Creation of Algorithms for Correction of Low-altitude Orbit Parameters for a Space Vehicle with Electrical Rocket Propulsion System

V.V. Salmin*, V.V. Volotsuev*, I.S. Tkachenko*

**Samara State Aerospace University, Samara, Russia (e-mail: sputnik@ssaul.ru)*

Abstract: Influence of design parameters of an electrical rocket propulsion system (ERPS) and algorithms of correction of low-altitude orbits for terms of useful functioning of a space vehicle (SV) is investigated. The method of a selection optimum ERPS and cyclogrammes of orbit correction low-altitude space vehicle is offered. Guaranteeing estimations of a propellant consumption on long intervals of time for various types ERPS on the basis of fixed plasma engines (FPE) are received.

Keywords: Computer-aided method, modeling, spacecraft

1. STATEMENT OF OPTIMIZATION PROBLEM

Difference of a task of a selection of design parameters of an electro-jet engine for correction of low-altitude orbits of an artificial satellite consists a universal purpose of the engine (ability to correct an orbit of different altitude under action of revolting force of an aerodynamic drag). Multimode character of implementation of each correction too is feature of this task (dispersion of necessary thrust force and a current consumption because of off-design conditions of flight, for example, because of oscillations of density of the upper atmosphere).

On low-altitude orbits with altitude nearby 300÷500 the km influence for the term of existence are rendered with the upper atmosphere of the Earth which leads to reduction{decreasing} of radius of an orbit and an input of space vehicle in dense layers.

The value of aerodynamic acceleration in a present situation of time can be determined under the formula

$$\vec{a}_{atm} = \sigma_{SV} \cdot \rho \cdot V \cdot \vec{V} \,. \tag{1}$$

Here ρ - atmospheric density, V - speed of space vehicle concerning a flow of an atmosphere, σ_{SV} - ballistic coefficient,

$$\sigma_{SV} = \frac{c_x \cdot S_M}{2 \cdot M_{SV}}, \tag{2}$$

where c_X - the factor characterizes character of interaction with an atmosphere of the Earth (depends on properties of a surface of space vehicle, $c_X = 2..2{,}5$), S_M - the area of cross-section of a space vehicle, which to perpendicularly flow (at non-directional flight approximately makes 25 % from all surface of space vehicle), M_{SV} - weight of a space vehicle in orbital flight.

Difference in this case is uncertainty of revolting force of an aerodynamic drag, because of inexact calculation of ballistic factor of space vehicle and an atmospheric density. For an estimation of aerodynamic acceleration static and dynamic models of density of the upper atmosphere of the Earth are used.

To describe relation of an average level of an atmospheric density to altitude it is possible to use static model of density

$$\rho_M = a_0 \cdot \exp\left[a_1 - a_2 \cdot (H - a_3)^{\frac{1}{2}}\right], \tag{3}$$

where ρ_M - modelling atmospheric density (average level), a_1, a_2, a_3, a_4 - the factors of model used for calculation of an atmospheric density at various values F_0 (the tabulated values), F_0 - the fixed value of an index of solar activity $F_{10.7}$ during the considered period of time, $F_{10.7}$ - index of solar activity equal to fluence of a radio emission on a wavelength 10.7 sm (frequency 2800 MGz).

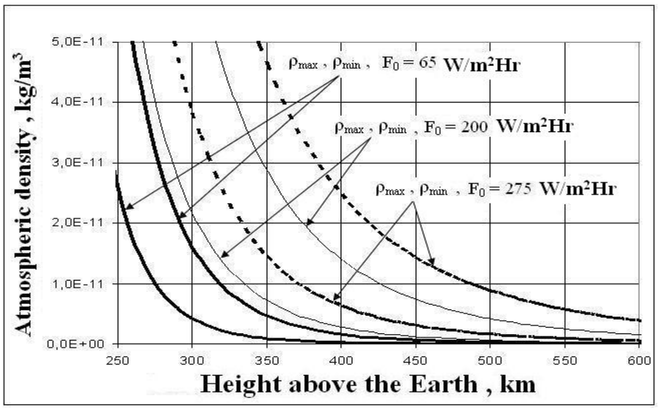

Fig. 1. Relation of limiting conditions of an atmospheric density to altitude at various levels of solar activity

The density can essentially change for short time under action of external factors. As a rule, a condition of an atmosphere characterize marginal levels of an atmospheric density on a present height at various levels of solar activity.

The value of density changes in the indicated limits during an investigated interval of time. If to consider a long range of time also it is necessary to consider semi-annual effects, geomagnetic disturbances, etc.

For example, average for an orbit value of the revolting acceleration a_{mid} describing influence from aerodynamic disturbances, is determined by a capture of an integral of atmospheric acceleration on an interval equal to a cycle time of space vehicle. Results of calculation are presented in Tab. 1 ($\sigma = 0{,}002$ m^2/kg).

Table 1 - Mean of "compensatory" jet acceleration

Orbit parameters		Minimum level of solar activity	Maximum level of solar activity
Altitude of a perigee, km	Altitude of apogee, km		
250	350	0.68	1.556
350	350	0.035	0.325

It is visible, that the dispersion of a level of compensatory acceleration is great enough.

As the reference model describing the flowing condition of an atmosphere, we shall use "dynamic" model of an atmospheric density:

$$\rho = \rho_M \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4 + \delta\rho\,, \qquad (4)$$

where ρ_M - modelling atmospheric density; k_1 - the factor considering influence of diurnal effect; k_2 - the correction coefficient considering semi-annual effect, k_3 - the factor considering difference of a mean diurnal index of solar activity from his average value for the period; k_4 - the factor considering correlation between an atmospheric density and a geomagnetic component; $\delta\rho$ - Random fluctuations.

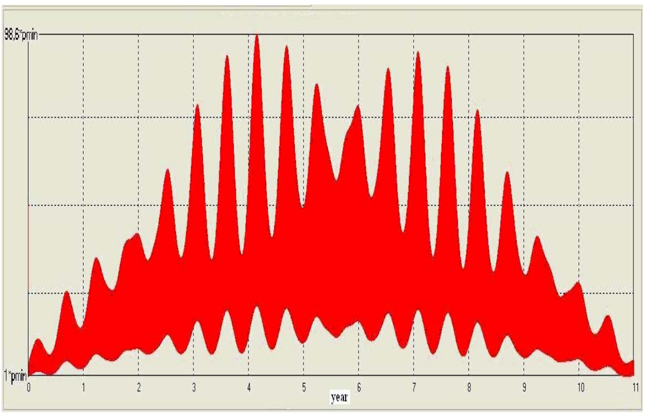

Fig.2. Character of oscillations of an atmosphere

Given model of density can be realized at imitating modelling the indignant motion of an artificial satellite, including process of correction of the orbital altitude, based on the solution of system of the equations in osculating elements a method of their numerical integration on the computer.

In case of modelling processes motion of space vehicle on a circular orbit with an inclination which is distinct from zero, it is necessary to consider, that the altitude above a surface of the Earth is not constant, and changes depending on argument of latitude.In view of compression of a terrestrial ellipsoid the altitude of space vehicle above a surface of the Earth is considered by formula:

$$H = r - R_E \cdot \left(1 - \varepsilon \cdot \sin^2 i \cdot \sin^2 u\right), \qquad (5)$$

where $r = H_{orb} + 6371$ km is radius of an orbit, R_E - radius of equator (6378,245 km), ε - factor of compression of a terrestrial ellipsoid ($\varepsilon \approx 0{,}0034$), i - orbit inclination, u - argument of latitude.

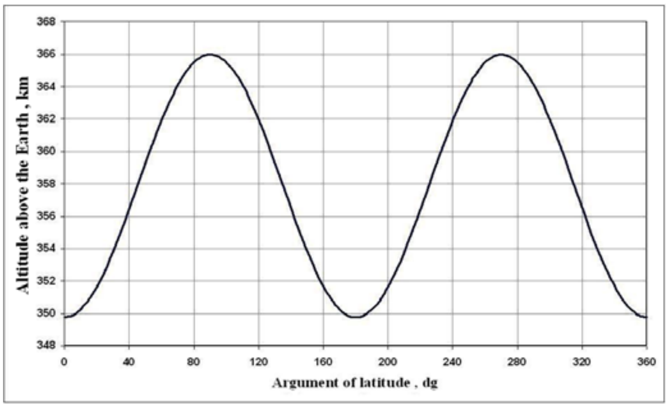

Fig.3. Relation of an orbital altitude of space vehicle to argument of latitude ($i = 60^0$)

It is visible, that the altitude of space vehicle for an orbit changes, and the atmospheric density changes during an orbit because of the Earth in the form of an ellipsoid.

2. MATHEMATICAL MODEL OF ORBIT CORRECTION ON LONG INTERVALS OF TIME

Task of maintenance of an orbit we shall formulate as a problem{task} of liquidation of disturbances of a cycle time during an orbit, formed because of action of force of an aerodynamic drag. Let's demand, that on an interval $[0, t_k]$ the cycle time was in a range $T_p + \delta T_{доп}$, where T_p - computational value of a cycle time; $T_{доп}$ - maximum deviation.

All interval of control we shall brad on N equal subintervals, each of which consists from m passive and n active orbits. On passive orbits the engine is switched off, and on active orbits ERPS creates constant on value jet transversal acceleration. As a result for m+n orbits the period is restored. Let's note, that on active orbits it is necessary not only to compensate influence of force of an aerodynamic drag, but also to liquidate the errors of the cycle time which has collected on passive orbits.

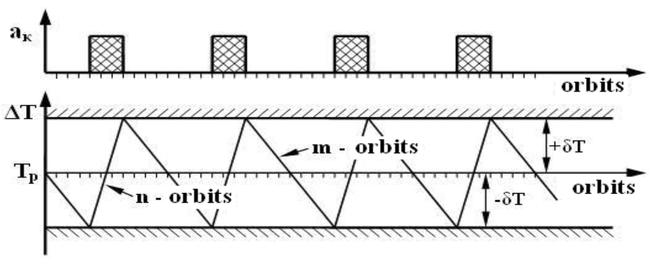

Fig. 5. The cyclogramme of actuation ERPS at correction of a cycle time

The model of a task of optimization of mission control of the satellite on a low-altitude orbit becomes enough difficult, and the ellipticity of an orbit demands the accurate description of "slow" change of aircraft attitude of an orbit on long intervals of time. In these tasks on the foreground there is a strategy of the guaranteed result, both at a selection of control laws, and by optimization of parameters of an approach-correcting propulsion system.

In the fundamentals of mathematical model the equation of motion of space vehicle in the vectorial shape which calculates by formula

$$M_{SV} \cdot \frac{d\vec{V}}{dt} = -M_{SV} \cdot \frac{\mu \cdot \vec{r}}{r^3} + \overrightarrow{F_V} \text{ , or}$$

$$\frac{d\vec{V}}{dt} = -\mu \cdot \frac{\vec{r}}{r^3} + \overrightarrow{f_V} \text{ ,} \qquad (6)$$

where M_{SV} - weight of a space vehicle in orbital flight, r - the flowing radius of an orbit, μ - Gravitational parameter of the Earth, F_V - resultant all revolting forces, f_V - resultant revolting accelerations.

The mathematical model of correction of a low, upper atmosphere of the Earth changing under action, orbits of space vehicle by means of ERPS receives, at association of model of weight and geometry of space vehicle (which is dependent on orbit parameters and control laws the satellite), of power model entering limitations, from the point of view of sufficiency of power for activity ERPS.

Having limited correction only plane elements of an orbit, and also a phase angle, we shall receive system of the equations (7), describing process of correction of a low-altitude orbit of mass,-geometrical and power model of space vehicle with ERPS.

$$\int_0^{T_{all}} \sigma_{SV} \cdot \rho(t) \cdot V^2(t) \cdot dt \le \int_0^{T_{all}} \frac{F_T}{M_{SV}} \cdot \alpha_{ERPS}(t) \cdot dt$$

$$M_{КА} = M_{БС} + M_{ЭУ} + \frac{1}{1-\alpha_К}\left(\gamma_Д \sqrt{\frac{2 \cdot N_{ЭРДУ} \cdot M_{РТ}}{T_М} \cdot \eta_Т \cdot \eta_{ПЭ}} + (1 + k_{СПХ}) \cdot M_{РТ}\right)$$

$$M_T = S_{SB} \cdot M_{ud.SB} + \frac{N_{m.day}}{E_{ud.ab}} + \delta M_{ab}$$

$$\sigma_{SV} = \frac{1}{T}\int_0^T \sigma(t)dt \text{ , } F_T = \frac{2 \cdot N_{ERPS}}{c} \cdot \eta_T \cdot \eta_{PE}$$

$$N_{m.day} = \frac{\sum_{i=1}^{n} N_i \cdot t_i}{T_{day}} + \frac{\sum_{k=1}^{m} \alpha_{ERPS}(t_k) \cdot N_{ERPS}}{T_{day}}$$

$$N_{m.day} = N_{m.SB}^{S} \cdot S_{SB} \cdot \cos\alpha_m$$

$$\frac{dA}{dt} = \frac{4a_K}{\pi}\sqrt{\frac{A^3}{\mu}}\left(\xi + \tfrac{\alpha-\pi}{2}\right) - 2\sigma_{SV}\rho(t)\sqrt{\mu A},$$

$$\frac{de}{dt} = \frac{a_K}{\pi}\sqrt{\frac{A}{\mu}}\left[-3e\left(\xi + \tfrac{\alpha-\pi}{2}\right) + 4\sin\left(\xi + \tfrac{\alpha}{2}\right)\cos\tfrac{\alpha}{2}\cos\eta - \frac{e}{2}\sin 2\left(\xi + \tfrac{\alpha}{2}\right)\cos\alpha\cos 2\eta\right] - 2e\sigma_{SV}\rho(t)\sqrt{\frac{\mu}{A}}$$

$$\frac{d\Delta u}{dt} = \sqrt{\mu}\,(A^{-1,5} - A_K^{-1,5})$$

$$\rho(t) = K_0(t) \cdot K_1(t) \cdot K_2(t) \cdot K_3(t) \cdot a_0 \cdot \exp\left[a_1 - a_2 \cdot ((r - 6371) - a_3)^{\frac{1}{2}}\right]$$

$$\rho(t) \in [p_{\min}(t), p_{\max}(t)]$$

In the given model the structure of control mode on an active orbit in view of limitations on power looks an orbit as follows: On active orbits actuation ERPS to be made on sites{segments} of optimum control by orbit parameters (figure 5) in view of presence of an accessible reserve of power for activity ERPS by the current moment. As a result general connection diagram ERPS on active orbits can be represented as in figure 5.

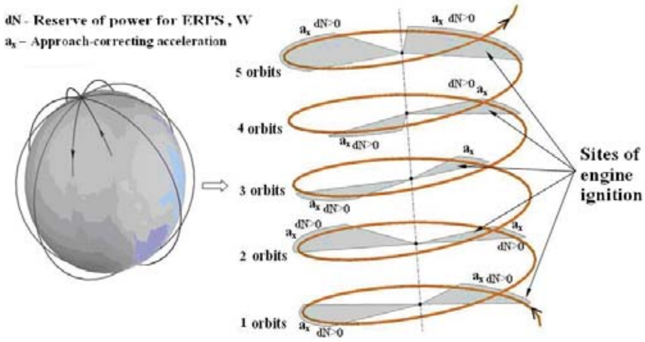

Fig. 5. The scheme of formation of cyclogrammes of correction on active orbits in view of limitations on power and uncertainty of a condition of an atmospheric density

For a selection of algorithms of correction of low-altitude orbits of a space vehicle with an electrical rocket propulsion system the scheme including listed below task is used:

• The general task of joint optimization of the space vehicle intended for set of dynamic maneuvers is formed. The design model of space vehicle, and also model of mass distribution on separate components of space vehicle is entered.

• From the general task of optimization its dynamic part (a selection of control modes with motion) is allocated. The dynamic task is solved consistently, with application of the models possessing a various degree of completeness, detailed elaboration and accuracy.

• The task of optimization of cyclogrammes of corrections together with synthesis of design parameters is solved. As a first approximation key parameters describing design shape of space vehicle, and also power of maneuver and control modes as a result get out.

• Influence of uncertain factors, both on results of the solution of a dynamic task, and on values of design parameters is investigated. The parameters providing a minimum of the maximum penalty in criterion of an optimality are in case of need determined, that is the guaranteeing approach is realized.

As a result we receive base cyclogrammes of orbit correction of the investigated satellite and we estimate expenses of a propulsive mass for correction at the most dense, least dense and dynamically changing condition of a residual atmosphere of the Earth of a space vehicle.

3 GUARANTEEING ESTIMATIONS OF A PROPELLANT CONSUMPTION ON LONG INTERVALS.

At a fixed atmosphere (for example, in the worst conditions - strategy of the guaranteed result) depending on altitude above a surface of the Terrestrial ellipsoid is required a level of thrust force ERPS, at least, such, that would be satisfied a necessary condition of correction. In image 6 relations of required thrust and required power ERPS for space vehicle with weight $M_{SV} \approx 6400$ kg and average ballistic factor of $\sigma_{mid} = 0{,}0036$ m^2/kg are constructed.

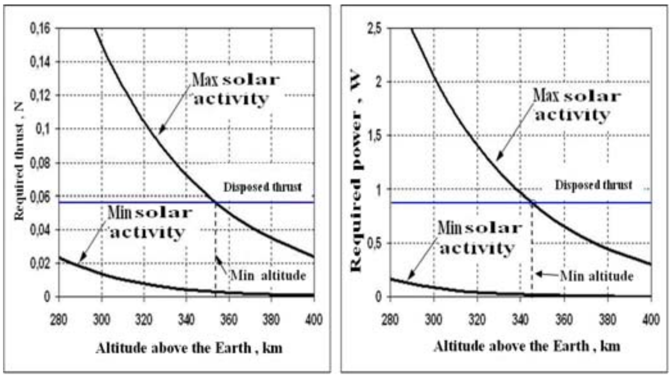

Fig. 6. Relation of the minimal required thrust and required power ERPS for implementation of correction from altitude of a perigee of an orbit at extreme levels of an atmospheric density

Under schedules it is visible, that for various altitudes of orbits depending on a level of solar activity it is required two levels of required thrust of a propulsion system and required power of power installation. And these levels can differ on the order. So, for an orbit in altitude 300км the maximum level of thrust, required for indemnification{compensation} of an aerodynamic drag, makes ~0,15 N, and minimal ~0,015 N; accordingly, the maximal{maximum} power is equal ~2 kw, and the minimal ~0,2 kw. At designing space vehicle conditions of its{his} flight are in advance unknown, including the expected density of the upper atmospheric slices, therefore an approach-correcting propulsion system calculates on the worst case.

Modelling of operated motion of the satellite implements a method of numerical integration of the differential equations describing changes of orbit parameters, closed by control laws actuations ERPS. In images 7-8 the example of modelling of process of correction of average altitude (cycle time) of a circular orbit is resulted. All modeled interval is divided into passive orbits on which there is a falling altitude and ERPS is not actuated, and on active orbits on which there is an increase in an orbital altitude due to actuation ERPS.

On the basis of the results received at modelling to be conducted the estimation of expenses of a propulsive mass on correction of a low-altitude orbit of space vehicle with set in the weight and in ballistic factor. Areas of the charge of a propulsive mass on orbit correction of space vehicle (Image 9) are under construction. The upper border of area shows the charge of a propulsive mass at an initial most dense condition of an atmosphere with the subsequent decay of an atmospheric density that is accompanied by reduction of value of an index of solar activity F_0 . The lower border of area shows the charge of a propulsive mass at an initial least dense condition of an atmosphere. As a result it is possible to determine the maximum and minimal propellant budget for the correction, demanded for maintenance of an orbit during planned term of existence of the satellite.

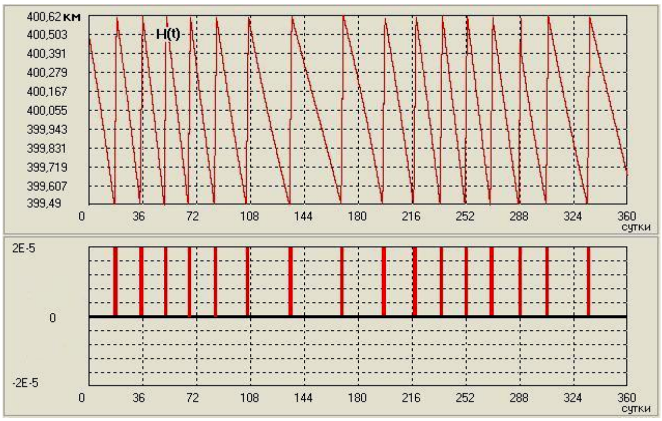

Fig. 7. Change of an orbital altitude of space vehicle from time and the cyclogramme of actuation ERPS at modelling on an interval of 360 day

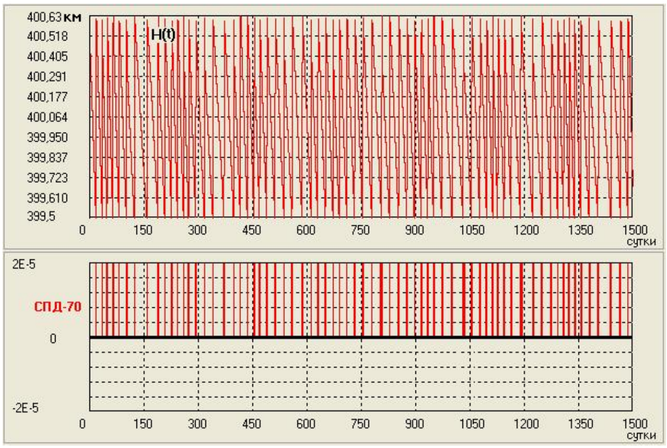

Fig. 8. Change of an orbital altitude of space vehicle from time and the cyclogramme of actuation ERPS at modelling on an interval of 1500 day

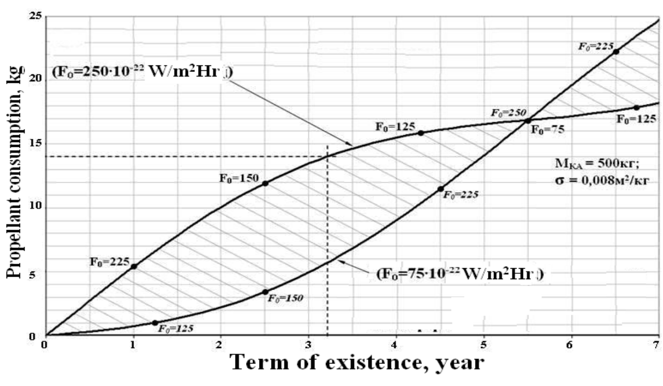

Fig. 9. Areas of expenses of fuel on orbit correction

4. CONCLUSION

In the article feasibility ЭРДУ for correction of low-altitude orbits of space vehicle is considered. The arrangement of sites of actuation ERP on an orbit, proceeding from reasons of an optimality of correction under the charge of a propulsive mass is chosen. It is offered to use sites of an orbit on which there is a redundant electrical power, according to cyclogrammes of actuation and re-energizing of onboard hardware. Standard cyclogrammes of orbit correction for various conditions of the atmosphere, described by various levels of density (and forces of an aerodynamic drag) are calculated. Guaranteeing estimations of a required level of thrust ERP and the charge of a propulsive mass for correction of various types of orbits on the set interval of time (exceeding 1-3 years) are received.

REFERENCES

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