# KINEMATICS ANALYSIS, DESIGN AND OPTIMIZATION OF A SIX DEGREES-OF-FREEDOM PARALLEL ROBOT

## Sergiu-Dan Stan, Vistrian Mătieş, Radu Bălan

Technical University of Cluj-Napoca Romania sergiustan@ieee.org, {matiesvistrian+radubalan}@yahoo.com

## Abstract

In this paper a mono-objective optimum design procedure for a six-degree of freedom parallel micro robot is outlined by using optimality criterion of workspace and numerical aspects. A mono-objective optimization problem is formulated by referring to a basic performance of parallel robots. Additional objective functions can be used to extend the proposed design procedure to more general but specific design problems. A kinematic optimization was performed to maximize the workspace of the mini parallel robot. Optimization was performed using Genetic Algorithms. It is concluded that the optimal design method is useful for the design of a six degree of freedom micro parallel robot.

# Key words

optimization, hexapod, 6 degree of freedom, Genetic Algorithms.

#### 1 Introduction

Parallel structures have inherent advantages for many applications: rigidity, accurate positioning, high velocities. Obtaining high performances requires two steps during the design process:

- 1. Choosing the appropriate mechanical structure;
- 2. Choosing the right dimensions.

The second point is very important as there is much more larger variation in the performances of parallel structures according to the dimensions than for classical, serial structures. Therefore the first stage in the design of a specified kinematic structure is establishing its architecture, i.e. the joint and link layout and the dimensions of the robot. Also, choosing the best kinematic-dimensions for a specified machining application is a difficult problem for many reasons. In most cases we are able to compute the performance criteria only for a given pose of the robot, which means local performances. To evaluate the robot, global performances are needed and therefore efficient algorithms.

In the literature, various methods to determine workspace of a parallel robot have been proposed using geometric or numerical approaches. Early investigations of robot workspace were reported by Gosselin [1], Merlet [2], Kumar and Waldron [3], Tsai and Soni [4], Gupta and Roth [5], Sugimoto and Duffy [6], Gupta [7], and Davidson and Hunt [8]. The consideration of joint limits in the study of the robot workspaces was presented by Delmas and Bidard (1995). Other works that have dealt with robot workspace are reported by Agrawal [9], Gosselin and Angeles [10], Cecarelli [11]. Agrawal [12] determined the workspace of in-parallel manipulator system using a different concept namely, when a point is at its workspace boundary, it does not have a velocity component along the outward normal to the boundary. Configurations are determined in which the velocity of the end-effector satisfies this property. Pernkopf and Husty [13] presented an algorithm to compute the reachable workspace of a spatial Stewart Gough-Platform with planar base and platform (SGPP) taking into account active and passive joint limits. Stan [14] presented a genetic algorithm approach for multi-criteria optimization of PKM (Parallel Kinematics Machines).

Most of the numerical methods to determine workspace of parallel manipulators rest on the discretization of the pose parameters in order to determine the workspace boundary [15, 16]. In the discretization approach, the workspace is covered by a regularly arranged grid in either Cartesian or polar form of nodes. Each node is then examined to see whether it belongs to the workspace. The accuracy of the boundary depends upon the sampling step that is used to create the grid. The computation time grows exponentially with the sampling step. Hence it puts a limit on the accuracy. Moreover, problems may occur possesses the workspace when singular configurations. Other authors proposed to determine the workspace by using optimization methods [14]. Numerical methods for determining the workspace of the parallel robots have been developed in the recent years.

In this paper, the optimization workspace index is defined as the measure to evaluate the performance of a six degree of freedom parallel micro robot.

Another contribution is the optimal dimensioning of the six degree-of-freedom parallel micro robot of type Hexapod for the largest workspace.

# 2 Optimal design of the six degree of freedom micro parallel robot based on workspace analysis

#### 2.1 Six DOF micro parallel robot

The micro parallel robot is a six degrees-of-freedom parallel manipulator comprising a fixed base platform and a payload platform, linked together by six independent, identical, open kinematic chains (Fig. 1). Kinematics of this structure is presented in Refs. [14].



Figure 1. Six degrees of freedom micro parallel robot with translation actuators



Figure 2. CAD model of the six degrees-of-freedom micro parallel robot

#### 2.2 Workspace index

One of the most important issues in the process of design of robot is their workspace. For parallel robots,

this issue may be more critical since parallel robots will sometimes have a rather limited workspace. Closed loop nature of the parallel robots limits their workspace. Also, in the context of design, the workspace determination procedure should be simple enough to be included in an optimization algorithm using for example Genetic Algorithms.

Because of this, applications involving these parallel robots require a detailed analysis and visualization of the workspace of these robots. The algorithm for visualization of workspace needs to be adaptable in nature, to configure with different dimensions of the micro parallel robot's links.

The workspace is discretized into square and equal area sectors. A multi-task search is performed to determine the exact workspace boundary. Any singular configuration inside the workspace is found along with its position and dimensions. The volume of the workspace is also computed by means of a computer program.

A type of parallel robot, namely Hexapod-type sixdegree of freedom robot is considered to demonstrate the effectiveness of the algorithm.

Workspace is another significant design criterion for describing the kinematics performance of parallel robots. Parallel robots use volume to evaluate the workspace ability. However, is hard to find a general approach for identification of the workspace boundaries of the micro parallel robots.

This is due to the fact that there is not a closed form solution for the direct kinematics of these parallel robots. That's why instead of developing a complex algorithm for identification of the boundaries of the workspace, it's developed a general visualization method of the workspace for its analysis and its design.



Figure 3. Workspace views of the HEXAPOD micro parallel robot with six degrees-of-freedom

The possible workspace of the robot is of a great importance for optimization of the parallel robots. Without the ability to solve the workspace is impossible to state that the robot can fulfill any work task.

The general analysis of the workspace consists in workspace determination using the described discretization method implemented in a MATLAB/SIMULINK environment.



Figure 4. Workspace views of the HEXAPOD micro parallel robot with six degrees-of-freedom



Figure 5. Graphical User Interface for determining the shape of the HEXAPOD micro parallel robot with six degrees-of-freedom

The workspace is the volume in the space case where the tool centre point (TCP) can be controlled and moved continuously and unobstructed.

The workspace is limited by *singular configurations*. At singularity poses it is not possible to establish definite relations between input and output coordinates. Such poses must be avoided by the control.

The robotics literature contains various indices of performance [19], [20], such as the workspace index W.











Figure 9. Section views through the workspace

#### 2.3 Performance evaluation

Beside workspace which is an important design criterion, transmission quality index is another important criterion.

The transmission quality index couples velocity and force transmission properties of a parallel robot, i.e. power features [21]. Its definition runs:

$$T = \frac{\|E\|^2}{\|J\| \cdot \|J^{-1}\|}$$
(1)

where E is the unity matrix. T is between 0 < T < 1; T=0 characterizes a singular pose, the optimal value is T=1 which at the same time stands for isotropy [22].

In isotropic configurations the Jacobian matrix has the condition number, as well as the determinant, equal to one and the robot performs very well with regard to its force and motion transmission capabilities.

A micro parallel robot is defined isotropic if it possesses at least one isotropic configuration [25] [26] [27].



Figure 10. Transmission quality index for HEXAPOD micro parallel robot with six degrees-offreedom (3D view)



Figure 11. Transmission quality index for HEXAPOD micro parallel robot with six degrees-offreedom (top view)

As it can be seen, the micro parallel robot presents better performances in the middle of its workspace, as presented in Fig. 10-11.

#### 2.4 Design optimization

The design of the robot can be made based on any particular criterion. The paper presents a genetic algorithm approach for workspace optimization of six-dof parallel micro robot. For simplicity of the optimization calculus a symmetric design of the structure of the six degree of freedom micro parallel robot was chosen.

In order to choose the robot dimensions L,  $q_{1min}$ ,  $q_{1max}$ ,  $q_{2min}$ ,  $q_{2max}$ ,  $q_{3min}$ ,  $q_{3max}$ ,  $q_{4min}$ ,  $q_{4max}$ ,  $q_{5min}$ ,  $q_{5max}$ ,  $q_{6min}$ ,  $q_{6max}$ , we need to define a performance index to be maximized. The chosen performance index is W (workspace). One objective function is defined and used in optimization. It is noted as W, and corresponds to the optimal workspace. We can formalize our design optimization problem as the following equation function of workspace:

$$Goal\_function=max(W)$$
 (2)

Optimization problem is formulated as follows: the objective is to evaluate optimal link lengths which

maximize (5). The design variables or the optimization factor is the ratios of the minimum link lengths to the base link length b, and they are defined by:

Constraints to the design variables are:

$$0 < L < 60$$
 (4)

$$q_{1min} = q_{2min} = q_{3min} = q_{4min} = q_{5min} = q_{6min},$$

$$q_{1max} = q_{2max} = q_{3max} = q_{4max} = q_{5max} = q_{6max},$$

$$q_{1max} = 1, 6q_{1min}, q_{2max} = 1, 6q_{2min}, q_{3max} = 1, 6q_{3min},$$

$$q_{4max} = 1, 6q_{4min},$$

$$q_{5max} = 1, 6q_{5min}, q_{6max} = 1, 6q_{6min},$$
(5)

For this example the lower limit of the constraint was chosen to fulfill the condition  $L \ge 30$ . For simplicity of the optimization calculus the upper bound was chosen the length  $L \le 60$ .

During optimization process using genetic algorithm it was used the following GA parameters, presented in Table 1. A genetic algorithm (GA) optimization method is used because of its advantages like robustness and good convergence properties.

Table I. GA Parameters

1	Population	50
2	Generations	100
3	Crossover rate	0,08
4	Mutation rate	0,005

The GA approach has the clear advantage over conventional optimization approaches in that it allows a number of solutions to be examined in a single design cycle. The traditional optimization methods searches optimal points from point to point, and are easy to fall into local optimal point. In genetic algorithms, the term chromosome typically refers to a candidate solution to a problem, often encoded as a bit string. The "genes" are either single bits or short blocks of adjacent bits that encode a particular element of the candidate solution (e.g., in the context of multi parameter function optimization the bits encoding a particular parameter might be considered to be a gene). An allele in a bit string is either 0 or 1; for larger alphabets more alleles are possible at each locus. Crossover typically consists of exchanging genetic material between two single chromosome haploid parents. Mutation consists of flipping the bit at a randomly chosen locus (or, for larger alphabets, replacing *a* the symbol at a randomly chosen locus with a randomly chosen new symbol).

Using a population size of 50, the genetic algorithm was run for 100 generations. A list of the best 50 individuals was continually maintained during the execution of the genetic algorithm, allowing the final selection of solution to be made from the best structures found by the genetic algorithm over all generations.

We performed a kinematic optimization in such a

way to maximize the workspace index *W*. It is noticed that optimization result for Hexapod when the maximum workspace of the 6 DOF micro parallel robot is obtained for *L*=60 mm. The used dimensions for the 6 DOF micro parallel robot were:  $q_{1min}=0$  mm,  $q_{1max}=100$  mm. Maximum workspace of the micro parallel robot with six degrees-of-freedom was found to be W= 45493 mm<sup>3</sup>.

And the shape of the optimized workspace of the parallel micro robot is shown in Fig. 10. The results show that GA can determine the architectural parameters of the robot that provide an optimized workspace. Since the workspace of a parallel robot is far from being intuitive, the method developed should be very useful as a design tool.



Figure 12. Workspace of the optimized parallel micro robot with six degrees-of-freedom.

However, in practice, optimization of the parallel robot geometrical parameters should not be performed only in terms of workspace maximization. Some parts of the workspace of micro parallel robot are more useful considering a specific application.

Indeed, the advantage of a bigger workspace can be completely lost if it leads to new collision in parts of it which are absolutely needed in the application. However, it's not the case of the presented structure.

#### 8 Conclusion

In this paper a mono-objective optimum design procedure for parallel robot was outlined by using optimality criterion of workspace and numerical aspects. A mono-objective optimization problem was formulated by referring to a basic performance of parallel robots. A kinematic optimization was performed to maximize the workspace of the 6 degrees-of-freedom micro parallel robot. Together with other optimization oriented toolboxes from MATLAB, the GAOT Toolbox provides a uniform environment for the mechanical engineer to experiment with and apply GAs to problems in optimization of micro parallel robots. It is concluded that the optimal design method is useful for the design of a six degree of freedom micro parallel robot with translation actuators.

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