MODELING VOLCANOMAGNETIC DYNAMICS BY RECURRENT LEAST-SQUARES SUPPORT VECTOR MACHINES

Stanislaw Jankowski[1], **Gilda Currenti**[2], **Rosalba Napoli**[2], **Zbigniew Szymanski**[1], **Luigi Fortuna**[3], **Ciro Del Negro**[2]

[1)] Warsaw University of Technology,
Poland
sjank@ise.pw.edu.pl

[2)] Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Catania,
Italy

[3)] Dipartimento Di Ingegneria Elettrica Elettronica e dei Sistemi Università di Catania,
Italy

Abstract

Nonlinear dynamic systems can be described by means of statistical learning theory: neural networks and kernel machines. In this work the recurrent least-squares support vector machines are chosen as learning system. The unknown dynamic system is a mapping of past states into the future. The recurrent system is implemented by special data preparation in the learning phase. The next iterations can be calculated but the convergence is usually not guaranteed. Due to the fact that the predicted trajectory can diverge from the real trajectory the semi-directed mode can be applied, i.e. after several prediction steps the system is updated by using the current values of the considered process as new initial conditions.
The idea was tested on the data generated by the chaotic dynamic system – the Chua's circuit. The methodology was then applied to real magnetic data acquired at Etna volcano.

Key words

recurrent ls-svm, volcanomagnetic dynamics

1. Recurrent Least-Squares Support Vector Machine

Least-squares support vector machine (LS-SVM) originates by changing the inequality constraints in the support vector machine (SVM) [1] formulation to equality constraints with objective function in the least squares sense [2, 3, 4].
The learning set for the regression task consists of pairs: input vectors and the target function value $D = \{(\mathbf{x}_i, t_i)\}\ \mathbf{x}_i \in X \subset R^d,\quad t_i \in R$. The model is expressed as: $f(\mathbf{x}) = \mathbf{w}\phi(\mathbf{x}) + b$.
The LS-SVM can be formulated as the following constraint optimization problem:

$$L = \frac{1}{2}\|\mathbf{w}\|^2 + \gamma\sum_{i=1}^{l}[t_i - \mathbf{w}f(\mathbf{x}_i) - b] \tag{1}$$

Kernel function:

$$K(\mathbf{x},\mathbf{x}') = \phi(\mathbf{x})\cdot\phi(\mathbf{x}') \tag{2}$$

The solution can be expressed as the linear combination of kernels weighted by the Lagrange multipliers:

$$f(\mathbf{x}) = \sum_{i=1}^{l}\alpha_i K(\mathbf{x}_i,\mathbf{x}) + b \tag{3}$$

Hence, the learning of this system is performed by solving the system of linear equations

$$\begin{bmatrix} \mathbf{K}+\gamma^{-1}\mathbf{I} & \mathbf{1} \\ \mathbf{1}^T & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{\alpha} \\ b \end{bmatrix} = \begin{bmatrix} \mathbf{t} \\ 0 \end{bmatrix} \tag{4}$$

We use the RBF kernel:

$$K(\mathbf{x},\mathbf{x}') = \exp\{-\eta\|\mathbf{x}-\mathbf{x}'\|^2\}$$

The global minimizer is obtained in LS-SVMs by solving the set of linear equations (instead of quadratic programming in case of SVM). However the sparseness of the support vectors is lost. In SVM, most of the Lagrangian multipliers α_i are zeros while in LS-SVM the Lagrangian multipliers α_i are proportional to the errors e_i.
In order to obtain the recurrent model of a given dynamic data we adopt the LS-SVM system for learning of dynamical feedback systems.
The learning algorithm consists of 2 phases. In the phase 1 the model state inputs are delayed measured output values of the process for the

input-output representation. Based on the learning data set

$$\{\mathbf{x}_k^{(0)}, y_k^p\}_{k=1}^N,\quad \mathbf{x}_k^{(0)} = [y_{k-1}^p, y_{k-2}^p, y_{k-3}^p] \tag{5}$$

the LS-SVM model is prepared

$$\{\alpha_k^{(0)}, \mathbf{x}_k^{(0)}\}_{k=1}^N,\ \text{N=1000.} \tag{6}$$

In the learning phase 2 the measured output values are replaced by the estimated output values of the predictor before performing the new learning phase. The procedure is described by following pseudo-code:

```
for n=1 to 2 do
  Estimate output values:
    y_k^(n) = simLSSVM({α_i^(n-1), x_i^(n-1)}_{i=1}^N,
    x_k^(n-1)), k = 1..N
  Prepare new learning set:
    {x_k^(n), y_k^(n)}_{k=1}^N,
    x_k^(n) = [y_{k-1}^(n), y_{k-2}^(n), y_{k-3}^(n)]
  Create new LS-SVM model:
    {α_k^(n), x_k^(n)}_{k=1}^N,
```

The organization of the data preparation is explained in Tables 1 and 2.

Table 1. The structure of learning data for the recurrent LS-SVM – learning phase I.

X input of LS-SVM			Y output of LS-SVM
Y(0)	Y(1)	Y(2)	Y(3)
Y(1)	Y(2)	Y(3)	Y(4)
…	…	…	…
Y(i)	Y(i+1)	Y(i+2)	Y(i+3)

Table 2. The structure of learning data for the recurrent LS-SVM – learning phase II.

X input of LS-SVM			Y output of LS-SVM
Y'(3)	Y'(4)	Y'(5)	Y'(6)
Y'(4)	Y'(5)	Y'(6)	Y'(7)
…	…	…	…
Y'(i)	Y'(i+1)	Y'(i+2)	Y'(i+3)

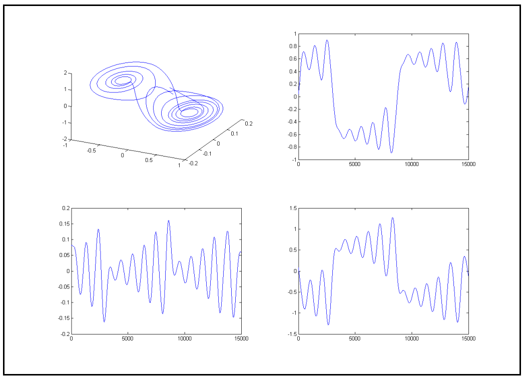
Y(i) – function value (measured)
Y'(i) - function value (predicted from recurrent LS-SVM)
Our approach differs from that presented in [4]. The organization of the learning phase follows the suggestion presented in [7].

2. Recurrent LS-SVM model of Chua's circuit

We tested our recurrent LS-SVM for artificial data set generated by the Chua's circuit. The dynamics of Chua's circuit can be described by a system of three nonlinear ordinary differential equations [8]:

$$\begin{aligned} \frac{dx}{dt} &= \alpha[y - x - f(x)] \\ \frac{dy}{dt} &= x - y + z \\ \frac{dz}{dt} &= -\beta y \end{aligned} \tag{7}$$

where the function $f(x)$ describes the electrical response of the nonlinear resistor, and its shape depends on the particular configuration of its components. The parameters α and β are determined by the particular values of the circuit components.



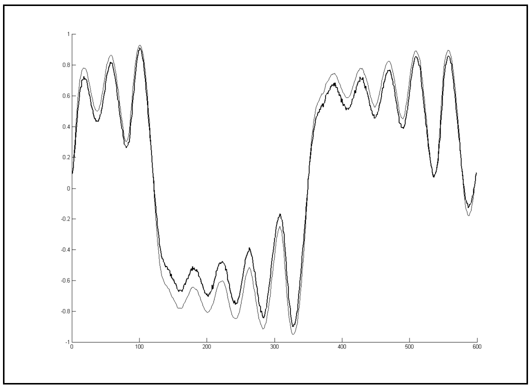


Figure 1 - Dynamics of the Chua's circuit (top). Measured values (solid line) and predicted values calculated by recurrent LS-SVM (thin line) - bottom.

We applied the recurrrent LS-SVM that learned the Chua's circuit dynamics based on 3 delayed values of the chosen component x(t). The results shown in Fig. 1 confirm the ability of this learning system to simulate the deterministic chaotic dynamics.

3. Magnetic data analysis on Etna volcano

Over the last decades different analyses have been devoted to reveal the presence of the chaotic motion in geomagnetic time series in volcanic areas [5,6]. The geomagnetic time series from the magnetic network on Etna volcano (Fig. 3, 4) are analyzed to investigate the dynamical behavior of magnetic anomalies. The predictability of the geomagnetic time series was evaluated to establish a possible low-dimensional deterministic dynamics.

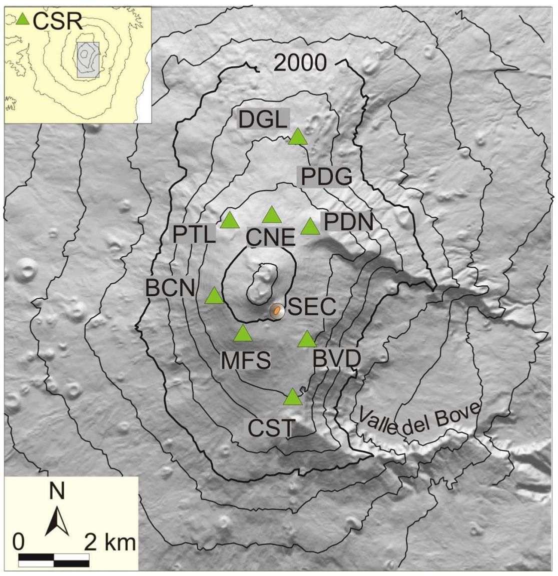


Figure 2. Magnetic monitoring network on Etna volcano.

The analysis of the 10-minutes differences at PDN with respect to the reference station CSR (located far away from the volcano edifice) shows prominent peaks centered around diurnal components at the period of 8, 12 and 24 h. After having removed the dominant periodic components, the filtered differences appear to be aperiodic and broadband (Fig. 3, 4). Therefore, we look at the recurrent least-squares support vector machine approach as a way to provide evidence on the mechanism generating the time dependent variations.

The data from PDN station was firstly normalized to the range [-1,1]. We used a learning data set from 7th to 14th January 2008. The testing data set spans from 15th to 21st January 2008.

The recurrent LS-SVM parameters are as follows: kernel type 'RBF' kernel, regularization parameter γ=2.13, kernel width σ=0.264. The current process value is calculated based on 3 previous measured values.

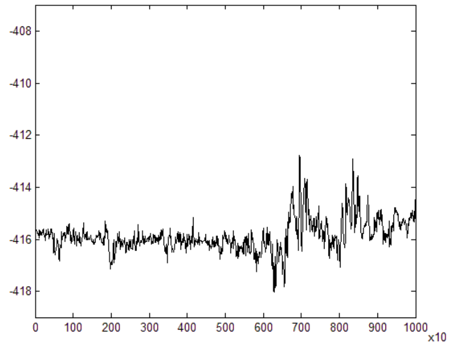


Figure 3. Learning data set from PDN station (7th to 14th January 2008).

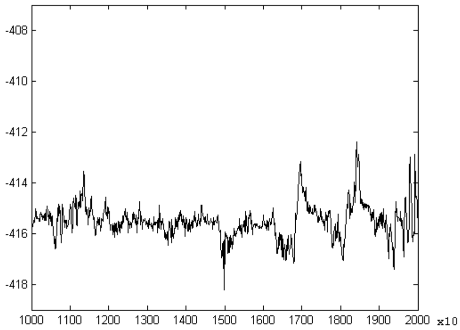


Figure 4. Testing data set from PDN station (15th to 21st January 2008).

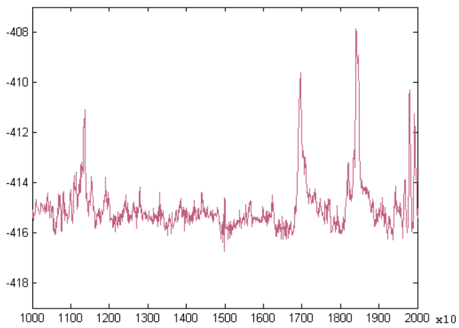


Figure 5. Results of simulation on the testing data set using recurrent LS-SVM.

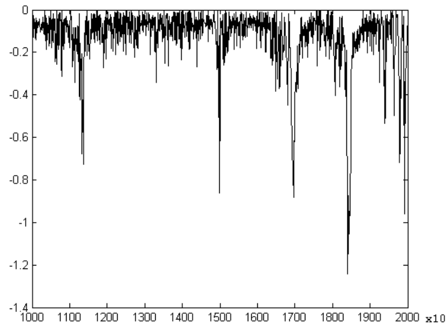


Figure 6. The Absolute Percentage Error on the testing data set.

The predictions are shown in fig. 5. The goodness of the model can be evaluated using Absolute Percentage Error (APE) defined as:

$$APE = \frac{\left| y_{actual} - y_{predicted} \right|}{y_{actual}} \cdot 100\% \qquad (8)$$

The APE is shown in Fig.6. The Mean Absolute Percentage Error (MAPE) equals -0.13 %, while the Root Mean Squared Error (RMSE) is 0.796.

The same recurrent LS-SVM was applied for the simulation of the data from 7th to 21th May 2008 (Fig. 7). On 13th May 2008 significant local magnetic field changes occurred which marked the resumption of the eruptive activity.

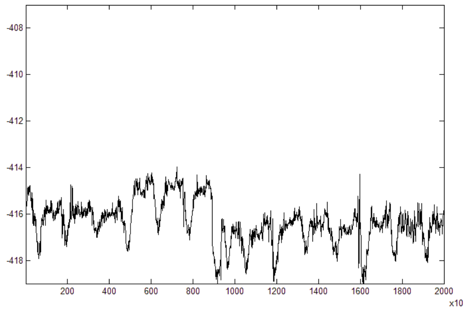


Figure 7. Testing data set from PDN station (7th to 21st May 2008).

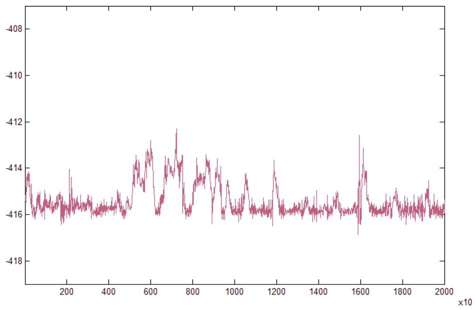


Figure 8. Results of simulation on the testing data set (7th to 21st May 2008).

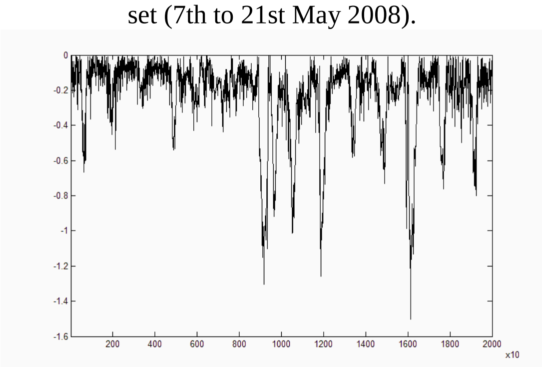


Figure 9. The Absolute Percentage Error on the testing data (7th to 21st May 2008).

We assumed that the performance of the recurrent LS-SVM trained on the data where no significant magnetic changes occurred would be lower when applied to the data where such changes occurred. Fig. 9 shows the APE values. The MAPE and RMS values (Fig. 10, 11) were calculated for the 25 hour periods. Increased error rate is clearly visible at the point corresponding to occurrence of significant magnetic changes.

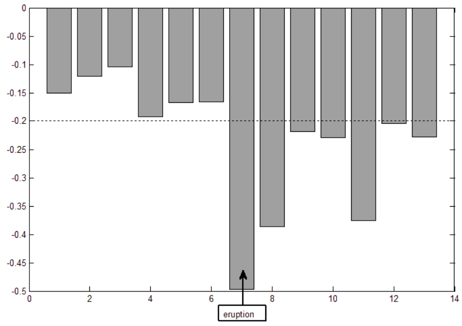


Figure 10. The Mean Absolute Percentage Error on the testing data (each bar represents 25 hour period). The data spans from 7th to 21st May 2008.

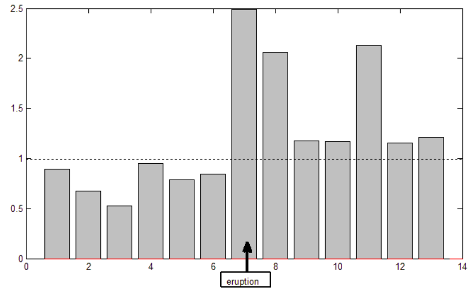


Figure 11. The Root Mean Square Error on the testing data (each bar represents 25 hour period). The data spans from 7th to 21st May 2008.

4. Conclusion

In this paper, identification methods are dedicated to understanding and describing the temporal dynamics of a geomagnetic time series gathered on Etna volcano. The results could have important implications on the study of the dynamical behavior of the volcanomagnetic signals. They underline that volcanomagnetic signals are the result of complex processes that cannot be easily predicted. The application of recurrent LS-SVM forecasting techniques has not provided strong evidence of nonlinear deterministic dynamics in volcanomagnetic data.

References

1. V.N. Vapnik: *Statistical Learning Theory*, Wiley Interscience, New York 1998
2. J.A.K. Suykens and J. Vandewalle: Least squares support vector machine classifier, *Neural Processing Letters*, **9**, 1999, 293-300
3. J.A.K. Suykens, T. Van Gestel, J. De Brabanter, B. De Moor, J. Vandewalle*:* *Least Squares Support Vector Machines*, World Scientific 2002
4. J.A.K Suykens, J. Vandewalle. Recurrent least squares support vector machines. *IEEE Transaction on Circuits and Systems*-I, **47** (2000), 1109-1114.
5. G. Currenti, C. Del Negro, L. Fortuna, A. Vicari, Nonlinear Identification of Complex Geomagnetic Models: An Innovative Approach, *Nonlinear Phenomena in Complex Systems*, 6,1 (2003) 524 – 533.
6. G. Currenti, C. Del Negro, L. Fortuna, R. Napoli, and A. Vicari, Non-linear analysis of geomagnetic time series from Etna volcano. *Nonlinear Processes in Geophysics* (2004) 11, 119-125.
7. M. Lucea: *Modélisation dynamique par réseaux de neurones et machines à vecteurs supports: contribution à la maîtrise des émissions polluantes de véhicules automobiles*, Ph. D. Thesis, Université Paris 6, 2006
8. L.O. Chua, T. Matsumoto, and M. Komuro. "The Double Scroll". *IEEE Transactions on Circuits and Systems* **CAS-32 (**8): 798-818, 1985
9. H.N. Qu, Y. Oussar, G. Dreyfus, W. Xu: Regularized Recurrent Least Squares Support Vector Machines, *Proc. 2009 Int. Joint Conf. Bioinformatics, Systems Biology and Intelligent Computing*, Shanghai, China 2009 (preprint)