Bounded error detection of the wind gusts
effect on a small-scaled helicopter

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Abstract This paper deals with the problem of estimating the effects of wind gusts on a small helicopter type UAV. A model of the UAV is first described, including potential effects of wind. Identification of the state vector representing the UAV is performed using a set estimation technique in order to characterize the variation boundaries of the flight induced by the presence of wind. Simulation results are presented to illustrate the detection performance.

Keywords: Helicopter modeling, bounding approach state estimation, atmospheric turbulence detection.

1. INTRODUCTION

Unmanned aerial vehicles (UAV) have been sought for a wide range of applications such as surface and utilities inspection, search and rescue operations, air pollution monitoring and environmental surveillance. However, in the case of small UAV’s, the flight domain is rapidly affected by atmospheric turbulence. Hence, disturbances such as wind gusts have a very significant nonlinear effect on the expected performance of the guidance and autopilot loop and therefore strongly affect the feasibility of the mission. In most descriptions of guidance loop synthesis for these vehicles, such an impact has not been taken into account. This contribution consists of describing a procedure to detect the effect of aerological disturbances on a small helicopter-type UAV so that the guidance loop could take this effect into account. The paper is organised as follows. We first describe the UAV and its dynamical model. The estimation procedure, based on the use of set-membership approach, is presented and the main steps of the algorithm are described. A simulated example is used to illustrate the obtained results.

1.1 UAV description

The UAV considered in this paper is a VARIO Benzin-Acrobat. The vehicle is powered by a 23 cm³ Zenoah engine connected through a shaft to the main rotor blades of diameter 1.80 m. Its nominal weight is 6.5 kg for the commercial version. Figure 1 presents the vehicle used for experimental purposes by the ONERA DCSD team in Toulouse. This helicopter has been equipped with several sensors including an IMU and a GPS.

1.2 Dynamic equations

The dynamics of the helicopter are modelled using a conventional six degree of freedom rigid body model driven by forces and moments that explicitly include the effects of the main and tail rotors. Let $\mathbf{\zeta} = (x y z)^T$ denote the relative position vector of the helicopter regarding the inertial frame ($I$). The velocity of the center of mass expressed in the inertial frame is denoted by $\mathbf{v} = \dot{\mathbf{\zeta}}$. Let $m$ be the total mass, $\mathbf{I} = diag(I_1, I_2, I_3)$ be the diagonal matrix representing the inertia of the helicopter and $(e_1 e_2 e_3)$ be a canonical basis of $\mathbb{R}^3$. The vector $\mathbf{\Omega} = (p q r)^T$ is the angular velocity of the vehicle in the body fixed frame ($\mathcal{B}$).

Let $R : (\mathcal{B}) \rightarrow (I)$ denote the rotation matrix representing the orientation of the body fixed frame ($\mathcal{B}$), where $R \in SO(3)$ is an orthogonal matrix defined by:

$$R =
\begin{pmatrix}
c_{\phi}c_{\theta} - s_{\phi}s_{\theta}c_{\psi} & c_{\phi}s_{\theta} + s_{\phi}s_{\psi} & c_{\psi}s_{\phi} \\
c_{\phi}s_{\theta} + s_{\phi}s_{\psi} & c_{\phi}c_{\theta} - s_{\phi}s_{\psi} & -c_{\psi}c_{\phi} \\
-s_{\phi} & c_{\phi} & 0
\end{pmatrix}
$$

(1)

where $c_{\alpha} = \cos \alpha$, $s_{\alpha} = \sin \alpha$ and $(\phi, \theta, \psi)$ represent the Euler angles.

The dynamic evolution of $R$ is given by (Murray et al.
\[ \dot{\mathcal{R}} = \mathcal{R}\Omega_x \]

where

\[ \Omega_x = \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix} \]

is the skew symmetric matrix such that \( \Omega_x u = \Omega \times u \)

Using the Newton-Euler formalism, the dynamical equations of the helicopter motion are written as:

\[
\begin{align*}
\dot{\zeta} &= v \\
mb &= \mathcal{F} + \mathcal{F}_{\text{wind}} \\
\dot{\mathcal{R}} &= \mathcal{R}\Omega_x \\
\dot{\Omega} &= -\Omega \times \Omega + \mathcal{M} + \mathcal{M}_{\text{wind}}
\end{align*}
\]

where \( \mathcal{F} \) is the resulting external force acting on the helicopter and \( \mathcal{M} \) is the resulting external torque applied to the helicopter at the center of mass, without wind. The terms \( \mathcal{F}_{\text{wind}} \) and \( \mathcal{M}_{\text{wind}} \) represent the force and torque due to the effect of the wind gusts respectively. The following paragraph describes the explicit expression of these terms.

**Resulting force \( \mathcal{F} \):** The different forces acting on the system are

- the thrust of the propellers \( F_p \),
- the weight \( mg \),
- the aerodynamic force of the fuselage \( F_f \).

The thrust of the propellers \( F_p \) results from the main rotor thrust \( F_{mr} \) and the tail rotor thrust \( F_t \). The resulting force \( \mathcal{F} \) is then obtained as

\[ \mathcal{F} = mge_3 + \mathcal{R}(F_{mr} + F_t + F_f) \]

The main rotor dynamics play a major role in terms of its contribution to the dynamics of the helicopter flight.

In the following, we assume that the three assumptions below are satisfied (Gavrilets et al. (2001)):

- the vertical flapping angle \( \beta \) is assumed to remain very small,
- the angle of attack of each blade is small,
- the angular accelerations are negligible compared to the angular rate of turn multiplied by the rotation speed.

Under these assumptions, the longitudinal and lateral blade flapping dynamics \( a_1 \) and \( b_1 \) can be modeled by the two following coupled first-order differential equations:

\[
\begin{align*}
\tau_f \dot{a}_1 + a_1 &= -\tau_f q + \frac{8}{\gamma \Omega^2} I_3 \dot{\beta} + K_{\text{lon}}(\delta_{\text{lon}} + K_a c_1) \\
\tau_f \dot{b}_1 + b_1 &= -\tau_f p + \frac{8}{\gamma \Omega^2} I_3 \dot{\beta} + K_{\text{lat}}(\delta_{\text{lat}} + K_b d_1)
\end{align*}
\]

where \( \gamma \) is the Lock number, \( k_3 \) is the rotor solidity ratio, \( I_3 \) is the blade moment of inertia about the flapping hinge and \( \tau_f \) is the effective rotor time constant (\( \tau_f > 0 \)). \( K_{\text{lon}} \) and \( K_{\text{lat}} \) are the effective steady-state longitudinal and lateral gains from the inputs to the Bell bar flap angles, \( c_1 \) and \( d_1 \) are the longitudinal and lateral stabilizer Bell bar states respectively. The dynamic equations can be written as (Gavrilets et al. (2001)):

\[
\begin{align*}
\tau_c \dot{c}_1 + c_1 &= -\tau_c q + C_{\text{lon}} \delta_{\text{lon}} \\
\tau_d \dot{d}_1 + d_1 &= -\tau_d p + D_{\text{lat}} \delta_{\text{lat}}
\end{align*}
\]

\( C_{\text{lon}} \) and \( D_{\text{lat}} \) are the effective steady-state longitudinal and lateral gains from the inputs to the Bell bar flap angles \( \tau_c \) is the stabilizer bar’s time (\( \tau_c > 0 \)).

Assuming that the longitudinal and lateral flapping angles \( a_1 \) and \( b_1 \) are small, the total thrust of the main rotor \( F_{mr} \) can be approximated by:

\[ F_{mr} \approx |F| \begin{pmatrix} -a_1 \\ b_1 \\ -1 \end{pmatrix} \]

where \( F \) is the norm of the thrust generated by the main rotor:

\[ F = mg + K_z(\delta_{\text{col}} - \delta_{\text{sta}}) \]

\( K_z \) can be experimentally obtained as the ratio between the vertical acceleration and the control value \( \delta_{\text{col}} \) and \( \delta_{\text{sta}} \) is the control value in stationary hovering flight condition.

**Tail rotor dynamics:** The tail rotor thrust representation does not take into account complex interactions between the main and tail rotors. As in Gavrilets et al. (2001) the thrust is expressed as being proportional to the lift of the tail rotor and the control yaw.

\[ F_{tr} = (0 K_{\text{ped}} \delta_{\text{ped}} 0) \]

where \( \delta_{\text{ped}} \) is the control input and \( K_{\text{ped}} \) is constant (\( K_{\text{ped}} > 0 \)).

**Fuselage force \( F_f \):** The fuselage force mostly originates from air drag. The drag force is defined along the body coordinate system axes and is assumed to act on the body and thus creates no torque. It can be approximated by (Gavrilets (2003)):

\[ F_f = -\frac{1}{2} \rho \begin{pmatrix} c_f S_f^2 V_\infty u_a \\ c_f S_f^2 V_\infty v_a \\ c_f S_f^2 V_\infty w_{in} \end{pmatrix} \]

where \( S_f^x, S_f^y, S_f^z \) are the effective drag areas of the fuselage in \( x, y, z \) directions; \( c_f \), \( c_f^x \) and \( c_f^y \) are the drag, lateral, and lift coefficients respectively; the velocity \( V_\infty \) is defined as follows:

\[ V_\infty = \sqrt{u_a^2 + v_a^2 + (w_a + w_{in})^2} \]

where \( (u_a, v_a, w_a) \) is defined as the difference between the velocity of the vehicle expressed in the body frame \( (B) \) and the wind velocity \( v_a = u - u_w; v_a = v - v_w; w_a = w - w_w \); \( w_{in} \) is the induced velocity in hover flight condition whose formula is given by:

\[ w_{in} = \sqrt{\frac{mg}{2 \rho \pi R_{mr}^2}} \]

\( R_{mr} \) is the main rotor radius.

**Resulting torque \( \mathcal{M} \):** The resulting torque created by the various forces can be represented by the expression:

\[ \mathcal{M} = b_{mr} \wedge F_{mr} + b_t \wedge F_t + \Delta_k \]

where \( b_{mr} \) and \( b_t \) are the moment arm of the main and tail rotors respectively; \( \Delta_k \) is the roll moment corresponding...
to the spring effect of the rotor hub (Mahony et al. (1999)). This term may be expressed as
\[ \Delta_k = k g \left( b_1 a_1 0 \right)^t \]

Effects of wind gusts: First the wind gust model will be described. The corresponding force \( F_{\text{wind}} \) and torque \( M_{\text{wind}} \) acting on the helicopter will be detailed afterwards.

Wind gust model: The mathematical model of wind gusts is a simple first-order model.

- Wind gust component along the x-axis:
  \[
  \begin{align*}
  u_w &= 0 & \text{for } x < x_{sg} \\
  u_w &= \frac{u_m}{2} (1 - \cos (\pi x / L_u)) & \text{for } x_{sg} < x < x_{eg} \\
  u_w &= 0 & \text{for } x_{eg} < x
  \end{align*}
  \tag{10}
  \]

- Wind gust component along the y-axis:
  \[
  \begin{align*}
  v_w &= 0 & \text{for } x < x_{sg} \\
  v_w &= \frac{v_m}{2} (1 - \cos (\pi x / L_u)) & \text{for } x_{sg} < x < x_{eg} \\
  v_w &= 0 & \text{for } x_{eg} < x
  \end{align*}
  \tag{11}
  \]

- Wind gust component along the z-axis:
  \[ w_w = 0 \tag{12} \]

- Wind gust velocity norm:
  \[
  V_w = \sqrt{u_w^2 + v_w^2 + w_w^2} \tag{13}
  \]
where \( x_{sg} \) and \( x_{eg} \) are respectively the distance between the helicopter location and the gust origin location before entering it and the distance between the helicopter location and the gust origin location after leaving it. \( L_u \) is the half length of the gust. The values \( u_m \) and \( v_m \) are the densities of the gust velocity.

Resulting force \( F_{\text{wind}} \) and torque \( M_{\text{wind}} \): In this section, we consider the effect of a lateral wind gust. This lateral perturbation acting on the fuselage causes a drag force \( F_w^f \) along the x and y body axes of the helicopter and a torque that is assumed negligible:
\[
F_w^f = - \frac{1}{2} \rho c_w^w c_y^w V_w u_w
\]
where \( c_w^w \) and \( c_y^w \) are the aerodynamics coefficients. The presence of a wind gust creates a destabilizing roll effect on the main rotor (Prouty (1990)) which is modeled as a torque \( M_{mr}^w \) about the axe \( z_b \):
\[
M_{mr}^w = \frac{1}{4k \beta} \rho S_{mr} c_{mr} R_{mr} \gamma V_w \sqrt{(0.75 R_{mr})^2 + b_2^2} \begin{bmatrix} 0 & 0 \ 0 & 0 \end{bmatrix}^t
\]
where \( S_{mr} \) is the area of the main rotor disk; \( c_{mr} \) is the thrust coefficient; \( \gamma \) is the azimuth angle of the main rotor blades.

The gust also creates disturbances on the tail rotor which are summarized as a force \( F_{tr}^w \)
\[
F_{tr}^w = \left( 0 \ 4 \rho \pi F_{tr}^2 R_{tr}^2 V_w w_w \right)^t
\]
and a torque \( M_{tr}^w \)
\[
M_{tr}^w = b_{tr} \wedge F_{tr}^w
\]
Thus, the resulting force and torque in the presence of wind gust are:
\[
\begin{align*}
F_{\text{wind}} &= R. (F_w^f + F_{tr}^w) \\
M_{\text{wind}} &= M_{mr}^w + M_{tr}^w
\end{align*}
\tag{14-15}
\]

2. ESTIMATION OF THE WIND GUST EFFECT

The aim of the detection method is not to exactly characterize the aerological perturbation but to evaluate if they have a global effect on the flight ability of the vehicle. This is why we are not looking for a single point estimate corresponding to the mean value evolution but for a characterization of the boundaries variation of perturbations.

Set estimation is therefore a well-suited technique for that purpose.

2.1 Ellipsoidal state estimation

We consider a dynamic discrete state space evolution of the form
\[
\begin{bmatrix} X_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} A_k & B_k \ end{bmatrix} \begin{bmatrix} X_k \\ z_k \end{bmatrix} + \begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{16}
\]
In the following, we make the assumption that the state perturbations and noise measurements are unknown but bounded without additional underlying probabilistic assumptions. We are therefore seeking the set of all values of the state vector that are consistent with this hypothesis in the sense that all errors fall within specified known bounds. The set of perturbations and noise bounds are expressed as
\[
\begin{align*}
w_k &\leq W_k^{-1} W_k \leq 1 \\
v_k &\leq V_k^{-1} V_k \leq 1
\end{align*}
\]
where \( W_k \) and \( V_k \) are given matrices. This approach corresponds to guaranteed estimation that was initially introduced by Schweppe (1968) and by Bertsekas and Rhodes (1971). Several methods have been developed to define the boundary of this set or to compute a set guaranteed to contain it. The main approaches compute the approximate set under the form of an ellipsoid, but other methods have been described that result in characterizing a polyhedral boundary or determine the union of intervals to which all state vectors belong (for example Milanese et al. (1996) for an overview).

The approach used in this work is based on the extension of the single-cut ellipsoid algorithm (Pronzato and Walter (1994)) initially designed for parameter estimation for state estimation problems as described in Maksarov and Norton (2002).

The initial phase of the algorithm defines an ellipsoid that is assumed to contain all values of the initial state vector \( X_0 \). The ellipsoid is characterized by its center \( X_0 \) and its associated matrix \( P_0 \) that defines its main axes and amplitude.

The estimation method is based on two steps very similar to those of Kalman filtering.

First, one produces a predicted ellipsoid on a given time horizon. This prediction set is centered at the predicted ellipsoid center, \( \hat{X}_{\hat{k}/k+1} = A_k \hat{X}_k + B_k \hat{u}_k \). Updates of the associated matrix must take into account the assumed state
perturbations. We operate \( n_w \) successive modifications \((n_w \text{ being the dimension of the state perturbation vector } w)\), of the ellipsoid, each corresponding to the uncertainty modification due to one process noise component. This procedure is suboptimal for non scalar perturbations but proves to be computationally efficient. Moreover, the perturbations are assumed to belong to an orthotope whose equations are:

\[
W_k = \bigcap_i \{ w_k : -b_i \leq c_i^t w_k \leq b_i \}
\]

where \( b_i \) is the bound associated with the eigenvector \( c_i \) of \( W_k \). The initial ellipsoid matrix \( P_{k-1} \) is obtained as \( A_k P_k A_k^T \). In addition, each modification operates the transformation:

\[
P_k = (1 + \rho_i) P_{k-1} + (1 + \rho_i^{-1}) b_i^2 G_k c_i^t G_i^t \]

\( i = 0, \ldots, l - 1 \)

where the value of \( \rho \) is the positive root of

\[
n \rho_i^2 + (n - 1) a_i \rho_i - a_i = 0
\]

where \( a_i = b_i^2 c_i^t G_i (P_{i-1}^{-1}) G_k c_i \) and \( n \) is the state dimension.

Finally \( P_{k-1} = P_k \).

When the prediction step has been realized, an update of the estimated set is performed by taking a new measurement into account. We first seek whether the predicted model error satisfies the assumption on the measurement outer bounds. We compute the normalized distance from the ellipsoid center to the new bounds by

\[
\alpha = \frac{e_k^t V_k^{-1} c_k - \sqrt{e_k^t V_k^{-1} c_k}}{h_k^t P_{k-1}^{-1} h_k}
\]

where \( e_k \) is the innovation \((e_k = y_k - F_k \hat{X}_{k+1})\) and \( h_k \) is the normal to the approximating hyperplane \((h_k = F_k e_k)\).

If \( \alpha \) is greater than 1, the intersection is empty and a specific procedure must be defined. If \( \alpha \) is lower or equal to \(-1/n\), the measurement does not carry any additional information and no update is required. In other cases, the state space estimated set must be updated. Define

\[
\tau_k = \frac{1 + n \alpha}{1 + \alpha}; \quad \sigma_k = \frac{2 \tau_k}{(1 + \alpha)}; \quad \beta_k = \frac{n^2 (1 - \alpha^2)}{n^2 - 1}
\]

The ellipsoid center \( \hat{X}_k \) is obtained as:

\[
\hat{X}_k = \hat{X}_{k+1} + \tau_k \frac{P_{k-1}^{-1} h_k}{\sqrt{h_k^t P_{k-1}^{-1} h_k}}
\]

and the symmetric definite positive matrix defining its shape is

\[
P_k = \beta_k (P_{k-1} - \sigma_k \frac{P_{k-1}^{-1} h_k h_k^t P_{k-1}^{-1}}{h_k^t P_{k-1}^{-1} h_k})
\]

It remains to define the procedure when the intersection is considered empty. The method implemented in the simulation operates a translation of the resulting ellipsoid toward the measurement image, halfway to the exact measurement image. This is of importance as the detection of aerological perturbations will be operated on the basis of a lack of consistency between predicted outputs and real measurements.

### 2.2 Detection of aerological perturbations

Our detection process of aerological disturbance is performed by comparing the prediction wind-free model to the measurements and thus deciding whether wind gusts react with the helicopter flight.

#### Choice of identification model:

The model used for identification purpose is described by equation (22).

\[
\begin{align*}
\dot{\zeta} &= v \\
\dot{m} &= F \\
\dot{\mathcal{R}} &= \mathcal{R}\Omega \\
\dot{\Omega} &= -\Omega \times \mathcal{I} + \mathcal{M}
\end{align*}
\]

#### Measurement vector:

The measurement vector \( y_m \) includes the sensor outputs, i.e., altitude \((m)\), velocity \((m/sec)\), Euler angles \((\text{deg})\), and angular velocity \((\text{rad/sec})\). The vector \( y_m \) is therefore defined by:

\[
y_m = (x, y, z, v_x, v_y, v_z, \phi, \theta, \psi, p, q, r)^t
\]

The unmeasured states \( a_1, b_1, c_1 \) and \( d_1 \) are estimated.

### 2.3 Simulation results

In this section, simulation results are presented to illustrate the ability of the bounded-error state estimation algorithm to detect the aerological disturbances. On each graphic, the curve \((-\) represents the predicted value without wind disturbance, the curve \((-,-\) corresponds to the ellipsoid center evolution, and the curve \((\_\) shows the boundary values of the estimated ellipsoid.

![Figure 2: Evolution of predicted and estimated z position](image)

The evolution of the position and velocity components are represented in Figures 2-3, and in Figures 4-5 respectively. The roll and yaw are exhibited in Figures 6-7 and the components of the angular velocities on Figures 8-9.

We can detect the impact of the lateral wind gust in the evolution of the position and the velocity in the x-y directions, and in the evolutions of roll, yaw, and p and r. Note that the position z, velocity v_z, pitch \(\theta\) and angular velocity \(q\) are not represented as the influence of aerological perturbation is very low. In this example, ellipsoidal bounding technique provides a suitable detection of longitudinal and lateral flapping angles and stabiliser bar Bell
Figure 3. Evolution of predicted and estimated $y$ position states which makes it possible to characterize the total thrust of the main rotor. Using a wind free model, we can observe that, before wind gust, the estimates converge to the "real" dynamical states. When a wind gust occurs, there is no longer consistency between predicted outputs and measurements for the set of values that are affected by the wind effect. The procedure described above to modify the ellipsoid when the measurements and predicted outputs are not consistent allows the tracking of the effect of the aerological perturbation. These results are of preliminary nature as it remains to be investigated their sensitivity to the hypothesis on the perturbation and noise bounds and to the reliability of the dynamical model.

3. CONCLUSION

In this paper, we have presented a method for detecting the effect of aerological perturbations on the flight performance of a small helicopter-like UAV. A model including a representation of the forces and torques induced by the presence of wind and aerological perturbations has been described. Identification procedure based on ellipsoidal characterization of the state vector variations has been applied on a simulated example of a UAV trajectory which is subjected to the influence of a wind gust after several seconds of wind-free flight. From the obtained results, it can be pointed out that it is possible to determine the time when there is no longer consistency between wind-free hypothesis and measurements and therefore to conclude on the presence of aerological effects. It remains to be studied the robustness of the resulting detection to both perturbation and noise bounds hypothesis and to the reliability of the model. Experimental work will be planned in the near future by using the Vario vehicle in the gust generator of the experimental area in ONERA. The final step is to define a suitable guidance law that modifies the UAV
Figure 8. Evolution of predicted and estimated angular velocity $\rho$

Figure 9. Evolution of predicted and estimated angular velocity $\sigma$

Figure 10. Evolution of predicted and estimated longitudinal and lateral blade flapping $a_1, b_1$

Figure 11. Evolution of predicted and estimated longitudinal and lateral stabilizer bar Bell $c_1, d_1$

trajectory whenever aerological effects are detected in order to satisfy both mission and security requirements.

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