Design Technique for High-quality Flight Control System

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Abstract: The energy approach to flight control has shown the high efficiency. On its basis the rational structure of energy control system at landing approach was created. However, the system with constant coefficients cannot provide acceptable control quality for maneuverable high-speed aircrafts over the whole flight envelope. Therefore the adjustment of energy control system parameters needed. For this purpose the new version of a modal method of system design with the required quality has been developed. With its help the optimum system parameters for the operational area were found. The loop of coefficients adjustment according to flight condition was entered into system. Modeling has shown invariance of handling qualities in a wide range of flight conditions

Keywords: modal method, energy control system, flight control, handling qualities, modeling

1. INTRODUCTION

The creation of high-quality automatic flight control systems took place throughout all history of aircraft evolution and remains actual now. There is a wide variety of flight control concepts.

One of nonconventional concept has been developed in Lambreghts (1983), Kurdjukov et al. (1994), Borisov et al. (1999) and has been called "The Energy Approach to Flying Objects (FO) Control".

Its essence consists in the following. The structure of traditional flight control systems can be submitted by the generalized circuit (Fig. 1).

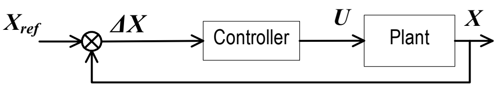

Fig.1

Control U is formed on the basis of error ΔX of a state variable vector X. The performance index gets out in class

$$Qx=Qx\ (U, X, \Delta X).$$

The nonconventional control concept (Fig. 2) in which a total motional energy E is controllable variable is offered instead.

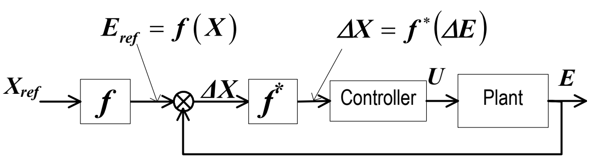

Fig.2

The performance index is also set at the form

$$Q_E=Q_E(U, E, \Delta E).$$

The basic quantitative relation of the approach is established by the *energy balance equation* (Kurdjukov et al., 1994)

$$\Delta H_E = \Delta H_E^{Engine} - \Delta H_E^{Drug} - \Delta H_E^{Wind}$$

This equation is written in the form of deviations of specific energy $H_E^{(\cdot)} = E^{(\cdot)}/mg$, i.e. the energy normalized by weight of the FO.

Terms in the right side note the engine work, the aerodynamic force work, and the wind work, accordingly. Integral expressions for each term are received.

The energy balance equation reflects interconnections of all energy sources and consumers in system «an aircraft — a power-plant — an environment».

PI-algorithms of engine thrust and elevator control are received from the energy balance equation

$$\frac{\Delta P}{mg} = k_P\left(\frac{\dot V}{g} + \frac{\dot h}{V}\right) + k_I\int\left(\frac{\Delta \dot V}{g} + \frac{\Delta \dot h}{V}\right)dt,$$

$$k_e\Delta\delta_e = k_P\left(\frac{\dot V}{g} - \frac{\dot h}{V}\right) + k_I\int\left(\frac{\Delta \dot V}{g} - \frac{\Delta \dot h}{V}\right)dt$$

where $\Delta\dot V$ and $\Delta\dot h$ are control errors on longitudinal acceleration and vertical velocity.

The energy control system (ECS) showed the excellent handling quality of transport aircrafts at landing approach

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under strong atmospheric perturbation, engine failures, abrupt glides, etc. (Pavlov et al., 2003). In all situations the structure and coefficients of the ECS remained constant for aircraft of different classes. Similar achievements of foreign researchers are known also (Akmeliawati and Mareels, 2002; Voth and Uy-Loi, 1991). However, the system with constant coefficients cannot provide acceptable control quality for maneuverable high-speed aircrafts over the whole flight envelope. Therefore, the adjustment of ECS parameters is needed. The version of a modal synthesis method has been developed in the paper for searching the optimum coefficients adjustments.

Modal methods became widespread in problems of control systems design with the specific quality and stability requirements. Methods of synthesis of single-loop control systems are the most developed and covered in the literature (Doll et al., 1997; Tutikov et al., 2004; Dylevskii et al., 2003) In the field of multi-loop systems the papers Ackermann (1985a), Akunov et al. (2003) are well known. In particular, Ackermann's formula (Ackermann, (1985b), gives the formal solution of the pole placement problem according to requirements of quality and stability.

This design method includes a matrix raising to (n-1)-th power which, in turn, demands its good conditionality. In Akunov et al. (2003) solution of a the feedback matrix is got at the form of product of the eigenvector matrix and some matrix possessing rather formal properties.

In this paper the control weights matrix is proposed for elimination of solution multiplicity in the case of incomplete control vector.

The system structure which is most appropriate to control destination in real technical object is configured with the help of this matrix. The algorithm for the calculation of the feedback coefficients matrix in an explicit form for one computing cycle without iterative procedures was found.

The optimal kits of coefficients for discrete set of flight modes in the whole operational area have been found with the help of this version of modal design.

Then these kits have been reduced by exception the coefficients which did not need to be adjusted or considering their smallness.

For the remaining adjusted coefficients analytical approximations have been found.

2. THE PROBLEM STATEMENT

Quality and stability of the closed control system are defined by its matrix eigenvalues placement. Suppose that that the desirable set of eigenvalues is a priori known.

The only but not sufficient requirement be stability of the closed-loop system.

The multivariable control system described by the matrix differential equation is considered as

$$\dot{x}(t) = Ax(t) + Bu(t),$$

where $A \in R^{n\times n}$ is the plant matrix, $x \in R^{n\times 1}$ is the state vector, $B \in R^{n\times m}$ is the control matrix, and $u \in R^{m\times 1}$ is the controls vector.

Let the pair matrixes ($A|B$) be controllable. The problem is to find control of the form

$$u = Kx$$

such, that the closed system

$$\dot{x} = (A + BK)x \qquad (1)$$

had a desirable set of eigenvalues. Here $K \in R^{m\times n}$ is the required feedback matrix.

3. PROBLEM SOLUTION

Let the desirable spectrum $\Lambda^* = \{\lambda_1^*,...,\lambda_n^*\}$ of the closed-loop system matrix $A+BK$ not contain multiple eigenvalues. The desirable spectrum Λ^* is achieved by a modal method by constructing the required matrix K.

Let $V = \{V_1,...,V_n\}$ be a set of eigenvectors of the closed-loop system matrix $A+BK$.

The matrix K in (1) is not unique for m> 1 as the spectrum correction can be achieved by various separate components of a control vector u_i or their linear combinations.

The control as the sum of weighted modal controls is offered (Borisov et al., 1990)

$$u = \sum_{i=1}^{n} q_i K_i x .$$

In other words the structure of matrix K becomes as follows

$$K = \sum_{i=1}^{n} q_i K_i . \qquad (2)$$

Here $q_i \in R^{m\times 1}$, $i = \overline{1,n}$ is assigned weights vector of a mode λ_i^*; $K_i \in R^{1\times n}$, $i = \overline{1,m}$ is required coefficients vector of a mode λ_i^* which the additional conditions

$$K_j V_i = 0 \quad \text{for } i \neq j \qquad (3)$$

providing independence of mode control are imposed on.

The equation for definition of eigenvector V_i of the closed-loop system (1) is

$$\lambda_i^* V_i = (A + BK)V_i = AV_i + BKV_i . \qquad (4)$$

Equation (4) having in view of (2) can be written in a form

$$\lambda_i^* V_i = AV_i + B\sum_{j=1}^{n} q_j K_j V_i . \qquad (5)$$

From (5) together with (3) it is followed

$$\lambda_i^* V_i = AV_i + Bq_i K_i V_i \qquad (6)$$

and

$$V_i = (I_n \lambda_i^* - A)^{-1} Bq_i K_i V_i$$

Multiplication of both sides of the equation by K_i from left gives

$$K_i V_i = K_i (I_n \lambda_i^* - A)^{-1} Bq_i K_i V_i .$$

Note that $K_i V_i$ is scalar and it is possible to reduce the equation to

$$1 = K_i (I_n \lambda_i^* - A)^{-1} Bq_i .$$

Multiplying *j-th* eigenvector V_j by K_i from left we receive

$$K_i V_j = K_i (I_n \lambda_j^* - A)^{-1} Bq_j K_j V_j = 0 .$$

It is possible to reduce by scalar $K_j V_j$ as above

$$0 = K_i (I_n \lambda_j^* - A)^{-1} Bq_j .$$

The system of n linear equations is received for definition K_i

$$\begin{array}{lll} 1 = K_i (I_n \lambda_i^* - A)^{-1} Bq_i & \textit{for i=j} & \textit{(one equation)} \\ 0 = K_i (I_n \lambda_j^* - A)^{-1} Bq_j & \textit{for } i \neq j & \textit{(n-1 equation)} \end{array} \qquad (7)$$

Taking into account that $K_i V_i$ is a scalar, define eigenvectors $\bar{V}_i$ as

$$\bar{V}_i = (I_n \lambda_i^* - A)^{-1} Bq_i \qquad (8)$$

and then the system of linear equations becomes

$$\begin{array}{l} 1 = K_i \bar{V}_i \\ 0 = K_i \bar{V}_j \end{array} . \qquad (9)$$

In expression (9) all variables are defined; therefore all eigenvectors $\bar{V}_i$ can be determined.

Let us denote the matrix of eigenvectors by $\bar{V}$

$$\bar{V} = (\bar{V}_1 \ldots \bar{V}_n) .$$

Multiplying the row-vector K_i from right on a matrix $\bar{V}$ and using constraints (9) the following expression can be written

$$K_i \bar{V} = (0 \ldots 1_i \ldots 0) .$$

Hence it is follows that

$$K_i = (0 \ldots 1_i \ldots 0) \bar{V}^{-1} . \qquad (10)$$

Substitute (10) in (2) results to condition

$$K = \left\{ \sum_{i=1}^{n} (0 \ldots q_i \ldots 0) \right\} \bar{V}^{-1} . \qquad (11)$$

Define the weight matrix Q as

$$Q = (q_1 \ldots q_n) , \qquad Q \in R^{m \times n} .$$

Then (11) can be written

$$K = Q\bar{V}^{-1} .$$

Elements q_i of the matrix Q are assigned so that their relative values would reflect a degree of influence of any control on each of plant controllable coordinate.

The control vector is more convenient for writing partly through derivatives.

For this purpose the following procedure is offered. The matrix C is introduced according to request of observability of matrix pair (A|C)

$$y = Cx$$

Usually the matrix C assigns controllable coordinates. The following relation is obvious

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} . \qquad (12)$$

Matrixes B and C should be the same dimension. Let us consider a block matrix

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} . \qquad (13)$$

Provided that the block matrix (13) is invertible, let us write its inverse of the form

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} = \begin{bmatrix} P_{11} P_{12} \\ P_{21} P_{22} \end{bmatrix} .$$

Having in view (12), we get

$$u = Kx = \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} P_{11} P_{12} \\ P_{21} P_{22} \end{bmatrix} \begin{bmatrix} \dot{x} \\ y \end{bmatrix} ;$$

and finally $\qquad u = KP_{11}\dot{x} + KP_{12}y .$

The received expression shows that control u can be calculated by derivatives $\dot{x}$ and a measured coordinates y.

4. DESIGN OF THE OPTIMUM FLIGHT CONTROL SYSTEM.

The approval of a modal method has been carried out during designing the flight control system of the hypothetical high-speed maneuverable aircraft. The problem to provide equally high flight characteristics of the aircraft over the whole operational area of flights has been set.

An aperiodic type reaction with duration no more than 30s was assumed for reference transient on velocity and height. Formally these system requirements were set by values of low-frequency roots. The linearized model contained three low-frequency roots. Values -0.5,-0.6, and -0.7 were assigned to them. The invariability of these roots was achieved due to adjusted feedback coefficients K_{ij} . The structure of an energy system with a correction loop is shown in Fig. 3.

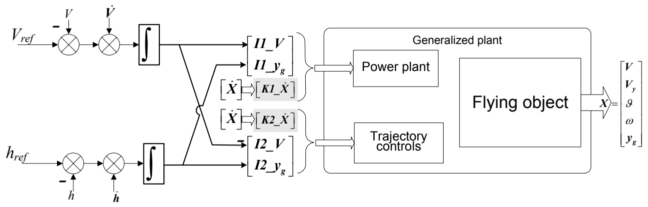


Fig.3. The structure of energy control system with modal correction

To establish limits of feedback coefficients K_{ij} values their calculation has been carried out along some specific trajectories over the whole operational area. The flight envelop and trajectories are shown in Fig. 4 with $\boldsymbol{H-M}$ coordinates.

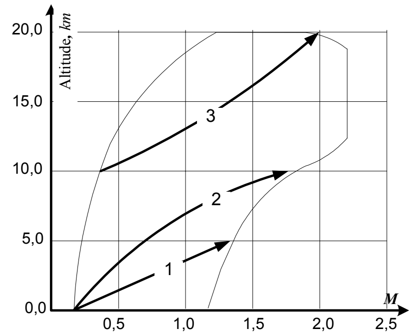


Fig.4. Investigated flight envelope

Some points have been chosen along all trajectories. Feedback coefficients in each point have been calculated by a modal method for thrust and trajectory channels.

Coefficients K_{1j} in thrust channel for six flight modes along 2-nd trajectory are resulted in Table 1. As is seen the change range of some coefficients is very wide. Therefore the circuit of coefficients K_{ij} adjustment according to flight conditions was entered into the structure of the ECS.

Table 1. Coefficients of matrix $\boldsymbol{K}$ in thrust channel

Mode **No**	1	2	3	4	5	6
K_{11}	-0.161	-0.145	-0.144	-0.133	-0.141	-0.142
K_{12}	-0.075	-0.010	-0.003	-0.004	-0.004	-0.004
K_{13}	-0.317	-0.038	-0.014	-0.014	-0.012	-0.012
K_{14}	-7.139	-1.835	-0.886	-1.255	-1.720	-2.044
K_{15}	-0.101	-0.020	-0.010	-0.009	-0.008	-0.008
K_{16}	0.000	0.000	0.000	0.000	0.000	0.000
K_{17}	0.000	0.000	0.000	0.000	0.000	0.000
K_{18}	0.000	0.000	0.000	0.000	0.000	0.000
K_{19}	-0.169	-0.166	-0.164	-0.154	-0.162	-0.163
K_{110}	-0.030	-0.011	-0.007	-0.006	-0.005	-0.004
K_{111}	-0.020	-0.021	-0.021	-0.021	-0.021	-0.021
K_{112}	-0.003	-0.001	-0.001	-0.001	-0.001	0.000

5. MODELLING THE CONTROL SYSTEM WITH ADJUSTED COEFFICIENTS

Test signals were simultaneous step commands on height and velocity.

On plots in Fig. 5-a,b the transients on height d_H and velocity d_V, and in Fig. 5-c,d reaction of an elevator d_Dle and engine thrust d_P in five points along the second trajectory, are presented. Flight conditions in these points are resulted in the same figure.

These records show that in the diversified flight conditions the aircraft practically equally carries out maneuvers.

Actions of system are presented very rational and economic.

This system property specifies the energy approach. And the property of behaviour invariance with respect to aircraft parameters variation is provided by adjustment of control system coefficients.

6. CONCLUSION

The technique of design of high-quality flight control systems is developed. The basic structural concept is the energy approach. On its base the rational structure of the energy control system (ECS) is received.

The ECS has shown excellent handling quality of transport aircrafts at landing approach in diversified conditions.

However, for maneuverable high-speed aircrafts the system with constant coefficients could not provide acceptable control quality over the operational area.

Therefore adjustment of its parameters was required for the use of the ECS over the whole flight envelope.

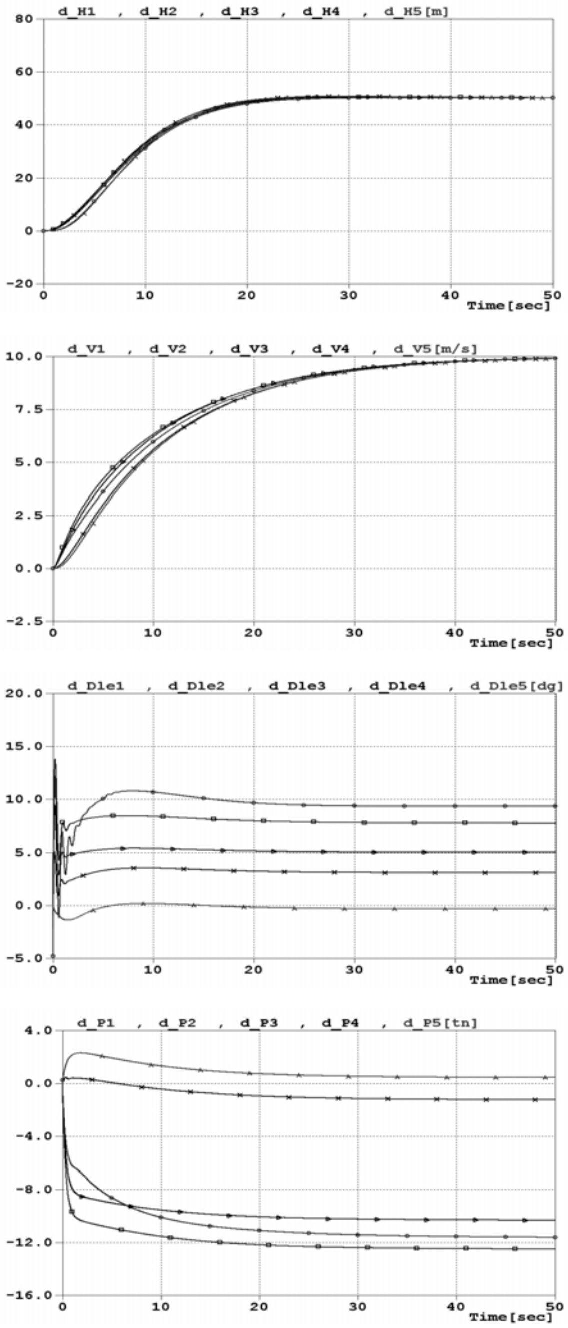
The version of a modal method for calculation of the matrix of feedback coefficients has been developed. In case of multivariable systems with an incomplete control vector the required feedback matrix was non-unique. For unique solution the control on each mode is offered as the sum of available controls with weight coefficients.

Synthesis of the flight control system for a high-speed maneuverable aircraft has been carried out.

Optimum feedback coefficients in discrete points of all operational area have been found.

The coefficients adjustment circuit depending on flight conditions was introduced into the system structure.

Modeling of the ECS with adjusted parameters has shown that transients had been invariant in various flight conditions and had been practically identical at speed from 100 up to 500 m/s and height from 0 up to 10000m.



Mode No	1	2	3	4	5
V (m/s)	100	200	300	400	500
H (m)	0	2500	5000	7500	10000

Fig. 5.Transitions records at various flight modes:

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