m(t)| < \delta \quad \text{at} \quad t > T \quad \text{for} \quad \forall \vartheta \in \Xi, \quad (3)$$

where $\delta > 0, T > 0$ is a transient time.

3 Control Law Design

Let us rewrite equation (1) as follows

$$y(t) = W_1(D)u(t) + W_2(D)f(t), \quad (4)$$

where $D = d/dt$is a differential operator, $W_1(D)$, $W_2(D)$ are transfer functions obtained by the transition from (1) to (4). Considering (2) and (4), rewrite the equation for the tracking error $\varepsilon(t) = y(t) - y_m(t)$ in form

$$\varepsilon(t) = W_1(D)u(t) + W_2(D)f(t) - y_m(t). \quad (5)$$

According to (5) we express $u(t)$as

$$\begin{aligned} u(t) = W_1^{-1}(D)\varepsilon(t) \\ +W_1^{-1}(D)\left[y_m(t) - W_2(D)f(t)\right]. \end{aligned} \quad (6)$$

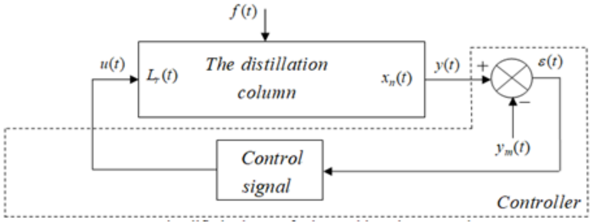

Figure 2. The proposed control scheme.

For compensation of disturbances we use the method [Furtat, 2014]. According to [Furtat, 2014], introduce the auxiliary loop

$$\bar{\varepsilon}(t) = W_m(D)u(t), \tag{7}$$

where $\bar{\varepsilon}(t) \in R$ is an output of the auxiliary loop, $W_m(D)$ is a stable minimum-phase transfer function with the relative degree γ. Let us rewrite the mismatch error $e(t) = \varepsilon(t) - \bar{\varepsilon}(t)$ to estimate proximity of the outputs (1) and (2)

$$e(t) = \varepsilon(t) - W_m(D)u(t). \tag{8}$$

Insert therein signal $u(t)$ that we obtained in (6)

$$\begin{gathered} e(t) = \left[1 - W_m(D)W_1^{-1}(D)\right]\varepsilon(t) \\ -W_m(D)W_1^{-1}(D)\left[y_m(t) - W_2(D)f(t)\right]. \end{gathered} \tag{9}$$

Introduce the control law as

$$u(t) = -W_m^{-1}(D)W_R(\mu, D)e(t), \tag{10}$$

where $W_R(\mu, \lambda)$ is a stable minimum-phase transfer function with the relative degree γ, as well as $\|W_R(\mu,\lambda)\|_\infty = 1$, $\mu > 0$ is a sufficiently small number.

Substituting (10) and (9) into (8) and then after expressing $\varepsilon(t)$, one obtains the closed-loop system in the form

$$\begin{gathered} \varepsilon(t) = \\ -\frac{(1-W_R(\mu,D))W_m(D)W_1^{-1}(D)}{1-(1-W_R(\mu,D))\left(1-W_m(D)W_1^{-1}(D)\right)} \\ \times\left[y_m(t) - W_2(D)f(t)\right]. \end{gathered}$$

Substituting (10) into (8), the control law $u(t)$ can be rewritten as follows

$$u(t) = \frac{W_m^{-1}(D)W_R(\mu, D)}{W_R(D) - 1}\varepsilon(t). \tag{11}$$

As a result, we obtain a simplified control law, see Fig. 2.

4 Simulations

Consider the model of distillation column with seven plates. It receives the feed stream F which arrives to the fourth feeding plate. The appropriate model is given in [Musch and Steiner, 1995; Bouyahiaoui et sl., 2005]. This column is used to separate a gasoline-toluic mixture.

According to [Musch and Steiner, 1995; Bouyahiaoui et sl., 2005], the reference model is defined by equation (2), where

$$A_{m1} = \begin{bmatrix} -0.0135 & 0.0063 & 0 & 0 & 0 \\ 0.029 & -0.0436 & 0.0168 & 0 & 0 \\ 0 & 0.029 & -0.0457 & 0.212 & 0 \\ 0 & 0 & 0.029 & -0.0502 & 0.029 \\ 0 & 0 & 0 & 0.027 & -0.0626 \\ 0 & 0 & 0 & 0 & 0.0356 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{12}$$

$$A_{m2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.049 \\ 0 & 0 & 0 & 0 & 0 & -0.0908 \\ 0 & 0 & 0 & 0 & 0 & -0.1369 \\ 0.0346 & 0 & 0 & 0 & 0 & -0.1176 \\ -0.0702 & 0.0446 & 0 & 0 & 0 & -0.1369 \\ 0.0356 & -0.0802 & 0.0548 & 0 & 0 & -0.124 \\ 0 & 0.0356 & -0.0904 & 0.0628 & 0 & -0.0892 \\ 0 & 0 & 0.0081 & -0.0157 & 0 & -0.0123 \\ 0 & 0 & 0 & -15.224 & -5.0086 & 299.42 \\ 0 & 0 & 0.0004 & 0.0283 & 0.0084 & -0.6868 \end{bmatrix},$$

$$A_m = [A_{m1}\ A_{m2}],$$

$$B_m = \begin{bmatrix} 0 \\ 0.533 \\ 0.0988 \\ 0.152 \\ 0.1653 \\ 0.1129 \\ 0.1023 \\ 0.0736 \\ 0.0102 \\ 0 \\ 0.0005 \end{bmatrix},$$

$$D_m = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -0.0005 & 0 & 0 & 0 & 0 \\ -0.009 & 0 & 0 & 0 & 0 \\ -0,0014 & 0 & 0 & 0 & 0 \\ -0.0019 & -0.1169 & 0.0086 & 0 & 0 \\ -0.0011 & 0.1129 & 0 & 0 & 0 \\ -0.001 & 0.1023 & 0 & 0 & 0 \\ -0.0007 & 0.0736 & 0 & 0 & 0 \\ -0.0001 & 0.0102 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6229 & 1.4409 \\ 0 & 0.0005 & 0 & 0 & 0 \end{bmatrix},$$

$$L = \begin{bmatrix} 0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \end{bmatrix}.$$

We define in a reference model (2): $r_m(t) = 0.1404$, $f_m(t) = 0.14\begin{bmatrix}1\ 0.2\ 1\ 1\ 1\end{bmatrix}^T$ and $x(0) = [1\ 0.8983\ 1\ 1\ 1\ 1\ 1\ 1\ 0.4878\ 1\ 1]^T$.

During the study of the distillation column's model, which is presented by the first equation in [Musch and Steiner, 1995] with parameters (12), it has been revealed that only transfer function for the output $y_m(t)$ and the input $r(t)$ is the minimum-phase one. Because of the assumption (3), in this paper we construct the control law for equation (1), i.e. distillate is regulated.

It is also obvious that the transfer function for the output $y_m(t)$ and the input $r(t)$ is stable with relative degree $\gamma = 1$.

The goal is to choose an algorithm that provides implementation of the condition (3). Chose in (7) $W_m(D) = \frac{1}{D+1}$ and $\alpha = 1$. The auxiliary loop is then defined as

$$\tilde{e}(t) = \frac{1}{D+1}u(t).$$

Assume in (11) $W_R(D,\mu) = \frac{1}{\mu D+1}$ and $\mu = 0,1$. Obviously, $\|W_R(D,\mu)\|_\infty = 1$. Then, the control law (11) that counterbalances the uncertainties (which act on the distillation column) is defined as

$$u(t) = -\frac{D+1}{0.1D}\varepsilon(t) \qquad (13)$$

Assume that parameters in the distillation column (1) are presented by

$$A_1 = \begin{bmatrix} -0.0135 & 0.0063 & 0 & 0 & 0 \\ 0.0823 & 0.0097 & 0.0701 & 0.0533 & 0.0533 \\ 0.0988 & 0.1278 & 0.0531 & 0.12 & 0.0988 \\ 0.152 & 0.152 & 0.181 & 0.1018 & 0.1018 \\ 0.1653 & 0.1653 & 0.1653 & 0.1923 & 0.1027 \\ 0.1129 & 0.1129 & 0.1129 & 0.1129 & 0.1485 \\ 0.1023 & 0.1023 & 0.1023 & 0.1023 & 0.1023 \\ 0.0736 & 0.0736 & 0.0736 & 0.0736 & 0.0736 \\ 0.0102 & 0.0102 & 0.0102 & 0.0102 & 0.0102 \\ 0 & 0 & 0 & 0 & 0 \\ 0.0005 & 0.0005 & 0.0005 & 0.0005 & 0.0005 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0533 & 0.0533 & 0.0533 & 0.0533 & 0.0533 & 0.0533 \\ 0.0988 & 0.0988 & 0.0988 & 0.0988 & 0.0988 & 0.008 \\ 0.152 & 0.152 & 0.152 & 0.152 & 0.152 & 0.0124 \\ 0.1999 & 0.1653 & 0.1653 & 0.1653 & 0.1653 & 0.0477 \\ 0.0427 & 0.1575 & 0.1129 & 0.1129 & 0.1129 & -0.024 \\ 0.1379 & 0.0221 & 0.1571 & 0.1023 & 0.1023 & -0.0217 \\ 0.0736 & 0.1092 & -0.0168 & 0.1364 & 0.0736 & -0.0156 \\ 0.0102 & 0.0102 & 0.0183 & -0.0055 & 0.0102 & -0.0021 \\ 0 & 0 & 0 & -15.224 & -5.0086 & 299.42 \\ 0.0005 & 0.0005 & 0.0009 & 0.0288 & 0.0089 & -0.6863 \end{bmatrix},$$

$$A = [A_1\ A_2],$$

$$B = 3\begin{bmatrix} 0 \\ 0.533 \\ 0.0988 \\ 0.152 \\ 0.1653 \\ 0.1129 \\ 0.1023 \\ 0.0736 \\ 0.0102 \\ 0 \end{bmatrix}$$

$$F = 2\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -0.0005 & 0 & 0 & 0 & 0 \\ -0.009 & 0 & 0 & 0 & 0 \\ -0.0014 & 0 & 0 & 0 & 0 \\ -0.0019 & -0.1169 & 0.0086 & 0 & 0 \\ -0.0011 & 0.1129 & 0 & 0 & 0 \\ -0.001 & 0.1023 & 0 & 0 & 0 \\ -0.0007 & 0.0736 & 0 & 0 & 0 \\ -0.0001 & 0.0102 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6229 & 1.4409 \\ 0 & 0.0005 & 0 & 0 & 0 \end{bmatrix},$$

$$x(0) = [0.5\ 0.8\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.4\ 0.5\ 0.5]^T,$$

$$f(t) = 0.14[1 + 2\sin t, 0, 2 + 2\sin 1.5t,$$
$$1 + 2\sin 2t, 1 + 2\sin 3t, 1 + 2\sin 0.5t]^T.$$

In Fig. 3 simulations of outputs $y(t)$ and$y_m(t)$ are shown. In Fig. 4 the simulations for error $\varepsilon(t)$ are given.

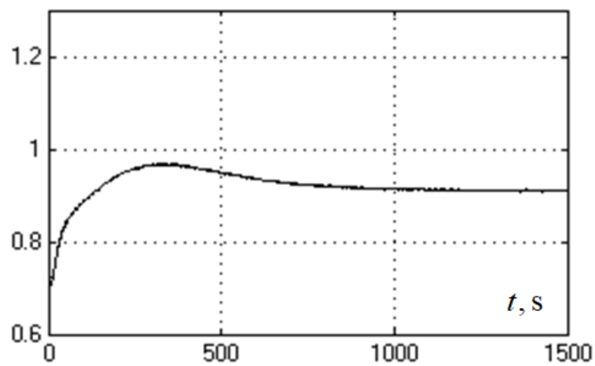

Figure 3. The graphs of $y(t)$ and $y_m(t)$.

The simulations have been shown that the proposed control law rejects parametric uncertainty and external disturbances and provides the goal (3) with given accuracy δ. Quality of transient responses depends on the choice of transfer function and coefficient α in auxiliary loop (7), and the value of μ in (10) (or (11)). It should be noted that implementation of the control law and calculation of its parameters are simpler compared with [Afanasyev, Kolmanovskii, and Nosov, 2003; Skogestad, Morari, and Doyle, 1988; Razzaghi and Shahraki, 2006; Yu, Poznyak, and Alvarez, 1999] .

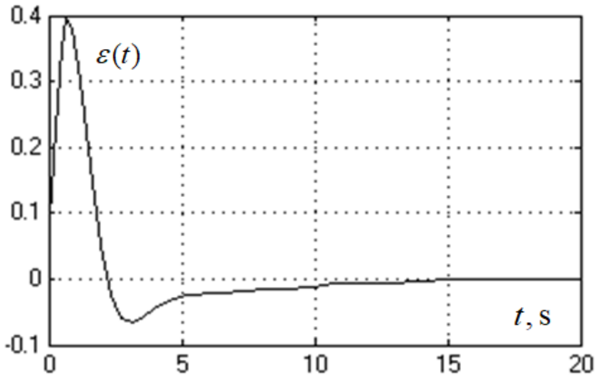

Figure 4. Results of simulation for error $\varepsilon(t)$.

5 Acknowledgements

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6 Conclusion

In this paper an algorithm for robust control of the distillation column is proposed. Column's model is represented by parametrically and functionally indefinite linear differential equation. The problem is solved by using the approach [Furtat, 2014]. The aim of control was the synthesis of continuous control law that provides tracking of output of the distillation column to the reference signal with the desired accuracy. The simulation showed good indicators of quality of transient responses and confirmed the results of analytical calculations. In contrast to [Afanasyev, Kolmanovskii, and Nosov, 2003; Skogestad, Morari, and Doyle, 1988; Razzaghi and Shahraki, 2006; Yu, Poznyak, and Alvarez, 1999] we propose an algorithm that is simpler in technical implementation and calculation of adjustable parameters. The algorithm provides better quality indicators of transient responses for bounded disturbances than ones from [Afanasyev, Kolmanovskii, and Nosov, 2003; Skogestad, Morari, and Doyle, 1988; Razzaghi and Shahraki, 2006; Yu, Poznyak, and Alvarez, 1999].

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