ROBUST MODEL PREDICTIVE CONTROL OF DISCRETE-TIME SWITCHED SYSTEMS

Patrizio Colaneri * **Riccardo Scattolini** *

* *Dipartimento di Elettronica e Informazione*
Politecnico di Milano
Piazza Leonardo da Vinci 32, 20133 Milano, Italy
e-mail: colaneri@elet.polimi.it, scattoli@elet.polimi.it

Abstract: This paper proposes a model predictive approach for H_∞ control of switched systems in discrete-time. A finite horizon dynamic game is set up for the computation of the control input and a switching strategy guaranteeing robustness at the face of bounded disturbances. A main point investigated in the paper is the existence of an auxiliary law useful to activate the predictive algorithm.

Keywords: Model predictive control, Switched systems, Riccati inequality.

1. INTRODUCTION

In a discrete-time setting, this paper tackles the problem of designing a control law for switched systems. To be precise, given a switched system, the problem consists in finding a control strategy and a switching signal that ensure the internal stability of the closed-loop system meanwhile ensuring a given attenuation level of the closed-loop input-output map.

Stability analysis of continuous time switched linear systems has been addressed by several authors, (Branicky, 1998), (Hespanha, 2004), (Ye et al., 1998), (Johansson, 1998), (Liberzon, 2003) and (Hockerman-Frommer et al., 1998) among others. On the contrary, discrete-time switched systems are less studied, since many possible interesting phenomena are not encountered, namely Zeno or chattering behaviors, see (Daafouz and Bernussou, 2001), (Xie et al., 2003), (Zhai, 2001). In this paper we take advantage of the result achieved in (Geromel and Colaneri, 2006) where a stabilizing switching strategy is devised based on coupled Lyapunov inequalities. In the present paper, these inequalities are given a Riccati-type formulation that permits the formulation and the solution of the H_∞ attenuation problem for switched systems.

On the other hand, the presence of constraints in the state and input of the system enforces the use of Model Predictive Control (MPC) based on the Receding Horizon (RH) paradigm, see e.g. the survey papers (Mayne et al., 2000) and (De Nicolao et al., 2000) on stabilizing MPC with constraints. The robustness properties inherent to stabilizing MPC algorithms designed for unperturbed systems, as well as the main approaches followed so far to develop new MPC methods with enhanced robustness characteristics, have been recently discussed in (Skokaert and Maine, 2004), (Magni and Scattolini, 2005), (Magni et al., 2006). In this paper, the H_∞ framework already studied in (Magni et al., 2003) is considered to design a stabilizing state-feedback H_∞-RH control law for constrained switched systems. The results of the first part of the paper, i.e. the design of a stabilizing switched control law for unconstrained systems based on a Riccati-type formulation, are then used to complete the MPC design coping with state and control constraints.

2. PRELIMINARIES

Consider the switched system

$$x(t^+) = A_{\sigma(t)}x(t) + B_{1\sigma(t)}w(t) + B_{2\sigma(t)}u(t) \quad (1a)$$
$$z(t) = C_{\sigma(t)}x(t) + D_{1\sigma(t)}w(t) + D_{2\sigma(t)}u(t) \quad (1b)$$

where $t \in Z$, $x(t) \in R^n$, $w(t) \in R^{m_1}$, $u(t) \in R^{m_2}$, $t^+ = t+1$ and, finally, $\sigma(t) \in \{1, 2, \cdots, M\}$ is an integer that selects the active model at time t between M given ones.

In order to adopt a model predictive viewpoint, we assume that the state variable x and the input variable u, at each instant of time, must fulfill the following constraints

$$x(t) \in X, \quad u(t) \in U \quad (2)$$

where X and U are two subsets of R^n and R^{m_2}, both containing the origin as interior point.

The problem is to determine a MPC state-feedback law and a switching policy

$$u(t) = \bar{k}^o(x(t)), \quad \xi(t) = \bar{\xi}^o(x(t)) \quad (3)$$

in such a way that the closed loop system with input w and output z has a finite L_2 gain, bounded by a positive attenuation level γ. Internal stability is then achieved for any disturbance w satisfying

$$\|w(t)\|^2 \le \gamma_d^2 \|z(t)\|^2 \quad (4)$$

with $\gamma\gamma_d < 1$. The set of admissible signals w satisfying (4) is denoted by $\mathcal{W}$.

The problem above stated is very complicate *per se* and becomes extremely demanding in view of the hard constraints (2). To avoid trivial solutions, we assume that the switched pair (A_σ, C_σ) is observable. This means that there not exist a initial state and a switching strategy resulting in a null free output.

3. AUXILIARY LAW

In order to extend the rationale underlying the model predictive control technique, we will discuss the existence of an auxiliary control law for the unconstrained H_∞ problem of switched systems. Consider a positive number γ and the coupled matrix inequalities, $\forall i, j$

$$P_i > A_i' P_i A_i + \sum_{j=1}^{M} \lambda_{ij} P_j + S_i L_i S_i' + C_i' C_i \quad (5a)$$
$$0 < \gamma^2 I - B_{1i} P_i B_{1i}' - D_{1i}' D_{1i} \quad (5b)$$
$$S_i = \left[\, A_i' P_i B_{2i} + C_i D_{2i}' \;\; A_i' P_i B_{1i} + C_i D_{1i}' \,\right] \quad (5c)$$
$$L_i = \begin{bmatrix} L_{i1} & L_{i2} \\ L_{i2}' & L_{3i} \end{bmatrix} \quad (5d)$$
$$L_{i1} = -(B_{2i}' P_i B_{2i} + D_{2i}' D_{2i}) \quad (5e)$$
$$L_{i2} = B_{2i}' P_i B_{1i} + B_{2i}' D_{1i} \quad (5f)$$
$$L_{i3} = \gamma^2 I - D_{1i}' D_{1i} - B_{1i}' P_i B_{1i} \quad (5g)$$

where λ_{ij} are suitable parameters such that $\sum_{j=1}^{M} \lambda_{ij} = 0$ and $\lambda_{ii} < 0$, for each $i = 1, 2, \cdots, M$. These equation are henceforth referred as Riccati-Metzler inequalities. We are now in a position to provide the following result.

Theorem 1. Suppose that there exist positive matrices P_i and a Metzler matrix $\Lambda = \{\lambda_{ij}\}$ such that inequalities (5a), (5b) are satisfied. Let

$$K_i = L_i^{-1} S_i \quad (6)$$

Then, consider the control law

$$u(t) = K_{\sigma(t)} x(t) \quad (7)$$

along with the switching rule

$$\sigma(t) = \xi(x(t)) = \arg\min_i x' P_i x \quad (8)$$

Finally, let

$$V(x) = \min_i x' P_i x \quad (9)$$

and the set

$$\Omega(K_\sigma, \xi, \gamma, \gamma_d) = \{x : \; V(x) \le \alpha\} \quad (10)$$

where α is a finite positive constant. Then the closed loop system under (8), (7) is asymptotically stable in $\Omega(K_\sigma, \xi, \gamma, \gamma_d)$ and it is such that

$$V(x(t^+)) - V(x(t)) < -\|z(t)\|^2 + \gamma^2 \|w(t)\|^2 \quad (11a)$$
$$\forall x \in \Omega(K_\sigma, \xi, \gamma, \gamma_d), \quad \forall w \in \mathcal{W} \quad (11b)$$

Proof First of all, let $\hat{A}_i = A_i + B_{2i} K_i$, $\hat{C}_i = C_i + D_{2i} K_i$. After cumbersome computations it is possible to rewrite (5a)-(5b) as

$$P_i > \hat{A}_i' P_i \hat{A}_i + \sum_{j=1}^{M} \lambda_{ij} P_j + \bar{S}_i \bar{L}_i \bar{S}_i' + \hat{C}_i' \hat{C}_i \quad (12)$$
$$0 < \gamma^2 I - B_{1i} P_i B_{1i}' - D_{1i}' D_{1i} \quad (13)$$
$$\bar{S}_i = \hat{A}_i' P_i B_{1i} + \hat{C}_i' D_{1i} \quad (14)$$
$$\bar{L}_i = (\gamma^2 I - D_{1i}' D_{1i} - B_{1i}' P_i B_{1i})^{-1} \quad (15)$$

Assume now that $i = \arg\min V(x(t))$. Then

$$\begin{aligned}
&V(x(t+1)) = \\
&\min_j (w(t)'B_i' + x(t)'\hat{A}_i')P_j(B_i w(t) + \hat{A}_i x(t)) \\
&\leq (w(t)'B_i' + x(t)'\hat{A}_i)P_i(B_i w_2(t) + \hat{A}_i x(t)) \\
&= w(t)'w(t) - z(t)'z(t) + 2w(t)'\bar{S}_i x(t) \\
&+ x(t)(\hat{A}_i' P_i \hat{A}_i + \hat{C}_i'\hat{C}_i)x(t) - w(t)'(\bar{L}_i)^{-1}w(t)
\end{aligned}$$

let now $\Delta V = V(x(t+1)) - V(x(t))$. Thanks to $\bar{L}_i > 0$, and by square completing we can write

$$\begin{aligned}
\Delta V \leq\ & \gamma^2\|w(t)\|^2 - \|z(t)\|^2 + 2w(t)'\bar{S}_i x(t) \\
& - w(t)'(\bar{L}_i)^{-1}w(t) - x(t)\sum_{j=1}^{M}\lambda_{ij}P_j x(t) \\
& - x(t)\bar{S}_i\bar{L}_i\bar{S}_i' x(t) \\
\leq\ & \gamma^2\|w(t)\|^2 - \|z(t)\|^2 \\
& - \|w^\star(t)\|^2 - x(t)'\sum_{j=1}^{M}\lambda_{ij}P_j x(t)
\end{aligned}$$

with $w^\star(t) = \bar{L}_i\bar{S}_i x(t)$. Since, for each j,

$$x(t)P_i x(t) \leq x(t)P_j x(t)$$

from $\lambda_{ij} \geq 0$, $i \neq j$, $\sum_{j=1}^{M}\lambda_{ij} = 0$, $\forall i$, it follows

$$\Delta V \leq \gamma^2\|w(t)\|^2 - \|z(t)\|^2$$

so that ΔV is negative if $\gamma\gamma_d < 1$. Asymptotic stability then follows. ■

Remark 1. The Riccati inequalities (5a)-(5b) are not LMI's in the variables P_i, and, moreover, depend on the choice of the design parameters λ_{ij}. As for this second problem, a simplification can be made at the price of a certain conservatism, i.e. the term $\sum_{i=0}^{M}\lambda_{ij}P_j$ can be substituted by the simpler term $\alpha(P_j - P_i)$ where α is a positive design parameter. Moreover, the inequalities (5a)-(5b) can be rewritten as LMIs in the variables $X_i = P_i^{-1}$. Premultiplying and postmultiplying (12) by X_i and letting $K_i = W_i P_i$, after cumbersome computations, it follows that matrices M_i, $i = 1, 2, \cdots, M$ are positive definite, where the single matrix M_i is

$$\begin{bmatrix}
X_i(1+\alpha) & X_i A_i' + W_i' B_{2i}' & X_i C_i' + W_i' D_{2i} & \sqrt{\alpha}X_i \\
\star & X_i - \dfrac{B_{1i}B_{1i}'}{\gamma^2} & -\dfrac{B_{1i}D_{1i}'}{\gamma^2} & 0 \\
\star & \star & \dfrac{D_{1i}D_{1i}'}{\gamma^2} & 0 \\
\star & \star & \star & X_j
\end{bmatrix}$$

These inequalities are easily solvable by standard LMI tools and a line search in α. Theorem 1 can be reformulated in the following way: Suppose that there exist positive definite matrices X_i, matrices W_i and a positive scalar $\alpha > 0$ such that $M_i > 0$. Let $K_i = W_i X_i^{-1}$ and consider the control law (7) along with the switching rule (8) with $P_i = X_i^{-1}$. Then the closed loop system under (1a), (6), (8) is asymptotically stable in $\Omega(K_\sigma, \xi, \gamma, \gamma_d)$, see (6), and it is such that (11) is met with.

4. MODEL PREDICTIVE SWITCHING CONTROL

In a model predictive control context, we will consider a finite time-interval $[t, t+N+1]$. At a given time t the designer has to chose a vector of strategies

$$\chi(t,N) = \left[\, \xi^0(x(t)) \ \cdots \ \xi^{N-1}(x(t+N-1)) \,\right]$$

and a vector of control values

$$\mathcal{K}(t,N) = \left[\, k^0(x(t)) \ \cdots \ k^{N-1}(x(t+N-1)) \,\right]$$

where the positive integer N is the *prediction horizon* and

$$\left(\xi^i : R^n \to \{1, 2, \cdots M\}, k^i : R^n \to R^{m_2}\right)$$

is the so-called *policy*. The sequence of disturbances chosen by "nature" is denoted as

$$\mathcal{Q}(t,N) = \left[\, w(t) \ w(t+1) \ \cdots \ w(t+N-1) \,\right]$$

In the classical Receding Horizon (RH) approach, only open loop strategies

$$\begin{aligned}
\chi(t,N) &= \left[\, \sigma(t) \ \sigma(t+1) \ \cdots \ \sigma(t+N-1) \,\right] \\
\mathcal{K}(t,N) &= \left[\, u(t) \ u(t+1) \ \cdots \ u(t+N-1) \,\right]
\end{aligned}$$

are considered. In order to take into account the variation of the state variable (due to the unpredictable behavior of the nature) we are well advised to consider closed-loop strategies and, consequently, minimize with respect to the sequence of policies. In general this problem is particular demanding since the policies in $\mathcal{K}(t)$ belong to an infinite-dimensional space. Concerning $\chi(t)$, there are a finite number (NM) of policies since $\xi^i(x)$ may assume only M values.

Now, assume that there exists an auxiliary law $\sigma^{aus}(t) = \xi(x(t))$, $u^{aus}(t) = K_{\sigma(t)}x(t)$, a domain of attraction $\Omega(K_\sigma, \xi, \gamma, \gamma_d)$ whose boundary is a level line of a positive function $V_F(x)$, with $V_F(0) = 0$ such that, $\forall x \in \Omega(K_\sigma, \xi, \gamma, \gamma_d)$ the constraints are satisfied (2) and $\forall w \in \mathcal{W}$, it results

$$V_F(x(t+1)) - V_F(x(t)) < -\|z(t)\|^2 + \gamma^2\|w(t)\|^2$$

Notably, this auxiliary control law can be the one developed in Section 3. The problem now consists in minimizing with respect to $(\chi(t,N),\mathcal{K}(t,N))$ and maximize, with respect to $\mathcal{Q}(t,N)$ the cost function

$$J(\bar{x},\mathcal{K},\chi,\mathcal{Q},N) = V_F(x(t+N) + \\ + \sum_{i=t}^{t+N-1} \|z(i)\|^2 - \gamma^2\|w(i)\|^2 \qquad (16)$$

subject to system (1) with $x(t)=\bar{x}$ and $x(t+N) \in \Omega(K_\sigma,\xi,\gamma,\gamma_d) \subset R^n$. If $(\bar{\chi}(t,N),\bar{\mathcal{K}}(t,N),\bar{\mathcal{Q}}(t,N))$ is the optimal solution of this min-max problem, according to the receding horizon principle, set

$$\xi(t) = \bar{\xi}^0(x(t)), \quad u(t) = \bar{k}^0(x(t)) \qquad (17)$$

The control law (17) turns out to be the MPC control law (3) in Section 2.

Theorem 2. Assume that the pair (A_σ, C_σ) is switching observable and that the auxiliary strategy exist. Let X^{MPC} the set of all states $\bar{x}$ such that the above min-max problem admits a solution. Then,

(i) X^{MPC} is a positively invariant set for the closed loop (1), (17) system.
(ii) $\Omega(K_\sigma,\xi,\gamma,\gamma_d) \subseteq X^{MPC}$, $\forall N$
(iii) The origin is asymptotically attractive for the closed loop system (1), (17) in X^{MPC}.

Proof. Define by $V(\bar{x},N)$ the optimal performance, i.e. $V(\bar{x},N) = J^o(\bar{x},\bar{\mathcal{K}},\bar{\chi},\bar{\mathcal{Q}},N)$. We now prove that X^{MPC} is a positively invariant set for the closed loop system (1), (17). If $\bar{x} \in X^{MPC}$, then there exist $\bar{\mathcal{K}}$ and $\bar{\chi}$ that bring the state in $\Omega(K_\sigma,\xi,\gamma,\gamma_d)$ at time $t+N$, i.e. $x(t+N) \in \Omega(K_\sigma,\xi,\gamma,\gamma_d)$. Hence, letting $t+i=t_i$ for short, consider the new policy at time t_1, i.e.

$$\hat{\chi}(t,N) = \left[\, \bar{\xi}^1(x(t)) \;\cdots\; \bar{\xi}^{N-1}(x(t_{N-1})) \;\; \sigma^{aus}(t_N) \,\right]$$
$$\hat{\mathcal{K}}(t,N) = \left[\, \bar{k}^1(x(t)) \;\cdots\; \bar{k}^{N-1}(x(t_{N-1})) \;\; u^{aus}(t_N) \,\right]$$

Since $\Omega(K_\sigma,\xi,\gamma,\gamma_d)$ is invariant with respect to the auxiliary law with $w(t) \in \mathcal{W}$, such a policy is still feasible. Moreover, being the auxiliary law feasible, it follows that $\Omega(K_\sigma,\xi,\gamma,\gamma_d) \subseteq X^{MPC}$ so that also the origin is included in X^{MPC}. Now observe that, letting $\mathcal{Q}(t,N)=0$, see (16), we can conclude that

$$J(\bar{x},\mathcal{K},\chi,0,N) = V_F(x(t+N)) + \sum_{i=t}^{t+N-1} \|z(i)\|^2 > 0$$

for each $\bar{x} \in X^{MPC}$, $\bar{x} \neq 0$. Moreover

$$V(\bar{x},N) = J(\bar{x},\bar{\mathcal{K}},\bar{\chi},\bar{\mathcal{Q}},N) \geq \min_{\mathcal{K}\times\chi} J(\bar{x},\mathcal{K},\chi,0,N)$$

for each $\bar{x} \in X^{MPC}$, $\bar{x} \neq 0$. In conclusion, for every $\bar{x} \in X^{MPC}$ it results:

$$V(\bar{x},N) > 0, \quad \bar{x} \neq 0$$

Now we prove that $V(\bar{x},N)$ is decreasing. Indeed, consider the horizon of length $N+1$ and the policies

$$\widetilde{\chi}(t,N+1) = \left[\, \bar{\chi}(t,N) \;\; \sigma^{aus}(t_N) \,\right]$$
$$\widetilde{\mathcal{K}}(t,N+1) = \left[\, \bar{\mathcal{K}}(t,N) \;\; u^{aus}(t_N) \,\right]$$
$$\widetilde{\mathcal{Q}}(t,N+1) = \left[\, \bar{\mathcal{Q}}(t,N) \;\; w(t_N) \,\right]$$

Therefore

$$J(\bar{x},\widetilde{\mathcal{K}}(t,N+1),\widetilde{\chi}(t,N+1),\widetilde{\mathcal{Q}}(t,N+1),N+1) = \\ \left(\sum_{i=t}^{t_N} \|z(i)\|^2 - \gamma^2\|w(i)\|^2\right) + V_F(x(t_{N+1}) \\ = V_F(x(t_{N+1})) - V_F(x(t_N)) + \\ + \|z(t_N)\|^2 - \gamma^2\|w(t_N)\|^2 \\ + V_F(x(t_N)) + \sum_{i=t}^{t_{N-1}} \|z(i)\|^2 - \gamma^2\|w(i)\|^2$$

Being

$$V_F(x(t_{N+1}) - V_F(x(t_N)) \leq -\|z(t_N)\|^2 + \gamma^2\|w(t_N)\|^2$$

it follows

$$J(\bar{x},\widetilde{\mathcal{K}}(t,N+1),\widetilde{\chi}(t,N+1),\widetilde{\mathcal{Q}}(t,N+1),N+1)) \\ \leq \left(\sum_{i=t}^{t_{N-1}} \|z(i)\|^2 - \gamma^2\|w(i)\|^2\right) + V_F(x(t_N))$$

Consequently,

$$V(\bar{x},N+1) \\ \leq \max_{w\in\mathcal{W}} J(\bar{x},\widetilde{\mathcal{K}}(t,N+1),\widetilde{\chi}(t,N+1),\widetilde{\mathcal{Q}}(t,N+1),N+1)) \\ \leq \left(\sum_{i=t}^{t_{N-1}} \|z(i)\|^2 - \gamma^2\|w(i)\|^2\right) + V_F(x(t_N)) \\ = V(\bar{x},N)$$

In conclusion

$$V(\bar{x},N+1) \leq V(\bar{x},N)$$

Furthermore,

$$V(\bar{x},N) = V(A_\sigma\bar{x} + B_{1\sigma}w + B_{2\sigma}u), N-1) + \\ + \|z\|^2 - \gamma^2\|w\|^2$$

so that

$$V(\bar{x},N) \geq V(A_\sigma\bar{x} + B_{1\sigma}w + B_{2\sigma}u), N) + \\ + \|z\|^2 - \gamma^2\|w\|^2$$

and finally, for each $\bar{x} \in X^{MPC}$:

$$V(A_\sigma \bar{x} + B_{1\sigma} w + B_{2\sigma} u), N) - V(x, N) \leq$$
$$\gamma^2 \|w\|^2 - \|z\|^2 \leq (\gamma^2 \gamma_d^2 - 1) \|z\|^2$$
$$-\epsilon \|z\|^2 < 0$$

The above strict inequality sign follows from the detectability assumption.

5. CONCLUSIONS

In the paper a new result on model predictive control of switched systems has been presented. In the authors' opinion this result can be the basis for further research on multilevel hierarchical control, see (Scattolini and Colaneri, 2007).

REFERENCES

M.S. Branicky (1998). "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems". *IEEE Trans. Automat. Contr.*, vol. 43, pp. 475-482.

J. Daafouz and J. Bernussou (2001). "Parameter dependent Lyapunov functions for discrete time systems with time varying parametric uncertainties". *Systems & Control Letters*, vol. 43, pp. 355-359.

G. De Nicolao, L. Magni, R. Scattolini (2000). "Stability and robustness of nonlinear receding-horizon control". In *Nonlinear Model Predictive Control*, F. Allgöwer and A. Zheng Eds., pp. 3-22, in Progress in Systems and Control Theory, Birkhauser Verlag.

J.C. Geromel, P. Colaneri (2006). "Stability and stabilization of discrete-time switched systems". *Int. J. Control*, 79, 7, pp. 719-728.

J.P. Hespanha (2004). "Uniform stability of switched linear systems : extensions of LaSalle's principle". *IEEE Trans. Automat. Contr.*, vol. 49, pp. 470-482.

J. Hockerman-Frommer, S.R. Kulkarni and P.J. Ramadge (1998). "Controller switching based on output predictions errors". *IEEE Trans. Automat. Contr.*, vol. 43, pp. 596-607.

M. Johansson, A. Rantzer (1998). "Computation of piecewise quadratic Lyapunov functions for hybrid systems". *IEEE Trans. Automat. Contr.*, vol. 43, pp. 555-559.

D. Liberzon (2003). *Switching in Systems and Control*, Birkhauser.

L. Magni, G. De Nicolao, R. Scattolini, F. Allgoewer (2003). "Robust model predictive control for nonlinear discrete-time systems". *International Journal of robust and nonlinear control*, 13, pp. 229-246.

L. Magni and D. Raimondo and R. Scattolini (2006). "Regional Input-to-State Stability for Nonlinear Model Predictive Control". *IEEE Trans. On Automatic Control*, Vol. 51, pp. 1548-1553.

L. Magni and R. Scattolini (2005). "Robustness and Robust Design of MPC for Nonlinear Discrete-Time Systems". Int. *Workshop on Assessment and Future Directions of NMPC*, Freudenstadt-Lauterbad, Germany, pp. 31-46.

D. Q. Mayne, J. B. Rawlings, C. V. Rao and P. O. M. Scokaert (2000). "Constrained model predictive control: Stability and Optimality". *Automatica*, Vol. 36, pp. 789-814.

R. Scattolini, P. Colaneri, (2007). "Hierarchical model predictive control". Int. Rep. *ICT Department*, Politechnic Institute of Milan.

P.O.M. Skokaert, D.Q. Maine (1998). "Min-max feedback model predictive control for constrained linear systems". *IEEE Transactions on Automatic Control*, Vol. 43, pp. 1136-1142.

A.J. Van der Schaft (1998). *L_2-gain and passivity techniques in nonlinear control*, Springer, Berlin.

G.M. Xie and L. Wang (2003). "Reachability realization and stabilizability of switched linear discrete-time systems". *Journal of Math. Analysis and Applications*, Vol. 280, pp. 209-220.

H. Ye, A.N. Michel and L. Hou (1998). "Stability theory for hybrid dynamical systems". *IEEE Trans. Automat. Contr.*, vol. 43, pp. 461-474.

G.S. Zhai (2001), "Quadratic stability of discrete-time switched systems via state and output feedback". *Proceedings of the 40th IEEE CDC*, pp. 2165-2166.

ACKNOWLEDGEMENTS

This paper has been partially supported by the Italian National Research Council (CNR) and by MIUR project "Advanced methodologies for hybrid control systems".