# ON THE POSSIBILITY OF COMBINED CONTROL SYNTHESIS FOR TRANSIENT PROCESSES RATE IN FREQUENCY SYNTHESIZER, WORKING IN WIDE BAND

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*Abstract* - **The procedure of combined control synthesis for frequency synthesizer under reference generator frequency, counter indexes, defining working basic and output frequency, and filter structure, using the main thesis of control theory for nonlinear systems with discrete time, is described. The possibility of using of geometrical imaginations, based on corresponding point mapping dependence on parameter, is noted.**

It is known, that satisfactory spectral and dynamical characteristics achievement for frequency synthesizers (FS) under fine size, energetic and other indexes of their construction is possible only when using hybrid phase synchronization systems (PSS), combining the main advantages of digital (the direct synthesis method) and pulse PSS [1]. For the purposes of optimal FS characteristics achievement the elaboration of FS structure control algorithm, allowing to use different partial methods for FS fast-action increase [2].

In current paper the synthesis procedure for combined control under reference generator frequency, indexes of counters, determining basic working and input FS frequency, and filter structure is presented. Pointed control elements choice is conditioned by their determining influence on fast-action of transient processes, arising in FS under frequency switching over fixed band.

Pointed problem solving became possible only after preliminary studies, as a result of which it was a success:

1) to work out mathematical apparatus, allowing to make automatic the process of obtaining information on transient processes duration under switching of control parameter [3];

2) to prove by means of worked out dynamical characteristics analysis methods, that under finite switch band size FS fast-action is limited [4];

3) to show analytically and by numerical experiment, that the minimal transient process duration under FS switching over fixed frequency band is achieved when using proportional-integrating filter (PIF) with parameters, being functions of switch band size [5].

It is necessary to mention, that the methods of synthesis of control action on tunable generator (TG) in FS by means of counter index switching through intermediate values row is known for a long time [6]. But such control opportunities research over the whole band is yet absent. It is obviously connected with the fact, that FS, constructed on the base of pulse-phase control hoop, is the essentially nonlinear system [7], so synthesizer controllability analysis under all possible switching over fixed frequency band by counter index commutation leads, according to Kalman theorem analog in the theory of nonlinear systems with discrete time controllability [8], to nonlinear functional equations solution problem.

In view of formulated problem complexity on the fist stage of quasi-optimal control synthesis problem it is assumed permissible control rang to be continuous. In order to realize the opportunities of qualitative analysis and to give the basis for numerical-analytical synthesis methods all the studies are executed under consequent complication of FS mathematical model (MM). On each research stage the evaluation of discreteness parameter influence on transient processes duration is done.

It is known [2], that practical realization of frequency putting methods, using optimal control theory requires complicated counting operations even in the case of the low order filters using. General statements of control theory for linear discrete systems [8] prompt, that for transient processes duration decrease we ought to decrease filter order and, maybe, even give its using up for the time of tuning process. Also it is shown [2], that the analogous statement on filter dimension role in frequency switching process duration forming for linear characteristics of FS elements remains without linearizing procedure over control (that is over counter index) for difference equations. So the main possibilities of synthesizer frequency control by counter index commutation is especially interesting.

Non-filter FS scheme dynamics is described by circle point mapping [12]:

$$\bar{\tau} = \tau_0 + \frac{\alpha}{g(\xi(\tau_0))}(mod\,1)$$
$$(0 < \tau_0 \le 1)\ , \qquad (1)$$

where $\tau_0, \bar{\tau}$ are the moments of preceding and following pulses appearance at counter output, $g(y)$ is the generator control characteristic, $\xi(\tau)$ – synchronizing signal, $\alpha$ – non-dimensional counter index.

Let $\alpha_0$ be counter index value, from which switching is carried out, $\alpha_n$ – new counter index value, corresponding to new frequency, $\alpha_{tr}$ – intermediate counter index value, corresponding to control process. Stationary values $\tau = \tau_0^*, \tau_n^*$, corresponding to $\alpha = \alpha_0, \alpha_n$ are to satisfy the correlation

$$g(\xi(\tau_0^*)) = \alpha_0,\ g(\xi(\tau_n^*)) = \alpha_n, \qquad (2)$$

and the process of phase $\tau_n^*$ be settled out of the phase $\tau_0^*$, by one period of counter work, using $\alpha_{tr}$ as the counter index, is described by the equations

$$\bar{\tau}_{tr}(\tau_0^*, \alpha_{tr}) = \tau_0^* + \frac{\alpha_{tr}}{g(\xi(\tau_0^*))} = \qquad (3)$$
$$= 1 + \tau_n^* = \bar{\tau}(\tau_n^*, \alpha_n),$$

so we may obtain the formula, connecting $\alpha_{tr}$ with $\alpha_0, \alpha_n$:

$$\alpha_{tr} = \alpha_0(1 + f(\alpha_n) - f(\alpha_0)), \quad (4)$$

where

$$f(\alpha) = \xi^{-1}(g^{-1}(\alpha)) \quad (5)$$

is defined, because $g'(y)>0,\ \xi'(\tau)>0$ for stationary regime. (Also $f'(\alpha) > 0$, that is why for $\alpha_n > \alpha_0$ $\alpha_{tr} > \alpha_0$ and for $\alpha_n < \alpha_0$ always $\alpha_{tr} < \alpha_0$). Thus $\alpha_{tr}$ is a nonlinear function of $\alpha_0, \alpha_n$, so for realization of full synthesizer controllability in fixed band $\Pi_\alpha = (\underline{\alpha} \le \alpha \le \bar{\alpha})$, it is necessary to guarantee the fulfillment of correlation

$$\bar{\alpha}_{tr} = \max_{\alpha_0, \alpha_n \in \Pi_\alpha} \alpha_{tr}(\alpha_0, \alpha_n) \le \bar{\alpha}, \quad (6)$$

$$\underline{\alpha}_{tr} = \max_{\alpha_0, \alpha_n \in \Pi_\alpha} \alpha_{tr}(\alpha_0, \alpha_n) \ge \underline{\alpha}. \quad (7)$$

So the problem of synthesizer controllability is reduced to the analysis of values from (6) and (7) concerning to fixed band value.

When generator control characteristics is linear

$$g(y) = 1 + sy \quad (s > 0, y > s^{-1}) \quad (8)$$

and stable fixed points of the mapping, corresponding to the main working regime, settle down the front of «saw» phase detector signal

$$\xi(\tau) = \gamma(2\tau a^{-1} - 1)\ (0 < \tau < a < 1), (9)$$

control function (4) has the form

$$\alpha_{tr} = \alpha_0(1 + \frac{a}{2\gamma s}(\alpha_n - \alpha_0)), \quad (10)$$

under the condition

$$0 < max\{1 - s\gamma, s\gamma a^{-1}\} < \alpha_0, \alpha_n < 1 + s\gamma. \ (11)$$

that is $\Pi_\alpha$ is in the region of main regime stability [12].

The analysis of $\alpha_{tr}$ as a function of $\alpha_0, \alpha_n$, which values are from $\Pi_\alpha$, allowed to find out the fact, that

$$\underline{\alpha}_{tr} = \begin{cases} \underline{\alpha}, \bar{\alpha} < 2\gamma s / a, \\ \bar{\alpha}(1 - \frac{a}{2\gamma s}(\bar{\alpha} - \underline{\alpha})), \bar{\alpha} > 2\gamma s / a, \end{cases} \quad (12)$$

$$\bar{\alpha}_{tr} = \begin{cases} \underline{\alpha}(1 + \frac{a}{2\gamma s}(\bar{\alpha} - \underline{\alpha})), \frac{1}{2}(\bar{\alpha} + 2\gamma s / a) < \underline{\alpha}, \\ \frac{a}{8\gamma s}(\bar{\alpha} + \frac{2\gamma s}{a})^2, \underline{\alpha} < \frac{1}{2}(\bar{\alpha} + 2\gamma s / a) < \bar{\alpha}, \\ \bar{\alpha}, \bar{\alpha} < 2\gamma s / a. \end{cases}$$

Thus $\alpha_{tr}$- change interval is identical to $\Pi_\alpha$, when

$$\bar{\alpha} < 2\gamma s/a\ . \quad (13)$$

Otherwise we have a widened band $\Pi_{\alpha tr} = (\underline{\alpha}_{tr} \le \alpha_{tr} \le \bar{\alpha}_{tr})$.for $\alpha_{tr}$. If this band belongs to capture zone of working regime

$$\underline{\alpha}_{tr} > max\{1 - \gamma s, \gamma s/a\}, \bar{\alpha}_{tr} < 1 + \gamma s. \ (14)$$

it is useful to name $\Pi_\alpha$ «potentially working band», and the problem of «potental controllability region» construction may be solved analytically [13].

Thus, under non-filter FS MM and control rang continuality it is a success to obtain analytical equations for the boundaries of controllability region, where FS transient processes last over one period of synthesized reference generator frequency, in the main parameters plain. The method of controlled parameters choice in this case assumes the simple qualitative representation by diagram change analysis for consequence function for point mapping of straight line.

In the case, when TG characteristics are linear and working basic frequency is fixed, the control function is also linear and uniquely defines intermediate values of basic counter index. Also requirement of intermediate values of basic counter index under all possible switching belonging to given band defines the complete controllability region in parameter space. If intermediate value of basic counter index leaves the complete controllability region, but it belongs to stability region, the potential controllability takes place.

In the case of working basis frequency obtaining out of reference generator frequency with the use of corresponding auxiliary counter control function defines no more, than the connection between the indexes of basic and auxiliary counters. So for obtaining of the unique solution of quasi-optimal control synthesis problem the additional condition is in need. In current work the requirement of maximum proximity of basic and auxiliary counter indexes division to the value, defined by control function under maximum of possible working basic frequency value, is used as such an additional condition. Taking into account discrete character of counter index allowed to prove the necessity to choose the closer to defined $\alpha_{tr}$ value in discrete set for maximal fast-action achievement.

As why as in reality counter indexes, forming working basic frequency and input frequency, change in discrete way, for optimal control synthesis it is necessary to work out recommendations on the choice of close to quasi-optimal control parameters values. In nonlinear FS this procedure may be realized by means of FS transient processes time formula analysis, taking into account the size of stationary regime environment, where transient process ends.

It is known, that under parameter choice in feedback for pulse-phase auto-correction systems [9] the thesis, that unlike the continuous systems, in pulse systems, optimal under fast-action, the finite transient process duration may be ensured [10], is used. Just thanks to this conclusion it is considered, that the optimal fast-action in FS is achieved in non-filter scheme under corresponding coefficient of amplification value choice [7].

However the coefficient of amplification value under FS working in fixed frequency band, that is under the robust stability conditions, in connection with counter index change, also changes essentially [2]. That is why the necessity of transient processes optimization in robust stable FS arises.

In [5] the attempt of pointed problem solution was made by the direct analysis of FS characteristic equation root dependence on counter index under different PIF parameter values with further numerical experiment data research.

However discussed problem importance not only for FS dynamical processes theory, but for pulse robust stable control systems in general, required both further working out pointed problem solution methods and exact connection between optimization criterion and transient processes duration determination.

In [11] optimization problem exact solution, using analysis results of D-laying boundaries dependence on PIF parameters and analytical representation of transient processes in FS under all possible switching of counter index, is adduced. The exact analytical studies of corresponding point mappings showed, that non-filter FS has a preference to FS with PIF only for the first control beat. Afterwards FS with PIF is more fast-acted.

The fact, that maximal synthesizer fast-action over the band is achieved by use of filter with specially chosen parameters [2], allows to consider the question of synthesizer controllability with first order PIF only [14]. As in that case system dimension increase, minimal transient process duration will consist of not one, as for non-filter synthesizer scheme, but two periods of counter work. The possibility of transient process duration over combined frequency control by filter use follows from discrete character of controlled parameters definition, what means «inaccuracy» of frequency setting by one interval of counter work (that means the necessity of additional time for determined accuracy achievement). In this case optimal filter use will allow to achieve necessary accuracy for working regime setting by shorter time.

As it was pointed out in [15], when using optimizing PIF type filter in FS scheme, MM has the form of cylinder surface point mapping. So in the case of controllability the counters, forming working frequency of reference generator and FS frequency, may give two intermediate values, and optimal transient process duration is received as a sum of two consequent intervals of working reference generator frequency control. Indeed, the point mapping, describing the dynamics of FS scheme with first order PIF has a form

$$\bar{\tau} = \tau_1 - E[\tau_1], \qquad (15)$$
$$\bar{x} = (x_0 - \xi(\tau_0))e^{-\mu(\tau_1 - \tau_0)} + \xi(\tau_0),$$

$$0 < \tau_0 \le 1, -\infty < x_0 < +\infty,$$

where $E$ means the integer part, $\tau$ and $x$ are non-dimensional control pulse phase and PIF capacity voltage, the value $\tau_1$, corresponding to following $\tau_0$ moment of control pulse appearance is achieved from the equation

$$\int_0^{\tau_1-\tau_0} g[\beta(x_0 - \xi(\tau_0))e^{-\mu\tau} + \xi(\tau_0)]d\tau = \alpha, \quad (16)$$

where $g(y)$ is the generator control characteristic, $\xi(\tau)$ – synchronizing signal, $\alpha$ – counter index, $\mu > 0, 0 \le \beta \le 1$ – filter parameters.

Let $\alpha_0$ be counter index value, from which switching is carried out, $\alpha_n$ – new counter index value, corresponding to new frequency, $\alpha_{1tr}, \alpha_{2tr}$ – intermediate counter index values, corresponding to control process. Stationary values $\tau = \tau_0^*, \tau_n^*$, corresponding to $\alpha = \alpha_0, \alpha_n$ are to satisfy the correlation

$$g(\xi(\tau_0^*)) = \alpha_0, \quad x_0^* = \xi(\tau_0^*), \quad (17)$$

$$g(\xi(\tau_n^*)) = \alpha_n, \quad x_n^* = \xi(\tau_n^*), \quad (18)$$

so the process of phase $\tau_n^*$ be settled out of the phase $\tau_0^*$, by one period of counter work, using $\alpha_{1tr}, \alpha_{2tr}$ as the counter indexes, is described by the equations

$$\alpha_{tr1} = \alpha_0(1 + \bar\tau - \tau_0^*), \quad (19)$$

$$\xi(\tau_n^*) = (\xi(\tau_0^*) - \xi(\bar\tau))e^{-\mu(1+\tau_n^*-\bar\tau)} + \xi(\bar\tau),$$

$$\alpha_{tr2} = \int_0^{1+\tau_n^*-\bar\tau} g[\beta(\xi(\tau_0^*) - \xi(\bar\tau))e^{-\mu\tau} + \xi(\tau)]d\tau.$$

The system (17) is the system of three equations with three unknown values: $\alpha_{1tr}, \alpha_{2tr}$ and $0 \le \bar\tau \le 1$. The problem of controllability region $\Pi_c$ construction, that is the region, where the values $\alpha_{1tr}, \alpha_{2tr}$ are from the capture zone of the main working regime, that is for

$$0 < max\{1 - s\gamma, \theta_c s\gamma a^{-1}\} < \alpha_0, \alpha_n < 1 + s\gamma,$$

$$\theta_c = 2max\{\frac{1}{2} - \frac{\beta}{\mu}\frac{e^\mu - 1}{e^\mu + 1}, \frac{\beta}{\mu} - \frac{1}{e^\mu - 1}\}, \quad (20)$$

the controllability region $\Pi_c$ satisfies the condition

$$max\{1 - s\gamma, \theta_c s\gamma a^{-1}\} < \alpha_{tr1}, \alpha_{tr2} < 1 + s\gamma. \quad (21)$$

Thus, in the case of optimizing PIF [16] the intermediate parameter values are defined by analytical correlation, allowing to determine controllability region form. And besides some visuality elements remain, and, in particular, it is possible to carry out the analysis of controllability region form and its transference about the band of keeping back and capture [12] under filter inertia increase.

However in the case of optimizing PIF use the analysis of counter index change discreteness role have no such a visual form, as in non-filter FS, and may be carried out mainly by means of numerical experiment use, which is reduced essentially to numerical finding of FS transient processes duration under discrete counter indexes values, close to calculate values, obtained by corresponding functional equations solving [14]. When organizing numerical experiment, the original methods of reliability evaluation for transient process finishing by means of Lyapunov functions apparatus application is used [17-20]. Thus under reference generator frequency control by means of setting of auxiliary counter index and corresponding intermediate basic counter commutation, defining TG oscillation full phase cover, not only transient process duration decrease is achieved, but also favorable phase correlation between synchronizing oscillations of reference generator and TG, which are necessary for narrow-band filter switching, completing general transient process in FS, are realized.

To conclude aforesaid let us mention, that the current paper presented a new and effective way of combined control synthesis under reference generator frequency, indexes of counters, determining basic working frequency and input FS frequency, and filter structure. In order to chose control elements in accordance with their influence on fast-action of transient processes, arising in FS under frequency switching over fixed band, consequence functions diagram behavior for point mappings, being studied system MM, must be analysed, with further taking into account the fact of parameter change discrete character.

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