VIRTUAL SENSORS FOR LINEAR DYNAMIC SYSTEMS: STRUCTURE AND IDENTIFICATION

Mario Milanese, Fredy Ruiz and Michele Taragna

***Abstract*— Consider a linear system with input u and outputs y and z. Assume that u^t and y^t are measured for all times t and that z^t is measured only for $t \leq \tau$, but it is of interest to know z^t for $t > \tau$. Such a situation may arise when the sensor measuring z fails and it is important to recover this variable, e.g. for feedback control. Another case arises when the sensor measuring z is too complex and costly to be used, except for an initial set of experiments. Assuming that z is observable from the couple (u, y), the standard approach consist of a two-step procedure: identify a model first, then design an observer/Kalman filter based on the identified model. Noticing that an estimator of z^t is a system with (u^t, y^t) as input that gives an estimate of z^t as output, the problem of directly identifying an estimator model from the available noisy data in the time interval $(0, \tau)$ is investigated. When stochastic noise is considered, the direct procedure can be carried out using standard techniques and performs better or like the two-step approach. In the case of Set Membership noise, a procedure for the identification of direct virtual sensors is presented. An example related to the vertical dynamics of vehicles with controlled suspension shows the effectiveness of the presented approaches.**

***Keywords*—direct virtual sensors, Kalman filter, system identification, estimation.**

I. INTRODUCTION

Consider a discrete-time, linear, time-invariant, multivariable, dynamic system S, initially at rest, described in state-space form as:

$$\begin{array}{rcl} x^{t+1} & = & Ax^t + B_1 u^t + B_2 d^t \\ y^t & = & C_1 x^t + v^t \\ z^t & = & C_2 x^t \end{array} \qquad (1)$$

where, for a given time instant $t \in \mathbf{N}$: $x^t \in \mathbf{R}^n$ is the system state; $u^t \in \mathbf{R}$ is a known deterministic input; $d^t \in \mathbf{R}^d$ is a (possibly vectorial) unknown disturbance input; $y^t \in \mathbf{R}$ is a known (measured) output; $z^t \in \mathbf{R}$ is a partially unknown output; $v^t \in \mathbf{R}$ is unknown measurement noise; A, B_1, B_2, C_1 and C_2 are constant matrices of suitable dimensions.

The assumptions about the system S described by (1) are the following ones:

- the matrices A, B_1, B_2, C_1 and C_2 are not known;
- the pair $[A, C_1]$ is observable and then, in absence of noise, $z^\tau = f(Y_1^\tau, U_1^\tau)$ for any τ, where
$$\begin{array}{rcl} U_1^\tau & = & [u^1,\ u^2,\ \ldots,\ u^{\tau-1},\ u^\tau]^T \\ Y_1^\tau & = & [y^1,\ y^2,\ \ldots,\ y^{\tau-1},\ y^\tau]^T; \end{array}$$
- measurements of u^t, y^t and z^t are available for $t = 1, 2, \ldots, \tau$;
- measurements of u^t and y^t are available also for $t > \tau$.

Considering that process and measurement noises affect equations (1), the filtering aim is to obtain a (possibly optimal in some sense) estimate $\hat{z}^t$ of z^t for $t > \tau$ using the available noise-corrupted input and output measurements.

Such a situation may arise when the sensor measuring z^t is too expensive and/or complex to be used, except for an initial set of experiments. Another case arises when the sensor measuring z^t fails and it is important to recover this variable, e.g. for feedback control.

The standard approach for the design of virtual sensors, assuming the system S is not known, consist of a two-step procedure: first, a model $\hat{M}$ is identified form the available noisy data set (u^t, y^t, z^t) for $t = 1, \ldots, \tau$; second, on the base of $\hat{M}$, a filter $\hat{K}$ is designed which gives an estimate $\hat{z}_K^t$ of z^t using as input (u^t, y^t) for $t > \tau$. An alternative approach, proposed in [1], is to use the data set (u^t, y^t, z^t) for $t = 1, \ldots, \tau$ to directly identify a filter $\hat{V}$ which gives an estimate $\hat{z}_V^t$ of z^t using as input (u^t, y^t) for $t > \tau$.

In a stochastic framework, the noises affecting (1) are supposed to be random sequences and optimality refers to minimizing the estimation error variance. In [1], it was shown that the direct approach gives, in the case of exact modeling, performances not worse than the two-step procedure and, more importantly, in the presence of modeling errors, the directly identified filter is the minimum variance estimator among the selected approximating filter class, while a similar result is not assured by the two-step design, whose performance deterioration due to modeling errors may be significantly larger.

In a Set Membership framework, the noises affecting (1) are supposed to be unknown but bounded sequences and optimality refers to minimizing a prefixed norm of the estimation error. To the authors' knowledge, no results exist about the design and performance evaluation of virtual sensors using a two-step approach in this framework. In this paper a methodology for the direct identification of filters is presented, minimizing the estimation error and guaranteeing some stability measure of the model being identified.

As a case study, the filter design problem is solved for an automotive application. The behavior of different linear virtual sensors for a quarter-car suspension system is presented, comparing the estimates provided by observers based on Kalman filter and the new Direct Virtual Sensor (DVS) design techniques. In the field of semiactive suspensions,

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many different control algorithms have been proposed, such as the well established "two state" Sky-Hook (see e.g. [2]) and "clipped" strategies (see e.g. [3], [4]) or Model Predictive Control techniques (see e.g. [5]). The computation of the control move requires to know, at each sampling time, the state of the suspension system. A typical configuration of sensors is based on accelerometers measuring the vertical accelerations of the chassis (sprung mass) and/or of the wheel (unsprung mass). In this example, the focus is on the estimation of the relative vertical speed between chassis and wheel, using data provided by just one accelerometer measuring the vertical acceleration of the chassis. This matter is important not only for cost reduction but also for safety reasons, for example when a sensor fails and the corresponding signal has to be recovered in some way for closing the feedback control loop. In order to investigate the achievable results in a quite realistic fashion, different Monte Carlo simulations have been carried out using standard "benchmark" road profiles employed in industrial tests, such as random, motorway, pavé, English track, short back and drain well profiles.

The paper is organized as follows. In Section II the main theoretical results on virtual sensors for stochastic settings are recalled and summarized. In the third section, the formulation of the direct virtual sensor identification for Set Membership setting is presented. In Section IV the presented approach is tested on a problem of filter design for the vertical dynamics of vehicles with controlled suspensions. In the last section, some concluding remarks are given.

II. Virtual Sensors Design for Stochastic Framework

In a stochastic setting, process and measurement noises corrupting (1) are assumed to be stochastic white sequences. For such a situation, a huge literature exists on the minimum variance filter design, assuming that the system S is known. In the present context, on the contrary, the system model is not known and the filter should be obtained from a noise-corrupted set of data generated by S in an initial experiment. Two different methodologies are considered to deal with this task.

A. Two-step Procedure

The usual solution to the proposed problem is a two-step procedure. First, an approximate system model $\hat{M}$ is obtained using standard identification methods. Note that the noise model should also be estimated as is needed for the filter design.

Given a linear model structure $M(\theta_M)$, $\theta_M \in D_M \subset \Re^{n_M}$, of the form

$$\begin{aligned} \hat{y}^t &= G_{yu}(q,\ \theta_M)u + G_{ye}(q,\ \theta_M)e_y \\ \hat{z}^t &= G_{zu}(q,\ \theta_M)u + G_{ze}(q,\ \theta_M)e_z \end{aligned} \tag{2}$$

where e_y and e_z are assumed to be white noise sequences and G_{yu}, G_{ye}, G_{zu} and G_{ze} are discrete time transfer functions in the standard forward shift operator q. A model is selected using a prediction error method as:

$$M(\hat{\theta}_M) = \arg\min \frac{1}{\tau} \det \sum_{t=1}^{\tau} \left[e_y^t,\ e_z^t\right]^T \left[e_y^t,\ e_z^t\right] \tag{3}$$

State-space model sets have been considered in this paper.

When $M(\hat{\theta}_M)$ or a suitable approximation is found, a (steady state) Kalman filter $\hat{K} = K(\hat{\theta}_M)$ is designed on the base of $M(\hat{\theta}_M)$ and the estimated noise properties. The estimator uses y^t and u^t to recover the state of the identified system (3) with minimum variance and uses it to obtain the desired output, thus giving the estimate $\hat{z}_K^t$ of z^t.

B. Direct Procedure

An alternative approach to the problem, as presented in [1], is to perform, starting from data (u^t, y^t, z^t) for $t = 1, \ldots, \tau$, the direct identification of a filter $\hat{V}$, named Direct Virtual Sensor (DVS), which gives an estimate $\hat{z}_V^t$ of z^t using as input (u^t, y^t) for $t > \tau$.

This aim can be pursued by selecting the filter $\hat{V}$ from a suitable family of parameterized predictor models $V(\theta_V)$, $\theta_V \in D_V \subset \Re^{n_V}$, of the form:

$$\begin{aligned} x_f^{t+1} &= A_f(\theta_V)\, x_f^t + B_{f1}(\theta_V)\, u^t + B_{f2}(\theta_V)\, y^t \\ \hat{z}^t &= C_f(\theta_V)\, x_f^t + D_{f1}(\theta_V)\, u^t + D_{f2}(\theta_V)\, y^t \end{aligned} \tag{4}$$

where $x_f \in \mathbf{R}^{n_f}$ is the estimator state, and the optimality criterion used in this paper to select the filter $\hat{V} = V(\hat{\theta}_V)$ is the error variance minimization:

$$\hat{\theta}_V = \arg\min_{\theta_V \in D_V} \frac{1}{2\tau} \sum_{t=1}^{\tau} \varepsilon^2(t, \theta_V)$$

where $\varepsilon(t, \theta_V) = z^t - \hat{z}^t$ is the prediction error of predictor models $V(\theta_V)$, see e.g. [6].

The interest for this new approach stems from the fact that even in the most favorable situation, e.g. when no modeling errors occur and the minimum variance filter is actually computable, the two-step procedure is proven to perform no better than the direct approach. The following theorem shows the statistical optimality properties of the two proposed approaches.

Theorem [1]. Let $\bar{E}$ denote statistical expectation. The following results hold with probability one as $\tau \to \infty$:

i) $\bar{E}\left[(z^t - \hat{z}_K^t)^2\right] \geq \bar{E}\left[(z^t - \hat{z}_V^t)^2\right]$

ii) If $S \in M(\theta_M)$, then $\hat{V}$ is a minimal variance filter among all linear causal filters mapping (u, y) into z.

iii) If $S \in M(\theta_M)$, the model structure $M(\theta_M)$ is globally identifiable and S is stable, then $\hat{K}$ also is a minimal variance filter so that:

$$\bar{E}\left[(z^t - \hat{z}_K^t)^2\right] = \bar{E}\left[(z^t - \hat{z}_V^t)^2\right]$$

■

This theorem states that, in general, the direct procedure offers better performances than the two-step procedure. Indeed, at best (e.g. under the exact modeling assumption $S \in M(\theta_M)$), the two approaches have asymptotically (w.r.t. τ) the same accuracy. However, in the presence of modeling errors, the directly identified filter, although not absolutely optimal, is the minimum variance estimator among all linear filters of the same order. A similar result is not assured by the two-step design, whose performance degradation caused by modeling errors may be significantly larger. Moreover, in the case of no modeling errors, result *ii)* shows that the directly identified filter $\hat{V}$ is optimal even if S is unstable, while this is not guaranteed by the Kalman filter $\hat{K}$.

III. Virtual Sensors Design for Set Membership Framework

In a Set Membership setting, the noise sequences are assumed to be bounded in some set. Moreover, the filter is assumed to belong to a class of systems with some guaranteed stability degree.

Under the assumed hypotheses about the system S and in particular the observability of the pair $[A, C_1]$, a family of stable estimators exists for the system (1), in the form of Luenberger observer:

$$\begin{aligned} x_e^{t+1} = &\quad Ax_e^t + B_1 u^t + L(y^t - C_1 x_e^t) \\ \tilde{z}^t = &\quad C_2 x_e^t + M(y^t - C_1 x_e^t) \end{aligned} \tag{5}$$

Each IIR filter in the form (5) is an exponentially stable dynamic system and can be written as:

$$\tilde{z}^t = \sum_{k=0}^{\infty} \alpha_k u^{t-k} + \sum_{k=0}^{\infty} \beta_k y^{t-k}$$

where the decay rate of its impulse response is bounded as:

$$\begin{cases} |\alpha_k| \leq L_u \rho^{-k}, & k \in [0, ..., \infty], L_u > 0, \rho > 1 \\ |\beta_k| \leq L_y \rho^{-k}, & k \in [0, ..., \infty], L_y > 0, \rho > 1 \end{cases} \tag{6}$$

The proposed methodology approximates the infinite impulse response of the above estimator with a long enough FIR filter $\hat{V}^{SM}$. The structure of the direct virtual sensor for the Set Membership framework (DVS-SM for short) is:

$$\hat{z}^t = \sum_{k=0}^{n_u} \alpha_k u^{t-k} + \sum_{k=0}^{n_y} \beta_k y^{t-k} \tag{7}$$

where n_u, n_y are given. The design parameters of this DVS-SM filter are L_u, L_y, ρ, n_u, n_y and can be suitably tuned in order to minimize the overall estimation error.

Notice that the estimation error $\delta^t = z^t - \hat{z}^t$ is bounded. In fact, $\delta^t = z^t - \hat{z}^t = (z^t - \tilde{z}^t) + (\tilde{z}^t - \hat{z}^t) = \tilde{\delta}^t + \hat{\delta}^t$. The term $\tilde{\delta}^t = z^t - \tilde{z}^t$ is the estimation error of filter (5), governed by the error dynamics resulting from the effect of noise and disturbances on the difference $\xi_x^t = x^t - x_e^t$ between the system and filter states. From (1) and (5), it follows that

$$\begin{aligned} \xi_x^{t+1} = &\quad (A - LC_1)\xi_x^t + B_2 d^t - Lv^t \\ \tilde{\delta}^t = &\quad (C_2 + MC_1)\,\xi_x^t - Mv^t \end{aligned} \tag{8}$$

and then $\tilde{\delta}^t$ is bounded since d^t and v^t are assumed to be bounded. Moreover, for any $\rho > 1$, the term $\hat{\delta}^t = \tilde{z}^t - \hat{z}^t = \sum_{k=n_u+1}^{\infty} \alpha_k u^{t-k} + \sum_{k=n_y+1}^{\infty} \beta_k y^{t-k}$ is bounded for any bounded input u^t and output y^t.

A measure of the estimation error is given by the following weighted p-norm:

$$WN = \|\delta_\tau\|_p^{W_\delta} = \left\|W_\delta^{-1}\delta_\tau\right\|_p = \left(\sum_{k=1}^{\tau} \left|w_{\delta,k}^{-1}\delta^k\right|^p\right)^{1/p}$$

with $\delta_\tau = \left[\delta^1, \ldots, \delta^\tau\right]^T$ and $W_\delta = \operatorname{diag}(w_{\delta,1}, w_{\delta,2}, \ldots, w_{\delta,\tau})$ a given weighting matrix where $w_{\delta,k} > 0\ \forall k$. By suitably choosing W_δ, it is possible to consider noise measures dependent on k : for example, $W_\delta = \operatorname{diag}(z^1,\ z^2,\ \ldots,\ z^{\tau-1},\ z^\tau)$ in the case of relative measurement errors.

For given L_u, L_y, n_u, n_y and ρ, an optimal filter $\hat{V}^{SM}$ of the form (7) can be selected by minimizing the above estimate quality measure, thus leading to the following identification problem:

$$\begin{gathered} [\hat{\alpha}_0, \ldots, \hat{\alpha}_{n_u}, \hat{\beta}_0, \ldots, \hat{\beta}_{n_y}] = \arg\min \|\delta_\tau\|_p^{W_\delta} \\ \text{such that} \end{gathered}$$

$$\begin{cases} \delta^t = z^t - \sum\limits_{k=0}^{n_u} \alpha_k u^{t-k} - \sum\limits_{k=0}^{n_y} \beta_k y^{t-k}, t \in [1, 2, \ldots, \tau] \\ |\alpha_k| \leq L_u \rho^{-k},\ k \in [0, \ldots, n_u] \\ |\beta_k| \leq L_y \rho^{-k},\ k \in [0, \ldots, n_y] \end{cases} \tag{9}$$

When disturbances and noise are assumed to be energy bounded signals, i.e. $d^t \in \ell_2(Z_+)$ and $v^t \in \ell_2(Z_+)$, the solution of the identification problem (9) with $p = 2$ and $W_\delta = I_{[\tau\times\tau]}$ leads to the minimization of the estimation error variance. In this case, the problem (9) can be efficiently solved by quadratic programming.

When disturbances and noise are assumed to be amplitude bounded signals, i.e. $d^t \in \ell_\infty(Z_+)$ and $v^t \in \ell_\infty(Z_+)$, the solution of the identification problem (9) with $p = \infty$ and $W_\delta = I_{[\tau\times\tau]}$ leads to the minimization of the worst-case estimation error. In this case, solution to problem (9) can be obtained by minimax optimization. Since minimax is a complex and not efficient procedure, a conservative but less demanding approximation for the $p = \infty$ case is:

$$\begin{gathered} [\hat{\alpha}_0, \ldots, \hat{\alpha}_{n_u}, \hat{\beta}_0, \ldots, \hat{\beta}_{n_y}, \sigma] = \arg\min \sigma \\ \text{such that} \end{gathered}$$

$$\begin{cases} \sigma \geq 0 \\ w_{\delta,t}^{-1} \left| z^t - \sum\limits_{k=0}^{n_u} \alpha_k u^{t-k} - \sum\limits_{k=0}^{n_y} \beta_k y^{t-k} \right| \leq \sigma, t \in [1, 2, \ldots, \tau] \\ |\alpha_k| \leq L_u \rho^{-k},\ k \in [0, \ldots, n_u] \\ |\beta_k| \leq L_y \rho^{-k},\ k \in [0, \ldots, n_y] \end{cases} \tag{10}$$

Solution to problem (10) is linear programming. Standard and efficient algorithms exist to solve it.

Regardless of the used norm, solution to problems (9) or (10) is a high order FIR filter, which in many cases is not well suited for practical use, for example in real time estimation. Since the found FIR filter is an approximation of the impulse response of a stable finite order filter, it is possible to perform a model order reduction, fitting the identified impulse responses with a stable and causal IIR filter $\hat{V}_n^{SM}$ of a prefixed order n.

The selection of the design parameters L_u, L_y, n_u, n_y and ρ can be made in different ways. For example, the direct procedure for stochastic framework may be applied to the available data set using different filter structures (orders) and then a suitable bound on the impulse responses of the resulting filters can be looked for. Another possibility is to solve the identification problem without constraints and then, starting from this solution, choose conservative bounds for L_u and L_y, and finally perform a line search on ρ varying n_u and n_y according to the selected decay rate.

IV. Virtual Sensors for Semiactive Suspensions

In this work, the virtual sensor methodologies are applied to the vertical dynamics of a road vehicle. The used model is a quarter-car semiactive suspension system, having the structure depicted in Fig. 1. The chassis and the wheels are modeled as rigid bodies and static linear characteristics are assumed for suspension. The parameters characterizing the model are:

- M_c: sprung (chassis) mass.
- M_w: unsprung (tire, wheel and other suspension components) mass.
- K_c: suspension spring constant.
- K_w: tire stiffness coefficient.

The variables describing the system are:

- x_r: road profile.
- x_w: wheel vertical position.
- x_c: chassis vertical position.
- $u(t)$: damping force.

The quarter-car model dynamics are given by the following set of differential equations:

$$\begin{aligned} M_c\ddot{x}_c &= u - K_c\left(x_c - x_w\right) \\ M_w\ddot{x}_w &= -u + K_c\left(x_c - x_w\right) - K_w\left(x_w - x_r\right) \end{aligned} \tag{11}$$

The parameter values used in the simulations are reported in Table I and have been taken from [5].

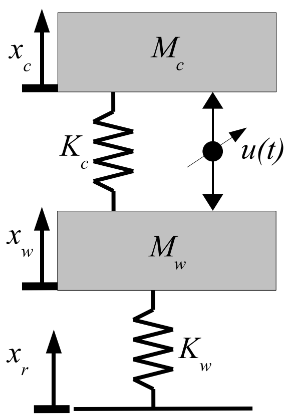

Fig. 1. Quarter-car suspension schematic

TABLE I

Parameters Values used in the Simulation

Parameter	Value
M_c	432.82 kg
M_w	40 kg
K_c	17200 N/m
K_w	200000 N/m

In semiactive suspension systems, the damping force is $u(t) = -\beta(t)\left[\dot{x}_c(t) - \dot{x}_w(t)\right]$, where the damping coefficient $\beta(t)$ is variable. At present, a widely used semiactive technique is the "On-Off Sky-Hook" control (see e.g. [7]), where the damper is adjusted at maximum or minimum damping to provide the following force:

$$u = \begin{cases} u_{i,max}\left(\dot{x}_c - \dot{x}_w\right) & \text{if } \dot{x}_c\left(\dot{x}_c - \dot{x}_w\right) \geq 0 \\ u_{i,min}\left(\dot{x}_c - \dot{x}_w\right) & \text{if } \dot{x}_c\left(\dot{x}_c - \dot{x}_w\right) < 0 \end{cases}$$

The maximum and the minimum curves $u_{i,max}$ and $u_{i,min}$ are represented in Fig. 2 as functions of the relative speed $v_{wc}(t) = \dot{x}_{wc} = \dot{x}_w(t) - \dot{x}_c(t)$.

The quarter-car model has been implemented in Simulink in order to obtain data simulating a possible experimental setup, characterized by type of road profile, control strategy, experiment length, measured variables and sensors accuracy.

It is considered that the road profile x_r is not known, the damping force $u(t)$ is known and corresponds to a "On-Off Sky-Hook" control, acceleration $\ddot{x}_c$ can be measured with a precision of 5%, the relative vertical speed $\dot{x}_{wc}$ can be measured only on an initial experiment with a precision of 5%.

Six data sets have been generated from the quarter-car model, all with a length of 13.7 seconds. Each data set corresponds to the system response to a "benchmark" road profile, subject to zero initial conditions, as described in [5]. The considered road profiles are among those used for the on-road tuning of the CDC-Skyhook (continuous damping control) system. These road profiles allow to test different dynamic conditions of the vehicle, in terms of frequencies and amplitudes:

- Random (shortened as RR): random road.
- Motorway (shortened as MW): level road.
- Pavé (shortened as PV): road with small amplitude irregularities.
- English Track (shortened as ET): road with irregularly spaced sequences of bumps and holes.
- Short Back (shortened as SB): impulsive road.
- Drain Well (shortened as DW): negative impulsive road.

Each data set consists of the values of u, $\ddot{x}_c$ and $\dot{x}_{wc}$, recorded with a sample time $T_s = 1/512s$.

The complete data set, formed by the six subsets of 7000 samples of each measured variable, has been partitioned as follows:

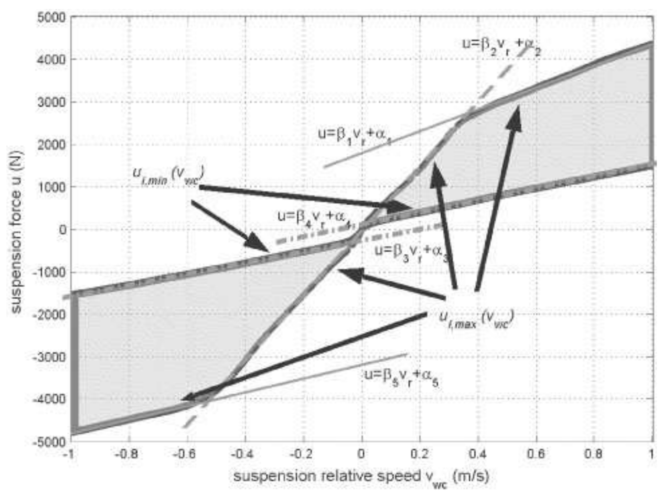

Fig. 2. Damper map

- identification set: the data corresponding to the first 6.8 seconds of the acquisition with a particular road profile;
- testing set: the data corresponding to seconds from 6.8 to 13.7 of the acquisition with the same road profile considered for the identification set, and the data corresponding to the 13.7 seconds of acquisition with the other five road profiles. This set has been used for testing the accuracy of identified models on data not used for identification.

The aim of a virtual sensor for the experimental setup presented above is to estimate the relative vertical speed $\dot{x}_{wc}$ of the quarter car model, using the available acceleration measurement $\ddot{x}_c$ and the applied suspension force $u(t)$. In fact, it was shown in [8] that the best trade-off between filter complexity and error signal ratio can be obtained using the chassis accelerometer instead of the wheel accelerometer, both for relative speed an position estimates.

Please observe that the quarter-car model described by equations (11) is undamped and this may lead to identification problems for both two-step and direct virtual sensor design techniques. To avoid that, it is enough to consider the suspension force $u(t)$ as the sum of two terms: a force $\beta_c \dot{x}_{wc}(t)$ and a known force $f(t) = u(t) - \beta_c \dot{x}_{wc}(t)$. By taking $f(t)$ instead of $u(t)$ as input, the overall suspension system is guaranteed to be asymptotically stable, since it contains a fictitious damper β_c between chassis and wheel. This way, all the assumptions of the Theorem reported in Section II are fulfilled – in particular, those required for the result *iii)* – and a fair comparison between the different methodologies may be performed. In this paper, the value $\beta_c = 3000$ Ns/m has been chosen, being approximately the average damping between the maximum and minimum curves represented in Fig. 2.

The correspondence between the virtual sensor problem formulated in Section I and the actual signals is: $u = f$, $d = x_r$, $y = \ddot{x}_c$ and $z = \dot{x}_{wc}$.

Two sets of identification experiments have been taken into account, with two different kinds of noise v corrupting the measurements in equations (1). In the first set of experiments, white random Gaussian noise corrupts the samples, such that the hypotheses of the stochastic framework of Section II are true. In the second one, amplitude bounded noise corrupts the measurements, thus meeting the assumptions of the Set Membership framework of Section III. A Monte Carlo simulation with 100 experiments has been performed for each noise type, using the same simulation conditions (road profile), but with different realizations of the noise sequences affecting the samples.

A. Virtual Sensors Identification Under Gaussian Noise

For each experiment, white Gaussian sequences corrupt the system outputs $\ddot{x}_c$ and $\dot{x}_{wc}$ with a noise to signal ratio of 5%.

The filter performance has been evaluated on the testing set, and the percent ratio between estimation error and signal, evaluated as:

$$ESR_{\%} = 100 \frac{\sqrt{\frac{1}{t_f - t_0 + 1} \sum_{t=t_0}^{t_f} \left(z^t - \hat{z}^t\right)^2}}{\sqrt{\frac{1}{t_f - t_0 + 1} \sum_{t=t_0}^{t_f} \left(z^t\right)^2}},$$

has been used as performance criterion, being t_0 and t_f the initial and the final time instants of any experiment, respectively. The average value of the $ESR_{\%}$ for the 100 obtained filters has been considered as a criterion to compare the three algorithms.

In the two-step methodology, for each one of the 100 experiments, a SITO (Single Input-Two Outputs) system $\hat{M}$ in the form (2), with force f as input and $\ddot{x}_c$ and $\dot{x}_{wc}$ as outputs, has been identified using standard identification methods. The `pem` routine of the MATLAB Identification Toolbox has been used to fit an initial model. On the base of this model, a steady state Kalman filter $\hat{K}$ has been designed as *two-step* virtual sensor.

In either direct methodology, for each one of the 100 experiments, a TISO (Two Inputs-Single Output) system with force f and acceleration $\ddot{x}_c$ as inputs and $\dot{x}_{wc}$ as output has been identified as *direct* virtual sensor.

For the stochastic framework, models $\hat{V}$ in the form (4) have been identified using standard identification methods; in particular, the MATLAB `pem` routine has been used.

For the Set Membership framework, filters $\hat{V}^{SM}$ in the form (7) have been identified by solving the identification problem (9), using $p = 2$ as norm and $W_\delta = I_{[\tau \times \tau]}$ as weighting matrix. To design in a suitable way the filter $\hat{V}^{SM}$, filters of orders 4 to 8 obtained with the direct methodology for the stochastic framework have been evaluated. Their impulse responses are plotted in figure 3 and a choice of $L_u = 3.75 \cdot 10^{-4}$, $L_y = 0.175$ and $n_u = n_y = 148$ appears reasonable. The choice of the ρ value is related to the considered performance criterion. The $ESR_{\%}$ has been evaluated for a single experiment identifying filters $\hat{V}^{SM}$ with different ρ in the range $[0.9, 1]$. In figure 4 the results are plotted and the best performances are obtained with $\rho = 0.965$: this suggested to maintain this choice of ρ throughout the overall Monte Carlo simulation. Reduced order filters $\hat{V}_n^{SM}$ have been identified by suitably approximating the FIR filter $\hat{V}^{SM}$.

Since in a practical situation the system order is not a priori known, filters of order 1 to 8 have been obtained and the dependence of the estimation quality on the model order has been evaluated for any methodology.

It is a common practice in automotive tests to use the data acquired with the random road profile as identification set. However, in [8] it turned out that the best results in terms of estimation error can be obtained with data acquired with the pavé profile; hence, either identification set has been considered in this paper. Table II reports the mean $ESR_{\%}$ achieved in the Monte Carlo simulations with all the road profiles in the testing set for filter structures of order $n = 1, \ldots, 8$.

Exploiting the physical insight, the two-step procedure naturally leads to design 4^{th} order Kalman filters. Instead, being the direct procedures essentially black-box approaches, DVS filters have to be chosen as the best trade-offs between estimation error and filter complexity. According to the results in Table II, for both the direct methodologies 3^{rd} order filters can be picked out, regardless of the data considered for the identification.

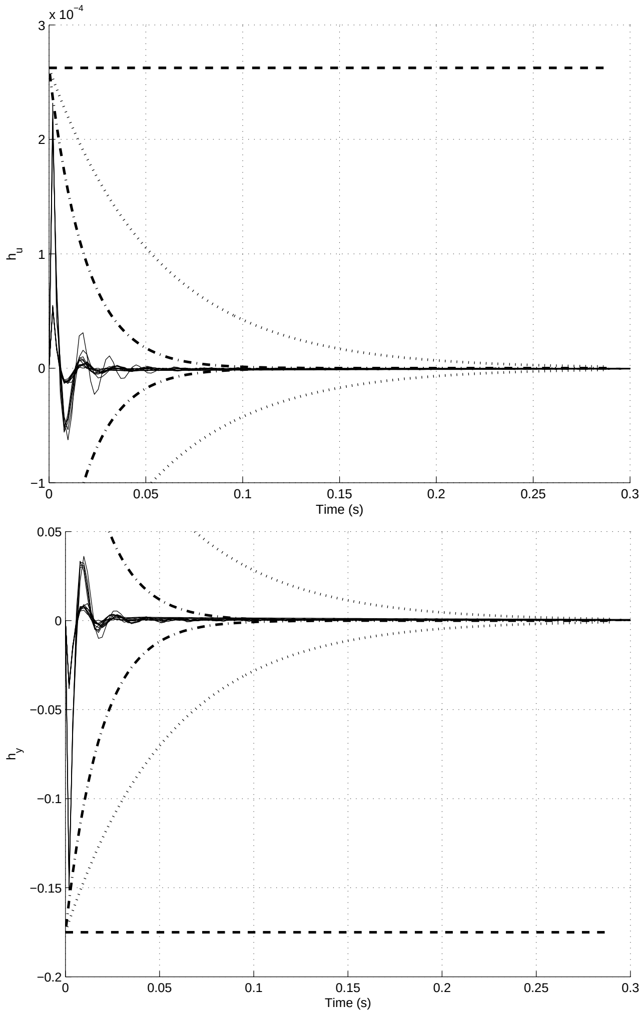

Fig. 3. Estimated DVS impulse responses (solid lines) and possible bounds for the DVS-SM using $L_u = 3.75 \cdot 10^{-4}$ and $L_y = 0.175$: $\rho = 1$ (dashed line), $\rho = 0.965$ (dotted line), $\rho = 0.9$ (dash-dotted line). Top, impulse response from applied suspension force f to relative speed $\dot{x}_{wc}$. Bottom, impulse response from chassis acceleration $\ddot{x}_c$ to relative speed $\dot{x}_{wc}$.

B. Virtual Sensors Identification Under Set Membership Noise

For each experiment, amplitude bounded noise sequences corrupt the system outputs $\ddot{x}_c$ and $\dot{x}_{wc}$. At each time instant, the noise is a realization of a random variable uniformly distributed into a range bounded by 5% of the instantaneous signal amplitude.

Note that the stochastic framework methodologies implicitly assume a Gaussian noise and then cannot take into account the information on the error boundedness. Nevertheless, these procedures have been applied in order to compare the achieved performances.

In the direct Set Membership methodology, filters $\hat{V}^{SM}$ in the form (7) have been identified by solving the identification problem (9), using $p = 2$ as norm and $W_\delta = \text{diag}(w_{\delta,1}, \ldots, w_{\delta,\tau}) = \text{diag}(Z_1^\tau)$, being $Z_1^\tau = [z^1, \ z^2, \ \ldots, \ z^{\tau-1}, \ z^\tau]^T$ the output measurements vector. The *a priori* information on the model class has been obtained in the same manner presented in the Gaussian case. Reduced order filters $\hat{V}_n^{SM}$ have been identified by suitably approximating the FIR filter $\hat{V}^{SM}$.

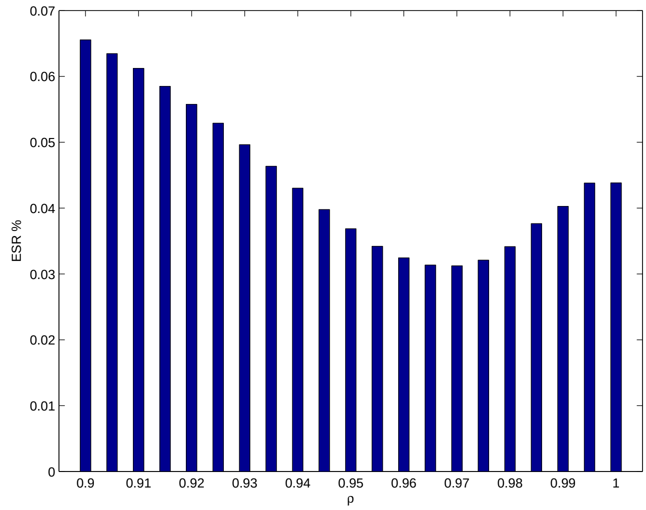

Fig. 4. $ESR_{\%}$ of filters $\hat{V}^{SM}$ versus the choice of ρ value.

TABLE II
MEAN $ESR_{\%}$ UNDER WHITE GAUSSIAN NOISE

	Random Road as Id Set			Pavé Road as Id Set		
n	$\hat{K}$	$\hat{V}$	$\hat{V}_n^{SM}$	$\hat{K}$	$\hat{V}$	$\hat{V}_n^{SM}$
1	233.7	21.9	21.9	207.3	21.9	21.9
2	47.4	10.0	8.5	37.5	8.5	8.3
3	19.5	4.8	4.1	15.8	3.9	3.8
4	12.2	4.6	4.3	4.9	4.0	3.8
5	8.3	4.5	4.2	4.8	3.9	3.8
6	11.2	4.5	4.3	4.8	3.9	3.8
7	8.1	4.4	4.3	6.3	3.9	3.8
8	7.5	4.4	4.3	6.2	3.9	3.8
148	-	-	3.4	-	-	3.1

The filter performances have been evaluated on the testing set, but in this case the weighted 2 norm of the estimation error, evaluated as:

$$WN = \left(\sum_{k=t_0}^{t_f} \left| w_{\delta,k}^{-1} \delta^k \right|^2 \right)^{1/2}$$

has been used as performance criterion, being t_0 and t_f the initial and the final time instants of any experiment, respectively, and $\delta^k = z^k - \hat{z}^k$. The average value of the WN for the 100 obtained filters has been considered as a criterion to compare the three algorithms.

For each noise realization, filters of order 1 to 8 have been estimated using the three reported methodologies. Data acquired with the random road and the pavé profiles have been considered as identification set, as explained in the stochastic case. Table III reports the mean WN achieved in the Monte Carlo simulations with all the road profiles in the testing set. Table IV reports the corresponding mean $ESR_{\%}$ achieved by the same filters, to allow a fear comparison with the results obtained in the Gaussian case.

TABLE III

MEAN WN UNDER SET MEMBERSHIP NOISE

n	Random Road as Id Set $\hat{K}$	Random Road as Id Set $\hat{V}$	Random Road as Id Set $\hat{V}_n^{SM}$	Pavé Road as Id Set $\hat{K}$	Pavé Road as Id Set $\hat{V}$	Pavé Road as Id Set $\hat{V}_n^{SM}$
1	904.9	176.9	81.9	1008.2	179.1	174.5
2	357.1	54.9	53.8	381.2	53.4	58.3
3	161.4	31.6	28.6	136.9	22.2	15.3
4	89.4	25.3	20.1	35.8	21.7	14.7
5	98.1	26.5	19.1	30.3	14.8	12.9
6	111.8	24.5	19.1	29.5	15.2	13.0
7	72.6	21.7	18.6	27.9	14.6	12.7
8	54.4	23.4	19.3	37.7	14.0	12.7
148	-	-	15.4	-	-	14.0

TABLE IV

MEAN $ESR_{\%}$ UNDER SET MEMBERSHIP NOISE

n	Random Road as Id Set $\hat{K}$	Random Road as Id Set $\hat{V}$	Random Road as Id Set $\hat{V}_n^{SM}$	Pavé Road as Id Set $\hat{K}$	Pavé Road as Id Set $\hat{V}$	Pavé Road as Id Set $\hat{V}_n^{SM}$
1	249.3	21.5	109.1	268.1	21.4	21.5
2	65.7	7.7	8.6	107.6	7.3	8.5
3	22.2	3.7	4.2	20.4	3.6	2.9
4	18.0	3.3	3.0	5.4	3.5	2.7
5	15.5	3.3	2.8	4.3	2.7	2.6
6	15.7	3.2	2.8	4.3	2.7	2.6
7	9.6	2.9	2.8	4.0	2.7	2.6
8	7.1	3.1	2.8	5.1	2.6	2.6
148	-	-	2.1	-	-	2.2

According to the results in Table III, on the base of the random road experiment DVS and DVS-SM of 4^{th} order can be picked out, while 5^{th} order DVS and 3^{rd} order DVS-SM might be reasonably chosen from the Pavé road experiment results.

Finally, in Table V the mean $ESR_{\%}$ achieved in the Monte Carlo simulations under Gaussian and Set Membership noises are summarized for the optimal virtual sensors selected according to the above criteria.

TABLE V

MEAN $ESR_{\%}$ FOR 4_{th} ORDER KALMAN FILTERS AND OPTIMAL DVSS

Road	Gaussian noise $\hat{K}$	Gaussian noise $\hat{V}$	Gaussian noise $\hat{V}_n^{SM}$	Gaussian noise $\hat{V}^{SM}$	Set Membership noise $\hat{K}$	Set Membership noise $\hat{V}$	Set Membership noise $\hat{V}_n^{SM}$	Set Membership noise $\hat{V}^{SM}$
RR	12.2	4.8	4.1	3.4	18.0	3.3	3.0	2.1
PV	4.9	3.9	3.8	3.1	5.4	2.7	2.9	2.2

V. CONCLUSIONS

From the presented results it turns out that, both the direct methodologies outperform Kalman filters and, even more important, do not suffer from performance degradation caused by under-modeling that may occur with Kalman filters. This allows, in the direct procedures, to use a sub-optimal solution of lower complexity that the optimal, with an acceptable loose of performance.

For any given order, the filters identified with the direct Set Membership methodology offer better performances than those identified with the stochastic one, in particular when the random road data is used. Furthermore, in order to achieve the performances of the DVS-SM, higher order structures should be used for the DVSs identified with the stochastic methodology. Note that in most of the cases the FIR DVS-SM perform better than DVSs and reduced order DVS-SM.

When amplitude bounded noise corrupts the measurements, the Kalman filters offer a poor performance, even using high order structures. Their WN is more than two times higher than that of the direct virtual sensors. This results remark the sensitivity of the two-step approach to the kind of noise corrupting the observations.

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