SURVEY OF ENCODING TECHNIQUES FOR QUANTUM MACHINE LEARNING

Siddhartha Sharma
Department of Computer Science
Indian Institute of Information Technology, Tiruchirappalli
India
siddharthasharma3110@gmail.com

Renugadevi N
Department of Computer Science
Tiruchirappalli
India
renugadevin@iiitt.ac.in

Article history:
Received 27.07.2024, Accepted 16.09.2024

Abstract
Quantum computing is a field of computation that processes information in a fundamentally different way compared to classical computers. Quantum computing is a rapidly expanding research field with ongoing investigations regarding applications of quantum computing in widespread domains such as cryptography, artificial intelligence, communications, etc. The core focus of this paper is quantum encoding which is a crucial branch of quantum computing. Quantum encoding involves mapping classical data into quantum states. This paper provides an in-depth analysis of prominent quantum encoding techniques with their strengths and weaknesses. Furthermore, this paper suggests development of hybrid encoding methods capable of emulating the concept of Euclidean distance by integrating amplitude and angle encoding.

Key words
Quantum Encoding, Angle Encoding, Amplitude Encoding, Basis Encoding, Quantum Computing, Hilbert Space.

1 Introduction

Quantum computing combines quantum mechanics, information theory, and aspects of computer science. The field of quantum information promises secure data transfer, dramatic increase in computing speed.[Ray, 2011] It is a novel method that uses the principles of quantum mechanics to handle highly challenging situations in a very short amount of time [Gill and Buyya, 2024]

Quantum machine learning represents a highly promising realm in contemporary physics and computer science research, with far-reaching implications spanning quantum chemistry[Peruzzo et al., 2014], artificial intelligence [Liu et al., 2024], and even high-energy physics [Andreassen et al., 2019]. It can be defined as the convergence of quantum computing and machine learning, wherein machine learning algorithms are executed on quantum devices[Wang and Liu, 2024]

Quantum encoding techniques play an essential role in quantum computing as they deal with accurate depiction of classical data in the Hilbert Space, which is the basis for quantum operations. Quantum encoding is used for Quantum machine learning[Srinivasan et al., 2018][Schuld et al., 2016][Biamonte et al., 2017][Orús et al., 2019][Ciliberto et al., 2018][Schuld et al., 2020], Quantum cryptography[Grover, 1996][Grover, 1997][Long et al., 1999], Quantum Optimization[Botsinis et al., 2013][Botsinis et al., 2015][Romero-Meléndez and González-Santos, 2017][Henao et al., 2015],Quantum Algorithms[Braine et al., 2021][Watanabe et al., 2023][Fuller et al., 2024][Liu et al., 2022b][O'Dwyer Boyle and Nikandish, 2024], etc. Encoding techniques like angle encoding, amplitude encoding and basis encoding are widely used for embedding classical data into quantum systems, allowing quantum algorithm to process classic data efficiently.

In quantum computer the fundamental unit of information is referred to as 'qubit'. The qubit is not binary (like classical bit) but quaternary in nature. A qubit can exist not only in a state corresponding to the logical state 0 or 1 as in a classical state bit but also in states corresponding to a blend of superposition of those classical states.

Quantum encoding techniques[Ding et al., 2024][Yano et al., 2021][Cheng et al., 2021] generally center around manipulating the properties of a qubit such as orientation[Rath and Date, 2023] and magnitude[Nakaji et al., 2022], etc.Encoding methods also use qubits to store bits representing classical data[Weigold et al., 2021]. However, simple quantum encoding techniques such as angle encoding, amplitude encoding and basis encoding experience performance degradation when the size and complexity of the classical dataset increases[Pande, 2024].

To achieve a better representation of classical data in the Hilbert space, some authors such as Abrar et al.[Abrar-Ul-Haq, 2020], Bhabhatsatam et al.[Bhabhatsatam, 2023] among others formulated new quantum encoding techniques. These new techniques,[Abrar-Ul-Haq, 2020], [Bhabhatsatam, 2023] are combinations of various simple quantum encoding strategies like basis encoding, angle encoding, etc. The new methods work well for datasets of data elements within the range of zero to one. However, these new techniques have higher time complexities compared to simple encoding methods as they require knowledge of data before processing.

Quantum fidelity[L'Abbate et al., 2024] is a measure of similarity between two quantum states[Nielsen and Chuang, 2010] and is estimated by methods such as SWAP test[Liu et al., 2022a], Quantum state tomography[Innan et al., 2024], direct fidelity estimation[Wang et al., 2023], etc. In the classical domain, Euclidean distance is used to measure the similarity between two data elements[Abrar-Ul-Haq, 2020].

This paper aims for the following:

- Provide an in-depth analysis of prevalent quantum encoding techniques, highlighting their advantages and disadvantages
- Propose a novel encoding technique that facilitates complex data representation with low time complexity and reduced computational requirements.

The rest of the paper is structured as follows.

- Section II provides a brief overview of basic quantum computing techniques.
- Section III provides a literature review detailing popular quantum encoding techniques along with their advantages and disadvantages.
- Section IV contrasts the encoding methods discussed in Section III.
- Section V suggests a hybrid encoding method capable of emulating the concept of Euclidean distance.

2 Preliminaries

Comparative performance of the different quantum encoding techniques under varying conditions remains an open area that warrants further research. This section aims to provide a brief review of the relative strengths and weaknesses of basic quantum encoding techniques.

Hilbert spaces are the closest generalization to infinite dimensional spaces of the Euclidean spaces [Loaiza, 2017].Hilbert Space is a significant component of Quantum Mechanics and it can be denoted as the complete space of inner product. Hilbert space can play a central role in order to determine the interpretation of the wave function.[Das and Islam, 2021]

Basis Encoding

Basis Encoding is a fundamental quantum encoding method that direct maps classical bits to qubits. The mathematical representation of basis encoding is as follows[Khan et al., 2024]:

$$X \approx \sum_{i=-k}^{m} b_i 2^i \rightarrow |b_m ... b_{-k}| \tag{1}$$

Where,

X : numeric input data,
m : original bit string length,
k : the degree of precision.

For Example, numerical data like '12' is initially converted to its binary equivalent 1100_2. The classical binary equivalent is embedded in Hilbert space.

$$1100_2 \rightarrow q_0 q_1 q_2 q_3$$

Where,

q_0 corresponds to $|1\rangle$,
q_1 corresponds to $|1\rangle$,
q_2 corresponds to $|0\rangle$,
q_3 corresponds to $|0\rangle$.

Basis encoding is used for quantum algorithms[Collins et al., 1998], [Shor, 1997], quantum cryptography[Li et al., 2016], quantum machine learning[Bhabhatsatam, 2023] and other fields related to quantum computing.

The high degree of precision, simplicity and ease of implementation are major advantages of basis encoding. However, basis encoding can only embed simple and discrete data elements. This limitation arises as each qubit stores a single bit of data, which does not leverage the sparsity of the data. Other encoding methods such as amplitude encoding, angle encoding, QRAM(Quantum random access memory)[Weigold et al., 2021], dense angle encoding[LaRose and Coyle, 2020] etc are better for compact and resource-efficient data representation.This advantage arisis because methods like dense angle encoding and QRAM exploit quantum properties such as relative phase degree of freedom and superposition of states respectively[Jaques and Rattew, 2023].Formulae for QRAM and Dense angle are mentioned below:

$$X \xrightarrow{qram} \sum_{n=0}^{n-1} \frac{1}{\sqrt{n}} |x_i\rangle$$

$$|x\rangle \xrightarrow{dense}$$

$$\bigotimes_{i=1}^{\lfloor N/2 \rfloor} \left(\cos(2\pi x_{2i-1})|0\rangle + e^{2\pi i x_{2i}} \sin(2\pi x_{2i-1})|1\rangle \right. \quad (2)$$

Where,

x_i : denotes i-th basis states

Amplitude Encoding
Amplitude encoding is a technique that embeds classical data into probability amplitudes of a quantum state[Schuld and Petruccione, 2018]. This is achieved by normalizing features of classical data points and then representing these normalized features as amplitudes of basis states. Mathematically, amplitude encoding can be represented as[Mashtura et al., 2023] :

$$|\psi_s\rangle = \sum_{i=1}^{2^n} c_i |i\rangle \quad (3)$$

Where,

$|\psi_s\rangle$ denotes the prepared quantum state from a data point having 2^n dimensions,
$|i\rangle$ denotes i-th computational basis state,
c_i is the i-th element of data point C.

Consider a 2D classical cartesian coordinate C = (1,2). After normalization, the point transforms to $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$ and will be embedded as follows:

$$(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}) \xrightarrow{Embedding} \frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$$

Amplitude encoding is used in quantum neural networks[Chalumuri et al., 2021], [Silver et al., 2022][Oliveira et al., 2008], quantum support vector machines(QSVM)[Nakaji et al., 2022], quantum data compression[Majji et al., 2023], and other quantum computing applications. The advantage of Amplitude encoding is that it can represent compact data with space complexity of $log_2 n$. Further, robust scalability is another advantage of amplitude encoding, which makes this method ideal for handling big data. However, there are challenges associated with amplitude encoding, such as noise susceptibility, need for normalization, lack of fault tolerance, etc hamper the accuracy of amplitude encoding. For example,(10,20) and (1,2), which are distinct points on a cartesian plane are encoded as $\frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$ in amplitude encoding.
Other encoding methods like angle encoding, dense angle encoding, etc were developed to understand the relationship between features of a datapoint.

Angle Encoding
In angle encoding, classical data is embedded into rotation angles of a qubit using quantum rotation gates[Shi et al., 2024]. Mathematically angle encoding can be formulated as[LaRose and Coyle, 2020]:

$$|x\rangle = \otimes_{i=1}^{[N/2]} cos(\pi x_{2i-1})|0\rangle + e^{2\pi i x_{2i}} sin(\pi x_{2i-1})|1\rangle \quad (4)$$

Where,
x denotes feature vector x = $[x_1, ...x_n]^T \in R^N$.

We can also use simple linear mapping functions to achieve angle encoding.For example: To encode numerical value $C = \frac{1}{2}$ with the mapping function $\theta = \pi x$, C would be encoded as:

$$\theta_c = \pi \mathrm{x} \frac{1}{2} = \frac{\pi}{2}$$

$$|C\rangle = cos(\frac{\theta}{2})|0\rangle + sin(\frac{\theta}{2})|1\rangle$$

$$|C\rangle = cos(\frac{\pi}{4})|0\rangle + sin(\frac{\pi}{4})|1\rangle$$

$$|C\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|0\rangle$$

Angle encoding is used for quantum-inspired tensor networks[Stoudenmire and Schwab, 2017], supervised and unsupervised machine learning, Quantum neural networks[Landman, 2021] et al. Advantages of angle encoding include simple state preparation, the ability to capture relations between attributes of a data point, simple circuit implementation among others. Nonetheless, problems such as measurement overhead, limited suitability, scalability issues, etc limit the effectiveness of angle encoding. The major disadvantage of angle encoding is configuring an appropriate mapping function as the mapping function should fit all data values within the range of 0 to π. Configuring such mapping functions requires complete knowledge of data before processing.

To overcome the issues faced with simple quantum encoding methods, new quantum encoding methods[Mashtura et al., 2023] were developed by combining multiple simple encoding methods. These new methods not only resolved the difficulties of individual simple encoding methods, but also amplified their advantages. Other alternate quantum encoding schemes were also developed in parallel which utilize concepts such as fourier transforms[Musk, 2020][Tan et al., 2015], quantum annealing[Mahmud et al., 2022], [Havlíček et al., 2019],[Bar et al., 2023], etc.

3 Encoding Techniques for Quantum Machine Learning

This section covers research papers on quantum encoding methods and outlines the benefits and drawbacks of the mentioned techniques.

To provide analysis of errors at readout in IBMQ's Bogota device, Quiroga et al.[Quiroga et al., 2021] implemented a quantum machine learning model based on the k-means algorithm. This paper recommends the use of either amplitude or angle encoding to embed classical data into Hilbert Space. For angle encoding, this paper applied the following formula:

$$\theta = arctan\tfrac{a_1}{a_0} \tag{5}$$

for a 2D input vector, $a = (a_0, a_1)$.

The mapping function used in this paper is flexible as the range of arctan is $(-\infty, \infty)$. However, with the increase in values, the gradient of the arctan function becomes almost zero. Another disadvantage of angle encoding is that it fails to extract knowledge about magnitudes. Hence this encoding function cannot be used for data with attributes of different magnitudes. An example of the same is illustrated below.

$$P_i = (0.1, 2.5)\ P_j = (10, 300)\ P_k = (0.2, 6)$$

$$\theta_{Pi} = arctan(\frac{2.5}{0.1}) = 1.53\ radians$$

$$\theta_{Pj} = arctan(\frac{300}{10}) = 1.53\ radians$$

$$\theta_{Pk} = arctan(\frac{6}{0.2}) = 1.53\ radians$$

Although the data points are distinct, despite that they are encoded with the same θ value

Modi et al.[Modi et al., 2023] applied the quantum k-means clustering algorithm to decode M- Quadrature Amplitude Modulation (M-QAM) signals. This paper uses angle encoding with the following mapping function:

$$x_i^{"} = \frac{\pi}{2}(x_i^{'} + 1) \qquad y_i^{"} = \frac{\pi}{2}(y_i^{'} + 1) \tag{6}$$

Where, $(x^{'}, y^{'})_i = \frac{(x,y)_i}{r_{max}}$,

$r_{max} = max_i\sqrt{x_i^2 + y_i^2}$.

Due to normalization, this encoding method can capture magnitudes of attributes belonging to a data point. Also, this method extracts relationships between magnitudes of disparate data points. As a result, this encoding method achieves an average accuracy of about 95 percent. Nevertheless, this encoding method is unsuitable for large and multidimensional datasets due to the high time complexity of normalization.

Abrar et al.[Abrar-Ul-Haq, 2020] implemented quantum clustering to classify cells into different types of cancer. The encoding method followed in this paper uses normalized data for angle encoding with the mathematical formula :

$$x^{'} \leftarrow \frac{x}{max(x^{\rightarrow})} \tag{7}$$

For two normalized features $x_0^{'}$ and $x_1^{'}$, Mapping will be as follows:

$$\theta = (x_0^{'} + 1)\frac{\pi}{2}$$

$$\phi = (x_1^{'} + 1)\frac{\pi}{2}$$

In this encoding method, the normalization is achieved using a simpler formula than the one used by Modi et al.[Modi et al., 2023]. As a result, this method captures magnitudes with a simple circuit design. However, this encoding method requires normalized data similar to the encoding scheme implemented by Modi et al.[Modi et al., 2023]. Hence, this method is not suitable for processing large and multidimensional datasets.

Bhabhatsatam et al.[Bhabhatsatam, 2023] introduced the Bit-Partition hybrid quantum encoding method to efficiently store and process classical data in quantum systems. This hybrid method combines amplitude and basis encoding. For the problem statement defined in this paper, the encoding was achieved using the following formulae:

$$|\psi_x\rangle = \sqrt{x_{norm}}|0\rangle + \sqrt{1 - x_{norm}}|1\rangle$$

$$|\psi_b\rangle = |b\rangle$$

$$|\psi_{XB}\rangle = |\psi_x\rangle \otimes |\psi_b\rangle \tag{8}$$

Where,

S = [x, b],
x is an integer in the range 0 to N-1,
$x_{norm} = \frac{x}{N-1}$,
b is either 0 or 1.

This hybrid encoding method not only amplifies the advantages but also diminishes the issues of amplitude and basis encoding. Normalized amplitude encoding captures the information regarding the magnitude of attributes associated with a data point. Basis encoding enables this hybrid encoding method to understand finer details of attributes belonging to data points. Hence, this encoding method can represent high dimensional continuous data robustly. Nonetheless, this method suffers from high space complexity due to the integration of basis encoding in the hybrid encoding scheme. As a result, this hybrid encoding method has limited applicability.

Mashtura et al.[Mashtura et al., 2023] proposed a new quantum encoding structure that combines angle encoding and amplitude encoding in parallel. The parallel utilization of angle and amplitude encoding results in better information extraction about the magnitude and relation between attributes of a data point. As a result, this encoding structure performs slightly better than amplitude and angle encoding. Nevertheless, the better performance of this method comes at the cost of higher circuit complexity. This encoding structure requires as many qubits as the sum of qubits required for amplitude and angle encoding. The time complexity of this encoding structure is similar to the time complexity of angle and amplitude encoding. Due to these limitations, only small quantum neural networks can be designed with this encoding methodology.

Other encoding methods utilize quantum circuits such as Instantaneous Quantum Polynomial Circuit[Kyriienko and Magnusson, 2022], ZFeatureMap, ZZFeatureMap, PauliFeatureMap and manually configured Feature Maps[Umeano and Kyriienko, 2024] etc. However, such approaches suffer issues such as limitation in expressivity, Limited Scalability, Noise and error sensitivity, etc when compared to simple encoding methods like basis, amplitude and angle encoding.

4 Challenges and Future Discussion

A major bottleneck experienced by quantum encoding methods is normalization.Normalization increases the time complexity of the encoding method as it requires scanning of the entire dataset. However, for the current encoding methods normalization of dataset is needed for extracting information regarding the magnitudes.
To mitigate the normalization requirement, quantum encoding methods should attempt to emulate the concept of Euclidean distance. For this, encoding should be done relative to classical data points in consideration. Relative encoding has no requirements of normalization and also provides flexibility in the representation of data points. For example:

$$C_i = 15, \quad C_j = 10 \quad C_k = 25$$

$$\theta_{pq} = atan(\frac{C_p}{C_q})$$

$$\theta_{ij} = atan(\frac{15}{10}) = 0.98 \text{ radians}$$

$$\theta_{ik} = atan(\frac{15}{25}) = 0.54 \text{ radians}$$

Such encoding methods can be developed independent of the dataset. The development of hybrid encoding schemes combining amplitude and angle encoding is an avenue to achieve relative encoding. Integrated usage of amplitude and angle encoding will enable the comparison of magnitudes and ratios for data points under scrutiny.

5 Conclusion

Quantum computing is a nascent and rapidly evolving field that plays a significant role in determining the success rate of any quantum algorithm. Quantum encoding techniques are an integral part of all quantum machine learning algorithms. This research paper has reviewed major quantum encoding techniques highlighting their advantages and disadvantages. It has explored the potential of hybrid encoding by discussing significant research works in the field. This paper suggests the integration of amplitude and angle encoding technique to enhance the overall performance. Continued research and innovation are essential for unlocking the full quantum advantage and realizing the potential of quantum computers and algorithms.

Table 1: Analysis of Quantum Encoding algorithms

Ref	Aim	Algorithm/s	Advantages	Disadvantages
[Abrar-Ul-Haq, 2020]	Implementing quantum clustering technique for classification of cancer cells	Amplitude Encoding $$\theta = (x_0^{\mid} + 1)\frac{\pi}{2}$$ $$\phi = (x_1^{\mid} + 1)\frac{\pi}{2}$$ Where, $$x_0^{\mid} = \frac{x_0}{Max(x_0)}$$ $$x_1^{\mid} = \frac{x_1}{Max(x_0)}$$ For (x0, x1) in a dataset containing n such data items	•High Classification accuracy. •Requires fewer iterations than its classical counterpart	•As the encoding method requires Normalization, leads to higher space complexity of O(n). •Suffers from issues such as precision and noise sensitivity
[Bar et al., 2023]	Suggests hybrid quantum classical approach for utilizing quantum computers for image classification	Multiple angle encoding	•Outperforms pre-existing methods in terms of accuracy •Provides a general model for image classification	Dynamic kernel size of the technique performs poorly for images with greater information.
[Bhabhatsatam, 2023]	Applies Bit-Partition hybrid quantum encoding methods to store and process classical data in quantum systems efficiently	Hybrid Quantum Encoding • Basis Encoding • Amplitude Encoding $$x_{norm} = \frac{x}{N-1}$$ $$b_{norm} = b$$ For (x0, x1) in a dataset containing n such data items	•Outperforms constituent encoding schemes •Balances the strengths and weaknesses of constituent encoding schemes	•Comparatively more space intensive due to the incorporation of basis encoding in the hybrid encoding method •Performance suffers on increase in information and magnitudes of attributes and datapoints.
[Mahmud et al., 2022]	Proposes time-efficient methods for C2Q data encoding and Q2C data decoding for quantum algorithms	Quantum Wavelet transform	•Improved time efficiency •Competitive with the state-of-the-art model with optimized circuit depth	•Higher complexity when compared with encoding strategies such as the one implemented by Quiroga et al. [Quiroga et al., 2021] •Fidelity suffers with an increase in information about the image/data

[Mashtura et al., 2023]	Proposes architecture for classification of classical data	Parallel utilization of angle and amplitude encoding	•Higher Accuracy •Leverages combined strength of several encoding techniques	•High Complexity •Encoding method not very robust •Specific to certain types of data
[Modi et al., 2023]	Implementation of quantum K means clustering	Angle encoding	•Simple Encoding strategy with high accuracy •Lower space complexity when compared to encoding strategies in similar domain	•Time complexity is high due to the requirement of scanning through the whole database before setting the parameters for encoding
[Quiroga et al., 2021]	Implementation of quantum k means clustering to discriminate quantum states at readout	Angle Encoding $\theta = arctan(\frac{a_1}{a_0})$ For (a1, a0) Amplitude Encoding $a^{\vert}_n = \frac{a_n}{\vert\vert a\vert\vert}$ For $a_n = (a_0, a_1, \ldots a_k)$	•Few of the highest accuracy rates among quantum encoding algorithms •Simpler than prevalent algorithms •Independent of the dataset	Failure rate increases briskly when the magnitude of the data increases.

From this study and Table 1, it is inferred that, while basis encoding is simple and effective, it is resource-intensive. Amplitude encoding is a powerful method but requires normalization and fails to capture relationships between attributes within a dataset. Angle encoding is compact and versatile, though it comes with complexities and resource overheads. Hybrid encoding outperforms the existing methods by combining the strengths of simple quantum encoding schemes. The implementation of hybrid encoding techniques, specifically, the integration of amplitude and angle encoding can enhance the performance.

6 Acknowledgements

The authors would like to thank and express our sincere gratitude to the editors and anonymous reviewers for their meticulous review and their helpful feedback, which greatly improved the quality of this paper.

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