IMPULSE POSITION CONTROL ALGORITHMS IN SYSTEMS WITH DELAY

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Abstract

The article is devoted to the formalization of concept of impulse-sliding regimes generated by positional impulse control. We have defined the notion of impulse-sliding trajectory as a limit of network element Euler polygons generated by a discrete approximation of the impulse position control. The equations describing the trajectory of impulse-sliding regime are received.

Key words

Impulse position control, systems with delay, an impulse-sliding regime, the Euler polygons.

1 Introduction

Usually, the positional control algorithms are introduced as a result of change in program control initial time and initial position to an arbitrary point of time and an arbitrary position. Such change may lead to the fact that it will take to realize impulse action at every instant. This give rise to some sliding impulse. Such phenomenon from the point of view of the theory of differential equation needs formalization. Furthermore, such motion type in the space position originates the motion sliding on the functional diversity. An impulse-sliding regimes in systems without delay were considered in [Zavalishchin and Sesekin, 1983; Finogenko and Ponomarev, 2013]. An impulse-sliding regimes for a linear systems with delay were considered in [Andreeva and Sesekin, 1997]. The reaction of nonlinear systems with delay to impulse action is understood in the sense of paper [Fetisova and Sesekin, 2005]. The definition of the solution for nonlinear systems with delay from [Fetisova and Sesekin, 2005] is the generalization for the notion of solution for the systems without delay from [Zavalishchin and Sesekin, 1997; Sesekin, 2000].

2 Formalizing impulse-sliding regime

Consider a dynamic system with impulse control

$$\dot{x}(t) = f(t, x(t), x(t-\tau)) + B(t, x(t))u, \;\; t \in [t_0, \vartheta], \tag{1}$$

with the initial condition

$$x(t) = \varphi(t), \;\; t \in [t_0 - \tau, t_0]. \tag{2}$$

where $f(\cdot,\cdot,\cdot)$ is the function with value in R^n, $B(\cdot,\cdot)$ matrix functions of the dimension $m \times n$. Elements of f and B are continuous functions and are satisfy to the conditions, which cover the existence and uniqueness of solution for any summable $u(t)$, $x_t(\cdot)$ — function-prehistory $x_t(\cdot) = \{x(t+s);\ -\tau \leq s < 0\}$. The function $\varphi(t)$ is a function of bounded variation at $t \in [t_0 - \tau, t_0]$.

We will assume that, the function $B(t, x)$ satisfies Frobenius condition

$$\sum_{\nu=1}^{n} \frac{\partial b_{ij}(t,x)}{\partial x_\nu} b_{\nu l}(t,x) = \sum_{\nu=1}^{n} \frac{\partial b_{il}(t,x)}{\partial x_\nu} b_{\nu j}(t,x). \tag{3}$$

This condition, according to [Fetisova and Sesekin, 2005; Zavalishchin and Sesekin, 1997] , ensures uniqueness the solution of the system (1) at the general control $u(t)$ (the generalized derivative of bounded variation function). We note that, there are various ways of defining a solution for equation (1), which, broadly speaking, shows that trajectories [Zavalishchin and Sesekin, 1997] to have different execute. We will use the definition that is based on the closure of the set of smooth trajectories in the space of functions of bounded variation [Zavalishchin and Sesekin, 1997]. Such definition is more natural from control theory standpoint. This is because the impulse control are often the some control idealizations operating on a short period of time and with greater intensity.

By an impulse positional control we shall mean operator $t, x_t(\cdot) \longrightarrow U(t, x(t))$, mapping the extended phase space $t, x(t)$ into the space of m-vector-valued distributions

$$U(t, x(t)) = r(t, x(t))\,\delta_t. \tag{4}$$

In this work it is assumed that the delay has only in $f(t, x(t), x(t-\tau))$, and control action of the delay does not contain. Here $r(t, x(t))$ m-dimensional vector-functional, δ_t - is the Dirac impulse function concentrated at the point t. The reaction of the system to the impulse position control $U(t, x(t))$ (which we call an impulse-sliding regime) is defined to be the set of Euler polygons $x^h(\cdot)$, $h = \max(t_{k+1} - t_k)$ corresponding to the set of decompositions $t_0 < t_1 < ... < t_p = \vartheta$. The Euler polygons $x^h(\cdot)$ is constructed as left-continuous function of bounded variation such that this solutions satisfies to the equation

$$\dot{x}^h(t) = f(t, x^h(t), x^h(t-\tau)) +$$

$$+ \sum_{i=1}^{p} B(t, x^h(t)) r(t_i, x(t_i)) \delta_{t_i} \tag{5}$$

with the initial condition $x(t) = \varphi(t), \ \ t \in [t_0 - \tau, t_0]$. The Euler polygons will satisfy the equation

$$x^h(t) = \varphi(t_0) + \int_{t_0}^{t} f(\xi, x^h(\xi), x^h(\xi - \tau))\, d\xi +$$

$$+ \sum_{t_i < t} S(t_i, x^h(t_i), r(t_i, x(t_i))) \tag{6}$$

and the jump functions are defined by the equations:

$$S(t_i, x^h(t_i), r(t_i, x^h(t_i))) = z(1) - z(0), \tag{7}$$

$$\dot{z}(\xi) = B(t, z(\xi)) r(t_i, x^h(t_i))), \ \ z(0) = x^h(t_i). \tag{8}$$

The jump function $S(t, x, \mu)$ is the solution of the equation

$$\frac{\partial y}{\partial \mu} = B(t, y). \tag{9}$$

Assuming that equality

$$r(t, x(t) + S(t, x(t), r(t, x(t)))) = 0. \tag{10}$$

is correct

This equality means, after the action of an impulse at the system at time t, the phase $t, x(t)$ representation is on the manifold $r(t, x(t)) = 0$.

3 Properties impulse-sliding regime

Lemma 1. *Let for all value ranges t_1, t_2, x_1, x_2, y_1 and y_2 inequalities be carried out*

$$||f(t, x, y)|| \le C(1 + \sup_{[t_0 - \tau]} ||x(\cdot)||), \tag{11}$$

$$||S(t_1, x_1, r(t_1, x_1)) - S(t_2, x_2, r(t_2, x_2))||$$

$$\le L(|t_1 - t_2| + ||x_1 - x_2||). \tag{12}$$

Then, for all decompositions h and all $t \in [t_0, \vartheta]$ the set of Euler polygons $x^h(\cdot)$ is bounded, what means, that there exist constant M, that

$$||x^h(t)|| \le M. \tag{13}$$

Proof. Under (6) and (11) the following inequality holds

$$||x^h(t)|| \le ||\varphi(t_0)|| + C \int_{t_0}^{t} (1 + \sup_{[t_0 - \tau, \xi]} ||x^h(\cdot)||) +$$

$$+ \sum_{t_i < t} ||S(t_i, x^h(t_i), r(t_i, x^h(t_i))||. \tag{14}$$

Due to the fact that

$$S(t_{i-1}, x^h(t_{i-1} + 0), r(t_{i-1}, x^h(t_{i-1} + 0)) = 0,$$

in view of (12), we have chain of inequalities

$$||S(t_i, x^h(t_i), r(t_i, x^h(t_i))|| = ||S(t_i, x^h(t_i), r(t_i, x^h(t_i))|| -$$

$$- S(t_{i-1}, x^h(t_{i-1} + 0), r(t_{i-1}, x^h(t_{i-1} + 0)) \le$$

$$\le L(t_i - t_{i-1} + ||x^h(t_i) - x^h(t_{i-1} + 0)||) \tag{15}$$

At the same time, in view of (11),

$$||x^h(t_i) - x^h(t_{i-1}+0)|| \leq$$

$$\leq \int_{t_{i-1}}^{t_i} ||f(\xi, x^h(\xi), x^h(\xi-\tau))||d\xi \leq$$

$$\leq C(t_i - t_{i-1} + L\int_{t_{i-1}}^{t_i} (1 + \sup_{[t_0-\tau,\xi]} ||x^h(\cdot)||))d\xi \quad (16)$$

In consequence, from (14) in view of (15) and (16) the following inequality holds

$$||x^h(t)|| \leq ||\varphi(t_0)|| + (L+C)(t-t_0) +$$

$$+ L(1+C)\int_{t_0}^{t} (\sup_{[t_0-\tau,\xi]} ||x^h(\cdot)||))d\xi. \quad (17)$$

Similarly [Lukojanov, 2011], from the last inequality we get

$$\sup_{[t_0-\tau,t]} ||x^h(\cdot)|| \leq$$

$$\leq R + (L+C)(t-t_0) + L(1+C)\int_{t_0}^{t} \sup_{[t_0-\tau,\xi]} ||x^h(\cdot)||)d\xi \quad (18)$$

where

$$R = \sup_{[t_0-\tau,t_0]} ||\varphi(\cdot)||.$$

Applying the estimation for the solution of inequality from [Bellman and Cooke, 1963] for the inequality (18) we can write

$$\sup ||x^h(\cdot)|| \leq (R + (L+C)(\vartheta - t_0))e^{L(1+C)(\vartheta-t_0)},$$

which completes the proof of lemma 1.

Note that the constants M can take the value

$$M = (R + (L+C)(\vartheta - t_0))e^{L(1+C)(\vartheta-t_0)},$$

Let $D-$ bounded closed set which belong all items $x^h(\cdot)$. By continuity $f(t,x,y), B(t,x)$ and $r(t,x)$ are bounded.

Let us introduce the following notation

$$M_1 = \max_{[t_0,\vartheta]\times D\times D} ||f(t,x,y)||, M_2 = \max_{[t_0,\vartheta]\times D} ||B(t,x)||,$$

$$M_3 = \max_{[t_0,\vartheta]\times D} ||r(t,x)||. \quad (19)$$

Lemma 2. *Under the assumptions made above, from each confinal sequence of Euler polygons $x^h(\cdot)$ we can select a subsequence $x^{h_p}(\cdot)$, uniformly at $(t_0,\vartheta]$ converging to absolutely continuous function $x(\cdot)$. Moreover for all $t \in (t_0,\vartheta]$, $r(t,x(t)) = 0$ ($x(t) = \varphi(t)$ for $t \in [t_0-\tau,t_0]$), in other words, the limit element of the impulse-sliding regime moves over the manifold which is described by the equation $r(t,x(t)) = 0$.*

Proof. The proof of convergence of the network $x^h(\cdot)$ will use generalization Arcels lemma from [Filippov, 1971]. Let $x^{h_i}(\cdot)-$ be a confinally sequence. Then, according to the (6)

$$||x^{h_i}(t'') - x^{h_i}(t')|| \leq \int_{t'}^{t''} ||f(t, x^h(t), x^h(t-\tau)||ds +$$

$$+ \sum_{k=m(t')+1}^{m(t'')} ||S(t_k, x^{h_i}(t_k), r(t_k, x^{h_i}(t_k)))||, \quad (20)$$

where $m(t)-$ the nearest point at the left in partition which is produces of polygons $x^{h_i}(\cdot)$. In accordance to (6),

$$||S(t_k, x^{h_i}(t_k), r(t_k, x^h(t_k)))|| =$$

$$||S(t_k, x^h(t_k), r(t_k, x^h(t_k)))|| -$$

$$||S(t_{k-1}, x^{h_i}(t_{k-1}+0), r(t_{k-1}, x^{h_i}(t_{k-1}+0)))||$$

Considering (12), we get

$$||S(t_k, x^{h_i}(t_k), r(t_k, x^h(t_k)))|| \leq$$

$$L(t_k - t_{k-1} + ||x^{h_i}(t_k) - x^{h_i}(t_{k-1}+0)||)$$

At the same time

$$x^{h_i}(t_k) - x^{h_i}(t_{k-1}+0) = \int_{t_{k-1}}^{t_k} f(\xi, x^h(\xi))d\xi.$$

By taking into account (18), we obtain

$$||S(t_k, x^{h_i}(t_k), r(t_k, x^h(t_k)))|| \le L(t_k - t_{k-1}+$$

$$+M_1(t_k - t_{k-1})) = L(1 + M_1)(t_k - t_{k-1}) \quad (21)$$

From (20) and (21) it follows that

$$||x^{h_i}(t'') - x^{h_i}(t')|| \le (M_1 + L(1 + M_1))(t'' - t')+$$

$$+L(2 + M)(t' - t_{t_i h_i}) \quad (22)$$

where $t_{t_i h_i}-$ the nearest point at the left in partition h_i to the point t'. The last inequality allows to apply generalization Arcels lemma from [Filippov, 1971] and ensures the existence of a subsequence $x^{h_i}(\cdot)$ which uniformly converges to the function $x(\cdot)$.

Now, we pass to the limit in the inequality (22) at $i \to \infty$. As a result we have $||x(t'')-x(t')|| \le (M_1+L(1+M_1))(t'' - t')$.

This means that $x(t)$ is an absolutely continuous function at $(t_0, \vartheta]$.

Now show that the limit network element $x^h(\cdot)$ belongs to manifold $r(t, x) = 0$. Let $t_{m_t h_i}-$ be the nearest point at the left in partition h_i by the time t. The following inequality holds

$$||r(t, x(t)|| \le ||r(t, x(t))-r(t, x^{h_i}(t))+r(t, h^i(t))|| \le$$

$$\le ||r(t, x(t))-r(t, x^{h_i}(t))||+||r(t_{m_t h_i} x^{h_i}(t_{m_t h_i}+0))-$$

$$-r(t, x^{h_i}(t))|| \le L[||x(t) - x^{h_i}(t)|| + (t - t_{m_t h_i})+$$

$$+||x^{h_i}(t_{m_t h_i} + 0) - x^{h_i}(t)||] \le$$

$$\le L[||x(t) - x^{h_i}(t)|| + (L + M)(t - t_{m_i h_i})]$$

By the uniform convergence of a sequence $x^{h_i}(\cdot)$ the first term at right part at the last inequality tends to zero. The second tends to zero because of $i \to \infty$ $h_i \to 0$. This completes the proof of properties $r(t, x(t)) \equiv 0$ when $t \in (t_0, \vartheta]$.

Lemma 3. *Let $r(t, x)$ be continuously differentiable vector function on all variables. Then, the following equality holds*

$$S(t_k, x^h(t_k), r(t_k, x^h(t_k))) - S(t_{k-1}, x^h(t_{k-1} + 0),$$

$$r(t_{k-1}, x^h(t_{k-1}+0))) = \int_{t_{k-1}}^{t_k} \Bigg[\frac{\partial S(\xi, x^h(\xi), r(t, x^h(\xi)))}{\partial \xi}$$

$$+\frac{\partial S(\xi, x^h(\xi), r(\xi, x^h(\xi)))}{\partial x} f(\xi, x^h(\xi), x^h(\xi - \tau))+$$

$$+\frac{\partial S(\xi, x^h(\xi), r(\xi, x^h(\xi)))}{\partial r} \Bigg(\frac{\partial r(\xi, x^h(\xi))}{\partial \xi} +$$

$$+\frac{\partial r(\xi, x^h(\xi))}{\partial x} \dot{f}(\xi, x^h(\xi), x^h(\xi - \tau))\Bigg)\Bigg] \, d\xi. \quad (23)$$

The validity of the lemma follows from the differentiation formulas of composite function.

Theorem 1. *Let all the conditions given above hold. Then, an impulse-sliding regime on the set $(t_0, \vartheta]$ is described by the equation*

$$\dot{x}(t) = \frac{\partial S(t, x(t), r(t, x(t))))}{\partial t}+$$

$$+\frac{\partial S(t, x(t), r(t, x(t)))}{\partial r} \frac{\partial r(t, x(t))}{\partial t} + [E+$$

$$+\frac{\partial S(t, x(t), r(t, x(t)))}{\partial x} + \frac{\partial S(t, x(t), r(t, x(t)))}{\partial r}\times$$

$$\times\frac{\partial r(t, x(t)))}{\partial x}]f(t, x(t), x(t - \tau)), \quad (24)$$

$$x(t_0 + 0) = x(t_0) + S(t_0, x(t_0), r(t_0, x(t_0))).$$

Proof. According to (6) and Lemma 3, $x^h(t)$ satisfies equation

$$x^{h_i}(t) = \varphi(t_0) + \int_{t_0}^{t} f(\xi, x^{h_i}(\xi), x^{h_i}(\xi - \tau))d\xi+$$

$$+\int_{t_0}^{t_{mh_i}}[\frac{\partial S(\xi,x^{h_i}(\xi),r(t,x^{h_i}(\xi)))}{\partial \xi}+$$

$$+(\frac{\partial S(\xi,x^{h_i}(\xi),r(\xi,x^{h_i}(\xi)))}{\partial x}+$$

$$+\frac{\partial S(\xi,x^{h_i}(\xi),r(\xi,x^{h_i}(\xi)))}{\partial r}\times$$

$$\times\frac{\partial r(\xi,x^{h_i}(\xi))}{\partial x})f(\xi,x^{h_i}(\xi),x^{h_i}(\xi-\tau))+$$

$$+\frac{\partial S(\xi,x^{h_i}(\xi),r(\xi,x^{h_i}(\xi)))}{\partial r}\cdot\frac{\partial r(\xi,x^{h_i}(\xi))}{\partial \xi}$$

Passing to the limit at the last equation and bearing in mind that $x(t)$ is absolutely continuous function, we can see that the theorem is true.

4 Conclusion

The formalization of the impulse-sliding regime for a nonlinear system with time delay is made. The equation to describe the limiting element impulse-sliding regime is obtained.

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