OUTPUT CONTROLLABILITY AND STEADY-OUTPUT CONTROLLABILITY ANALYSIS OF FIXED SPEED WIND TURBINE

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Abstract

This paper deals with the concepts of output controllability and steady output controllability, it demonstrates that they are not equivalent of concepts. A linear system has been calculated from the nonlinear equations of the squirrel cage induction generator, supposing it connected directly to the grid and assuming a steady state operating point. The study of output controllability and steady-output controllability concepts of the introduced system is done.

Key words

Output Controllability, Steady-Output Controllability, Squirrel Cage Induction Generator, Linear System.

1 Introduction

In the theory of continuous linear time-invariant dynamical control systems the most popular and the most frequently used mathematical model is given by the following differential state equation and algebraic output equations

$$\left.\begin{array}{l}\dot{x}(t) = Ax(t) + Bu(t)\\ y(t) = Cx(t) + Du(t)\end{array}\right\} \quad (1)$$

where x is the state vector, y is the output vector, u is the input (or control) vector, $A \in M_n(\mathbb{R})$ is the state matrix, $B \in M_{n\times m}(\mathbb{R})$ is the input matrix, $C \in M_{p\times n}(\mathbb{C})$ is the output matrix, and $D \in M_{p\times m}(\mathbb{C})$ is the feedthrough (or feedforward) matrix.

Controllability is an important property of a control system, and the controllability property plays a crucial role in many control problems, such as stabilization of unstable systems by feedback, or optimal control (see [1], [4] for example).

Systematic study of controllability was started in the mid 20 century and theory of controllability is based on the mathematical description of the dynamical system.

Roughly, the concept of controllability denotes the ability to move a system around in its entire configuration space using only certain admissible manipulations. The exact definition varies slightly within the framework or the type of models applied. In the literature there are many different definitions of controllability which depend on the type of dynamical control system. In this paper it is considered the output controllability.

Output controllability is the related notion for the output of the system, the output controllability describes the ability of an external input to move the output from any initial condition to any final condition in a finite time interval. A controllable system is not necessarily output controllable, and an output controllable system is not necessarily controllable.

On the other hand, it is well known the recent increasing of wind power in the electrical network. Since, it can be interesting study and ensure the output-controllability of Fixed-Speed Wind Turbines (FSWT), which can affect directly the behavior of power systems.

This paper is organized as follows. In Section 2, it is introduced the concepts of controllability and output-controllability. The steady output-controllability is defined in Section 3. In the section 4, two examples are developed. In section 5, the system under study is presented, and linearized to obtain the linear system. The output controllability and steady output-controllability of the system is calculated in section 6. Finally, the conclusions are summarized in Section 7.

2 Controllability and output Controllability

The most frequently used fundamental definition of controllability for linear control systems with constant coefficients is the following.

Definition 2.1. *Dynamical system* (1) *is said to be controllable if for every initial condition* $x(0)$ *and every vector* $x_1 \in \mathbb{R}^n$*, there exist a finite time* t_1 *and control* $u(t) \in \mathbb{R}^m$, $t \in [0, t_1]$, *such that* $x(t_1) = x_1$.

This definition requires only that any initial state $x(0)$ can be steered to any final state x_1 at time t_1. However, the trajectory of the dynamical system between 0 and t_1 is not specified. Furthermore, there is no constraints posed on the control vector $u(t)$ and the state vector $x(t)$.

Controllability can be easily computed by means of the following algebraic criteria:
the system is controllable if and only if the matrix presented in the equation 2 has full rank.

$$C = \begin{pmatrix} B \; AB \; A^2B \ldots A^{n-1}B \end{pmatrix} \tag{2}$$

This matrix is called controllability matrix.

Theorem 2.1. *Dynamical system* (1) *is controllable if and only if rank* $C = n$.

Similar to the state controllability of dynamical control system, it is possible to define the so-called output controllability for the output vector $y(t) \in \mathbb{R}^p$ of dynamical system. Although these two concepts are quite similar, it should be mentioned that the state controllability is a property of the differential state equation , whereas the output controllability is a property both of the state equation and algebraic output equation.

Definition 2.2. *Dynamical system* (1) *is said to be output controllable if for every* $y(0)$ *and every vector* $y_1 \in \mathbb{R}^p$*, there exist a finite time* t_1 *and control* $u_1(t) \in \mathbb{R}^m$*, that transfers the output from* $y(0)$ *to* $y_1 = y(t_1)$.

Therefore, output controllability generally means, that we can steer output of dynamical system independently of its state vector.

For a linear continuous-time system, like (1), described by matrices A, B, C, and D, it is defined the output controllability matrix

$$oC = \begin{pmatrix} CB \; CAB \ldots CA^{n-1}B \; D \end{pmatrix} \tag{3}$$

and it is obtained the following result.

Theorem 2.2. *Dynamical system* (1) *is output controllable if and only if rank* $oC = p$.

It should be pointed out, that the state controllability is defined only for the linear differential state equation, whereas the output controllability is defined for the input-output description i.e., it depends also on the linear algebraic output equation. Therefore, these two concepts are not necessarily related.

Theorem 2.3. *The output controllability character is invariant under feedback.*

Proof. Let F be a matrix in $M_{m\times n}(\mathbb{R})$ Considering $(A+BF, B, C, D)$, it is easy to compute $(A+BF)^k$ obtaining

$$\begin{array}{l} C(A+BF)^kB = \\ CA^kB + \sum_{0\le\ell\le k-1} CA^{k-\ell-1}BF(A+BF)^\ell B \end{array}$$

Making the following column elementary transformations

$$c_j + c_{j-1}FB + c_{j-2}F(A+BF)B + \ldots + c_1F(A+BF)^{j-2}B$$

where c_ℓ indicates the ℓ column of the output controllability matrix of (A, B, C, D), it is obtained the output controllability matrix for $(A+BF, B, C, D)$.

3 Steady-output Controllability

Within the linear systems theory, it is often asked about the possibility that the state-steady outputs converge to a constant value.
In order to be able to analyze this concept, it is given the following definition.

Definition 3.1. *A vector is called constant steady-state output controllable if there exists an input constant vector* u *such that*

$$\lim_{t\to\infty} y(t) = K \tag{4}$$

where K *is a* $p \times 1$ *constant output vector.*

Taking Laplace transforms to the system

$$\left.\begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{array}\right\}, \tag{5}$$

reformulating this definition in the following manner.

Proposition 3.1. *A constant output vector* K *is steady-output controllable if there exists an input* $u(s) = \dfrac{k}{s}$ *such that*

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sy(s) = K \tag{6}$$

Clearly a necessary condition for constant steady-output controllability of the system is that the system be stable. The concept of stability is very important in systems theory.

Remember that a system is stable if and only if

$$\text{rank}\begin{pmatrix} s_0I_n - A & B \\ C & 0 \end{pmatrix} = \text{rank}\begin{pmatrix} sI_n - A & B \\ C & 0 \end{pmatrix}, \forall s_0 \in \mathbb{R}^+.$$

Proposition 3.2 ([5]). *A necessary and sufficient condition for constant steady-state output controllability of a stable system is*

$$\textit{rank}\begin{pmatrix} A & B \\ C & 0 \end{pmatrix} = n + \min\{m, p\} \tag{7}$$

No all systems are stable but some times, it is possible to stabilize them by means a feedback or/and an output injection, concretely, it can be said that a system as (5) is stabilizable if and only if there exist a feedback $F \in M_{m\times n}(\mathbb{R})$ or/and output injection $J \in M_{n\times p}(\mathbb{R})$ such that the system close loop system

$$\left.\begin{array}{l}\dot{x}(t) = (A + BF + JC)x(t) + Bu(t)\\ y(t) = Cx(t)\end{array}\right\} \qquad (8)$$

is stable.

Now, it can be analyzed the conditions for constant steady-state output controllability of a stabilizable system, having the following result.

Proposition 3.3. *A necessary and sufficient condition for constant steady-state output controllability of a stabilizable system is*

$$\textit{rank}\begin{pmatrix} A & B\\ C & 0\end{pmatrix} = n + \min\{m, p\}$$

Proof.

$$\text{rank}\begin{pmatrix} A+BF+JC & B\\ C & 0\end{pmatrix} =$$

$$\text{rank}\begin{pmatrix} I_n & J\\ 0 & I_p\end{pmatrix}\begin{pmatrix} A & B\\ C & 0\end{pmatrix}\begin{pmatrix} I_n & 0\\ F & I_m\end{pmatrix} =$$

$$\text{rank}\begin{pmatrix} A & B\\ C & 0\end{pmatrix}$$

Then

$$\text{rank}\begin{pmatrix} A & B\\ C & 0\end{pmatrix} = n + \min\{m, p\}$$

if and only if

$$\text{rank}\begin{pmatrix} A+BF+JC & B\\ C & 0\end{pmatrix} = n + \min\{m, p\}$$

4 Output controllability vs. Steady-output controllability

In order to demonstrate that both output controllability and steady output controllability are not equivalent concepts, two different examples are developed.

Example 4.1. *Let*

$$\left\{\begin{array}{l}\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}u\\ \\ y = \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\end{array}\right.$$

$$\text{rank}\begin{pmatrix}CB & CAB\end{pmatrix} = \text{rank}\begin{pmatrix}1 & 0\end{pmatrix} = 1 = p$$

Then the system is output-controllable.

$$\text{rank}\begin{pmatrix} A & B\\ C & 0\end{pmatrix} = \text{rank}\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} = 2 < n + \min(m, p) = 3$$

Therefore, the system is not steady-output controllable.

Example 4.2. *Let*

$$\left\{\begin{array}{l}\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}u\\ \\ \begin{pmatrix}y_1\\ y_2\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\end{array}\right.$$

$$\text{rank}\begin{pmatrix}CB & CAB\end{pmatrix} = \text{rank}\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix} = 1 < p = 2$$

Then the system is not output-controllable.

The system is stable because

$$\text{rank}\begin{pmatrix} s & 0 & 0\\ -1 & s & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix} = 3\ \forall s \in \mathbb{C}$$

and

$$\text{rank}\begin{pmatrix} A & B\\ C & 0\end{pmatrix} = \text{rank}\begin{pmatrix}0 & 0 & 0\\ 1 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix} = 3 = n + \min(m, p)$$

Then, the system is steady-output controllable.

5 Modeling of FSWT

The global analyzed system is a wind power generator connected directly to the grid. The controllability condition for the system described in this section can be found in [2].

The linear system is defined by means of the squirrel cage induction generator differential equations. The differential equations of the generator are time dependant. Its inputs are the voltage of the grid. The outputs are the active and reactive power delivered by the wind power generator.

Supposing the system to be in steady state. This hypothesi implies constant slip. Therefore, the system can be described as:

$$\underbrace{\frac{d}{dt}\begin{pmatrix}\Delta i_{sq}\\ \Delta i_{sd}\\ \Delta i_{rq}\\ \Delta i_{rd}\end{pmatrix}}_{\dot{X}} =$$

$$\underbrace{-\frac{1}{L_sL_r - M^2}\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4\\ -\alpha_2 & \alpha_1 & -\alpha_4 & \alpha_3\\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8\\ -\alpha_6 & \alpha_5 & -\alpha_8 & \alpha_7\end{pmatrix}}_{A}\underbrace{\begin{pmatrix}\Delta i_{sq}\\ \Delta i_{sd}\\ \Delta i_{rq}\\ \Delta i_{rd}\end{pmatrix}}_{X} \tag{9}$$

$$+\underbrace{\frac{1}{L_sL_r - M^2}\begin{pmatrix}L_r & 0\\ 0 & L_r\\ -M & 0\\ 0 & -M\end{pmatrix}}_{B}\underbrace{\begin{pmatrix}\Delta v_{sq}\\ \Delta v_{sd}\end{pmatrix}}_{U}$$

where

$$\begin{aligned}
\alpha_1 &= L_r r_s\\
\alpha_2 &= M^2\dot{\theta}_r + (L_sL_r - M^2)\dot{\theta}\\
\alpha_3 &= -Mr_r\\
\alpha_4 &= ML_r\dot{\theta}_r\\
\alpha_5 &= -Mr_s\\
\alpha_6 &= -ML_s\dot{\theta}_r\\
\alpha_7 &= L_s r_r\\
\alpha_8 &= (L_sL_r - M^2)\dot{\theta} - L_sLr\dot{\theta}_r
\end{aligned}$$

The α_i parameters have constant value. They are dependant of the machine parameters such as stator and rotor impedance. Moreover Δ indicates a little variation of the selected operating point.

5.1 Linearizing the System

The desired output signals, both active and reactive power are nonlinear functions, described as:

$$\begin{aligned}
Q_s &= \tfrac{3}{2}(v_{sd}i_{sq} - v_{sq}i_{sd})\\
P_s &= \tfrac{3}{2}(v_{sd}i_{sd} + v_{sq}i_{sq})
\end{aligned} \tag{10}$$

Then, it is necessary linearize these equations to obtain the linear system of the outputs.
Hence, applying Taylor's approximation around the steady state operating point to these equations.

$$\begin{aligned}
Q_{ss} = \tfrac{3}{2}(&\underbrace{(v_{sd0}i_{sq0} - v_{sq0}i_{sd0})}_{Q_{ss0}}\\
&+(v_{sd0}\Delta i_{sq} - v_{sq0}\Delta i_{sd} + i_{sq0}\Delta v_{sd} - i_{sd0}\Delta v_{sq}))\\
P_{ss} = \tfrac{3}{2}(&\underbrace{(v_{sd0}i_{sd0} + v_{sq0}i_{sq0})}_{P_{ss0}}\\
&+(v_{sd0}\Delta i_{sd} + v_{sq0}\Delta i_{sq} + i_{sd0}\Delta v_{sd} + i_{sq0}\Delta v_{sq}))
\end{aligned} \tag{11}$$

where the values with the 0-subscript are the constant values corresponding to the steady state operating point.
To simplify the calculations, it is used to linearize the system a small variation in the power values.

$$\begin{aligned}
\Delta Q_{ss} &= Q_{ss} - Q_{ss0}\\
\Delta P_{ss} &= P_{ss} - P_{ss0}
\end{aligned} \tag{12}$$

Then, the output system described as Y = CX + DU can be written as follows:

$$\begin{pmatrix}\Delta Q_{ss}\\ \Delta P_{ss}\end{pmatrix} = \begin{pmatrix}v_{sd0} & -v_{sq0} & 0 & 0\\ v_{sq0} & v_{sd0} & 0 & 0\end{pmatrix}\begin{pmatrix}\Delta i_{sq}\\ \Delta i_{sd}\\ \Delta i_{rq}\\ \Delta i_{rd}\end{pmatrix} + \begin{pmatrix}-i_{sd0} & i_{sq0}\\ i_{sq0} & i_{sd0}\end{pmatrix}\begin{pmatrix}\Delta v_{sq}\\ \Delta v_{sd}\end{pmatrix} \tag{13}$$

6 Output controllability and steady output controllability of FSWT

In the following section, it is studied output controllability and steady-output controllability of FSWT.

Applying the theorem 2.1 in the linearized system, it can be computed rank oC. Let $S \in Gl(n;\mathbb{R})$ be such that $C_1 = SC = \begin{pmatrix}I_2 & 0\end{pmatrix}$ it is easy to prove that

$$\text{rank}\, oC = \text{rank}\begin{pmatrix}C_1B & C_1AB & C_1A^2B & C_1A^3B & D\end{pmatrix} = 2.$$

Therefore, the system is output controllable.
With respect steady-output controllability, it is computed rank $\left(\begin{smallmatrix}A & B\\ C & 0\end{smallmatrix}\right)$ in order to apply proposition 3.3.
From the parameters of the generator, it can be guarantied $M^2 \neq L_rL_s$, thus the matrix B has also full rank.
Hence,

$$\text{rank}\begin{pmatrix}A & B\\ C & 0\end{pmatrix} = 6 = n + \min\{m, p\}$$

and the system is steady-output controllable.

7 Conclusion

This paper has presented the concepts of output controllability and steady output controllability. Moreover, by means of two different examples have been demonstrate the non equivalence of both concepts. Also, a linear system has been calculated from the nonlinear equations of the squirrel cage induction generator.
Output controllability and steady output controllability have been demonstrated using the A,B,C matrices.
Moreover, the demonstration is made with a generic

system. Therefore, it can be ensured not only for an example.

Due to the output controllability condition, it can be concluded that any output can be reached regulating the voltage inputs.

On the other hand, steady output controllability condition can ensure the output controllability on a long term.

References

C.T. Chen, "Introduction to Linear System Theory". Holt, Rinehart and Winston Inc, New York, (1970).

J.L. Domínguez-García (2010), " Modeling and control of squirrel cage induction generator with full power converter applied to windmills". Master Thesis, Oulu

Katsuhiko Ogata Modern Control Engineering (3rd ed.), Ed. Prentice-Hall, NJ, (1997).

J. Klamka, *Controllability of dynamical systems-a survey*. Archives of Control Sciences **2**, pp. 281-307, (1993).

D. Qiu, Q. Wang, Y. Zhou, *Steady-state output controllability and output controllability of linear systems*, Computational Intelligence and Industrial Applications, IEEExplore (2009). On page(s): 147 - 150