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# EXPERIMENTAL RESEARCH OF CONSECUTIVE COMPENSATOR APPROACH ON BASIS OF MECHATRONIC SYSTEMS

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**Abstract**

An experimental research of consecutive compensator approach on basis of mechatronic systems is considered. It is assumed that the plant is linear time-varying systems with unknown fast-changing bounded parameters and affected by unknown bounded disturbance. The method assuring the solution of tracking problem with prescribed accuracy is represented.

**Key words**

Adaptive control, Decision feedback, Output control, Single-input/single-output systems, Linear time-varying systems, Mechatronic systems.

## 1 Introduction

This work deals with experimental research of consecutive compensator approach (Bobtsov and Nikolaev, 2005) on basis of mechatronic systems (Mechatronic Systems, 2001) concerning problem of analysis of robust and adaptive output control of indeterminate linear time-varying systems. In this paper a method of output control proposed in (Bobtsov and Nagovitsina, 2007) is considered. In paper (Bobtsov and Nagovitsina, 2007) an approach of output control of linear time-varying systems with unknown bounded parameters was presented. An assumption, that the plant is affected by unknown bounded disturbance, was proposed. The proposed method is based on the results published in (Bobtsov and Nikolaev, 2005;), where the problem of stabilization of nonlinear and time-varying system was considered. Thus the represented algorithms can be applied both for nonlinear and linear time-varying systems.

It is necessary to mention that this paper deals with experimental researches of consecutive compensator approach on basis of mechatronic system. The Mechatronics Control Kit (Mechatronic Systems, 2001) includes three ready to assemble objects, a DC-motor, a Pendubot, and an Inertia Wheel Pendulum. The digital electronics is fully integrated and includes Texas Instruments DSP development system, the TMS320C6711 DSK Board (a DSP board with parallel port interface), a PWM/Optical Encoder data Acquisition Daughter Board, a PWM amplifier, 5 Volt and 24 Volt DC power supplies and all required cables. Additional hardware includes a 24 Volt DC motor with 1000 counts/rev optical encoder. A second 1000 counts/rev optical encoder and aluminum links and mounts to construct the above experiments.

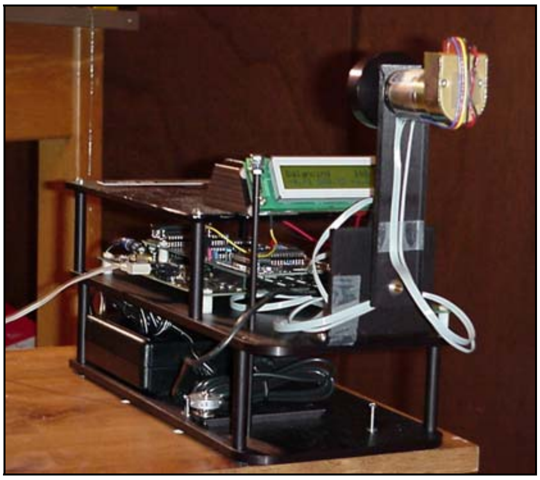

Fig.1 Mechatronics Control Kit

## 2 Problem statement

We consider linear time-varying system (Bobtsov and Nagovitsina, 2007)

$$\begin{cases} \dot{z} = Fz + L(u+w) + \theta(t)y(t-\tau), \\ y = Sz, \end{cases} \quad (1)$$

where $z(t) \in R^n$ is vector of state variables; $F$, $L$ and $S$ are $(n \times n)$, $(n \times 1)$ and $(1 \times n)$ unknown constant matrices; $\theta(t) \in R^n$ is vector of unknown time-varying parameters; $y(t) \in R$ is output variable; $w(t) \in R$ is bounded unknown disturbance.

Let us assume that only output variable is measured, but not its derivatives, the state $z(t)$ and disturbance $w(t)$ are not measured and parameters of vector $\theta(t) \in R^n$ are smooth and bounded functions. We also assume that transfer function $H(p) = S(pI - F)^{-1}L = \frac{b(p)}{a(p)}$ is minimum-phase, i.e. $b(p)$ is a Hurwitz polynomial.

Together with the plant we consider the command signal $y*$ which is measured and satisfies the condition

$$\left|\frac{d^i y*}{dt^i}\right| \le \overline{C} < \infty, \qquad (2)$$

where $i = \overline{0, \rho}$ and number $\rho = n - m$ (where $n$ and $m$ dimensions of $a(p)$ and $b(p)$ polynomials accordingly) is a transfer function $H(p) = \frac{b(p)}{a(p)}$ relative degree.

We define the purpose of control as the solution of the problem of synthesizing the algorithm which at any initial conditions ensures the boundedness of all system signals as well as the execution of purpose condition

$$|e(t)| < \Delta, \qquad (3)$$

for some $t \ge t_1$, where $e = y - y*$ is a tracking error, $\Delta$ is a number which can be decreased by control law selection.

We consider an additional purpose of control. We apply proposed algorithm for the mechatronic system in the tracing task. We use DC-Motor as a plant with rotation angle (output signal $y$) and various reference signals $y*$ (e.g. the second optical encoder) providing the control objective (3).

## 3 Control design

We rewrite the system (1) in the following form (Bobtsov and Nagovitsina, 2007)

$$\begin{cases} \dot{z} = Fz + L(u + w) + \sum_{i=1}^{n} D_i \theta_i(t) y(t - \tau), \\ y = Sz, \end{cases} \qquad (4)$$

where $D_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $D_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, …, $D_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$ – $(n \times 1)$ vectors; $\theta_1$, $\theta_2$, …, $\theta_n$ are components of the vector of unknown time-varying parameters $\theta(t) = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n-1} \\ \theta_n \end{bmatrix}$.

The state-space model (4) can be represented in the input-output form

$$\begin{aligned} y &= \frac{b(p)}{a(p)}(u + w) + \frac{c_1(p)}{a(p)}\theta_1(t) y(t - \tau) \\ &+ \frac{c_2(p)}{a(p)}\theta_2(t) y(t - \tau) + \ldots + \frac{c_n(p)}{a(p)}\theta_n(t) y(t - \tau) \\ &= \frac{b(p)}{a(p)}(u + w) + \sum_{i=1}^{n} \frac{c_i(p)}{a(p)}\theta_i(t) y(t - \tau), \end{aligned} \qquad (5)$$

where $p = \frac{d}{dt}$ is differentiation operator, transfer function $\frac{c_i(p)}{a(p)} = S(pI - F)^{-1} D_i$.

Choose the control law of the following form (Bobtsov and Nagovitsina, 2007)

$$u = -\phi(p)(k + \lambda)\hat{e}, \qquad (6)$$

where $k$ is a positive number; the positive parameter $\lambda$ is intended for compensation of the uncertainties $\sum_{i=1}^{n} D_i \theta_i(t) y(t - \tau)$ and $w(t)$; polynomial $\phi(p)$ is chosen for the polynomial $\beta(p) = \phi(p) b(p)$ to be Hurwitz and $(n-1)$ order; function $\hat{e}(t)$ is the estimate of signal $e(t) = y(t) - y*(t)$ which is calculated according to the following algorithm

$$\begin{cases} \dot{\xi}_1 = \sigma \xi_2, \\ \dot{\xi}_2 = \sigma \xi_3, \\ \ldots \\ \dot{\xi}_{\rho-1} = \sigma(-k_1 \xi_1 - \ldots - k_{\rho-1} \xi_{\rho-1} + k_1 e), \end{cases} \qquad (7)$$

$$\hat{e} = \xi_1, \qquad (8)$$

where number $\sigma > k + \lambda$ (calculation procedure of $\sigma$ is presented in (Bobtsov and Nagovitsina, 2007)), and parameters $k_i$ are calculated for the system (7), (8) to be asymptotically stable for input $e = 0$.

## 4 Experimental researches

In this part we consider realization of proposed approach for Mechatronic Control Kit. DC-motor is used as plant and it can be described by the following model:

$$\begin{cases} \dot{\omega}(t) = -a\cdot\omega(t) + b\cdot u(t), \\ \dot{\psi}(t) = \omega(t), \\ y(t) = \psi(t). \end{cases} \quad , \qquad (9)$$

where $y(t)$ is rotation angle (output signal), $\omega(t)$ is rate of turn, $u(t)$ is value of voltage applying to motor (input signal), $a, b > 0$ are unknown time-varying parameters. Furthermore the control channel has saturation and dead zone. So, the control signal belongs to the range $[-10; 10]$ where interval $[-D; D]$ is dead zone and $D = 1{,}5$.

Choose the control law based on (6) with some additional changes for input constraints rejection

$$u = \begin{cases} -10, \, if \;\; \bar{u} < -10, \\ \bar{u}, \, if \;\; -10 \le \bar{u} \le 10, \\ 10, \, if \;\; \bar{u} > 10. \end{cases} \qquad (10)$$

where $\bar{u}(t)$ is calculated control value, consisting of two parts

$$\bar{u}(t) = \bar{u}_c(t) + \bar{u}_d(t) \qquad (11)$$

$\bar{u}_c(t)$ is signal from consecutive compensator

$$\begin{cases} \bar{u}_c(t) = (p+1)\, k \cdot \hat{e}(t), \\ \dot{\xi} = \sigma(-\xi + e), \\ \hat{e} = \xi. \end{cases} \qquad (12)$$

and $\bar{u}_d(t)$ is dead zone compensation

$$\bar{u}_d(t) = D \cdot sign(\bar{u}_c(t)) \,. \qquad (13)$$

For switching minimization in $\bar{u}_d(t)$ component we choose this control component in the following form

$$\bar{u}_d(t) = \begin{cases} -D, \, if \;\; \bar{u}_c < -\delta, \\ D, \, if \;\; \bar{u}_c > \delta, \end{cases} \; , \qquad (14)$$

where $\delta$ is a small number defining from given precision (e.g. $\delta = 0{,}01$).

The results of experiment for motor with control parameters $k = 1$, $\sigma = 100$, $\delta = 0{,}01$ are presented in Fig. 2 – 7. The results of experiment with parameters $k = 0{,}1$, $\sigma = 10$, $\delta = 0{,}01$ are presented in Fig. 8 – 9.

In Fig. 10 mechatronic system consisting of encoder as reference source and motor as the plant is shown. An angle is referenced by the encoder and provided by the motor with allowable established error.

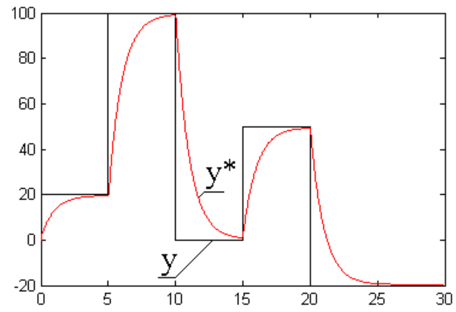


Fig. 2 Transients in control system (9) – (14) for variables $y(t)$ and stepped reference $y^*(t)$

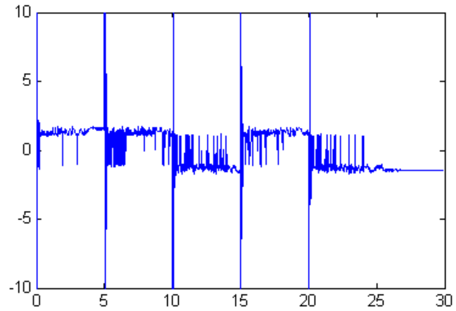


Fig. 3 Transients in control system (9) – (14) for variable $u(t)$ when $y^*(t)$ is stepped

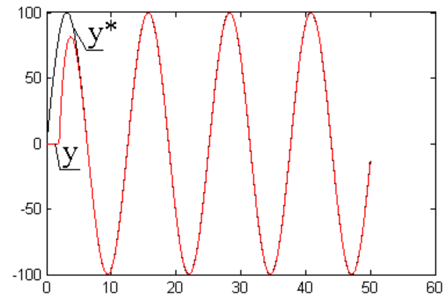


Fig. 4 Transients in control system (9) – (14) for variable $u(t)$ when $y^*(t)$ is stepped

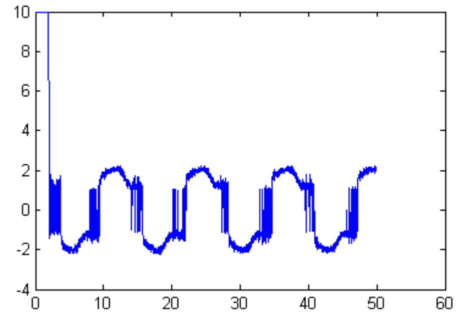


Fig. 5 Transients in control system (9) – (14) for variable $u(t)$ when $y^*(t)$ is sinusoidal

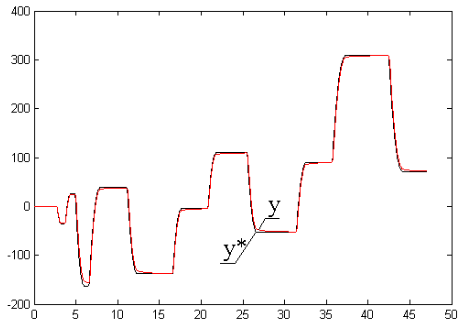


Fig. 6 Transients in control system (9) – (14) for variables $y(t)$ and $y^*(t)$ manually referenced by encoder

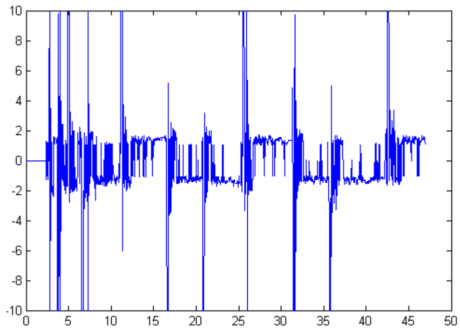

Fig. 7 Transients in control system (9) – (14) for variable $u(t)$ when $y^*(t)$ is manually referenced by encoder

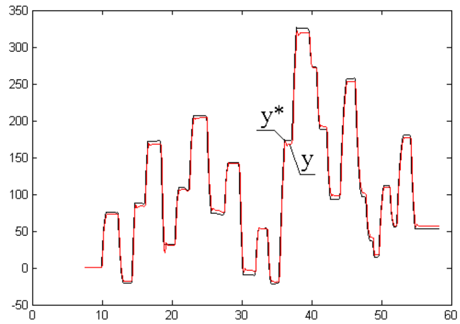


Fig. 8 Transients in control system (9) – (14) for variables $y(t)$ and $y^*(t)$ is manually referenced by encoder

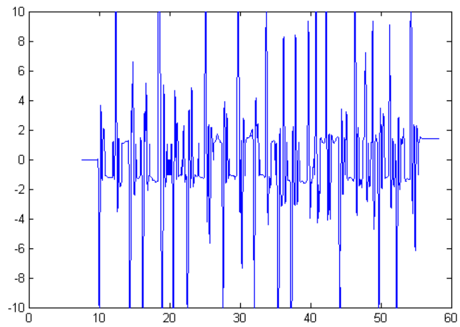

Fig. 9 Transients in control system (9) – (14) for variable $u(t)$ when $y^*(t)$ is manually referenced by encoder

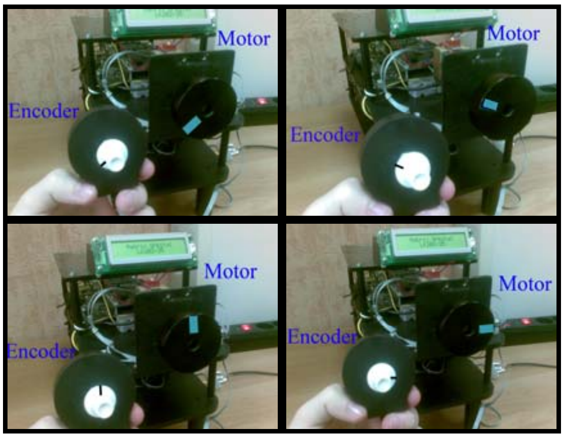


Fig. 10 Mechatronics system consisting of encoder as angle reference source and motor as plant

It is important to emphasize that consecutive compensator approach has been developed for very hard tasks such as control of nonlinear, time-varying linear, delayed systems with unknown parameters with unmeasured disturbance. At the same time this scheme is very easy for practical use. It is explained by very small dimension (it is equal to relative degree of plant).

Later listing of program used in mentioned above experiment is presented to demonstrate approach simplicity:

Initialization of parameters (one part of program):

```
k = 1;
sigma = 100;
D = 1.5;
delta = 0.01;
ksi[2]={0, 0};
Sample_Time=0,005.
```

Controller (other part of program, run with fixed step)

```
read_encoders((float*)&enc1, (float*)&enc2);
Command = enc1;
Feedback = enc2;
Error = Command - Feedback;
ksi[0] = ksi[1];
ksi[1] = ksi[0] + Sample_Time*sigma*(Error -
ksi[0]);
u=k*(ksi[0] + sigma*(Error - ksi[0]));
if (u >  delta)  u = u + D;
if (u < -delta) u = u - D;
if (u > 10.0) u = 10.0;
if (u < -10.0) u = -10.0;
out_control(u).
```

## 5 Conclusion

In this work the experimental research of consecutive compensator approach on basis of mechatronic systems has been demonstrated. An essential simplicity of this approach realization has been shown.

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