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About absolute stability of control valves

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Summary

The paper is devoted to some general tendencies in absolute stability of hydraulic control valves. The valves of very different designs and functions have some common elements. It allows to formulate a problem for some generalized valve and to consider the character of stability for typical cases. It is shown particularly which type of valve characteristic is preferable for different types of dissipative forces.

Key Words

Stability, valves, control, oscillations, damping

1. Introduction

Hydraulic control systems are applied in vehicles very widely. At the same time it is known from engineering experience that these systems tend to exhibit undesirable pressure oscillations. It is usual, that concrete variants of valves are checked for instability numerically with some simulation programs [Jelali and Kroll, 2003]. Hence the following questions arise.

- If numerical simulation with some initial conditions shows no oscillations, is it valid also for all initial conditions?
- If the numerical simulation shows instability of a concrete valve, how should be changed its design?

Thus, along with the numerical simulation it is important to have a general conception of the oscillation phenomenon on the base of classical analytical methods.

2. Formulation of Problem

There is a big variety of hydraulic valves [Handbook of Hydraulic Fluid Technology, 1999]. Some designs are presented in Fig.1.

The objective of a hydraulic control valve in general case is transforming a control force into some controlled pressure of oil. How is this achieved?

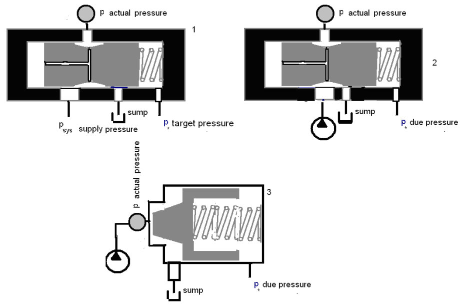

Figure 1. Typical control valves

A piston under the action of the control force and the controlled pressure moves and overlaps partially or wholly the oil flow through a variable control gap. It should change the controlled pressure such that the piston comes to equilibrium.

Thus, the valves of very different designs have some common elements:

1. Piston is a body with one degree of freedom. It has mass m and coordinate x(t).
2. Working cell is a vessel of oil with the controlled pressure p. The working cell has some compliance or so called hydraulic capacity c_h
3. Control surface of area A is some surface connected with the piston transversal to its direction of motion. Through this surface the controlled pressure acts on the piston.
4. Control force is a force of any nature (hydraulic, magnet, mechanical) that acts on the piston against the controlled pressure. We denote this force as Ap_* in order to introduce the target pressure p_*
5. There is always some flow of oil from a higher pressure source to the working cell

(input) and from the working cell to sump (output) through gaps of variable cross-section. The difference between input and output flows is denoted as Q. This quantity depends on the piston coordinate x and generally on the pressure p. As a rule, the last dependency is weak and we will not take it into account. The function Q(x) can be called characteristic of a valve. It is determined by the geometry of the control gaps and may be modified with the help of some notches (Fig.2)

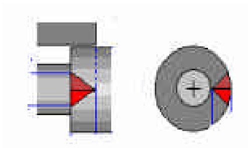

Figure 2. Notches

6. A <u>spring</u> with stiffness c connects the piston with a base.
7. <u>Dissipative forces</u> acting on the piston can be classified under three types:
- Viscous damping $-D_v$
- Coulomb friction - Fsign (v)
- Square resistance - $D_2 v^2$sign (v)

Here v=dx/dt, und D_v, F, D_2 are some coefficients.

The square resistance arises usually due to an orifice between the control cell and the control surface. The coefficient D_2 can be calculated as γ $(A/A_b)^2$. Here A und A_b are the areas of the control surface and the orifice, γ is a constant.

The described generalized valve is schematically illustrated in Fig.3 as a flowchart for SimulationX

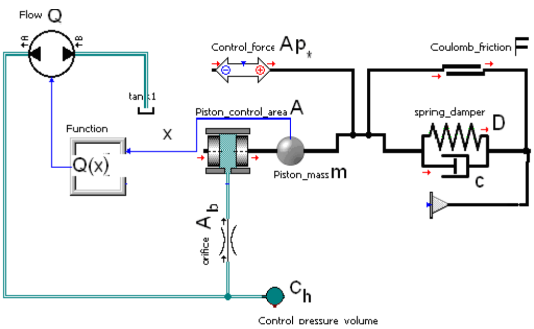

Figure 3. Flowchart for SimulationX

The differential equations corresponding to this generalized model can be transformed to the following dimensionless form:

$$\xi'' + B\xi' + f\,sign\,\xi' + k\xi'^2\,sign\,\xi' + \xi = -P \quad (1)$$

$$P' = q(\xi) + \alpha\xi' \quad (2)$$

Here the following dimensionless quantities are introduced.

$$\xi = xc/(Ap_*),$$
$$P = (p - p_*)/p_*,$$
$$\tau = t\sqrt{c/m},$$
$$B = D_v/\sqrt{c/m},$$
$$f = F/(Ap_*),$$
$$\alpha = A^2/(cc_h),$$
$$k = D_2 Ap_*/(mc),$$
$$q(\xi) = Q(x)\sqrt{m/c}/(c_h p_*) \quad (3)$$

We shall consider as allowed characteristics of valve q(ξ) such everywhere continuous functions for which q(0)=0, ξ q(ξ)>0 and integrals

$\int_{-\infty}^{0} q(\xi)d\xi$ and $\int_{0}^{\infty} q(\xi)d\xi$ are not divergent.

The main question to be answered is the question about the absolute stability of the static solution ξ=0, P=0

2. Lurie problem

First we consider the system (1),(2) without Coulomb friction and square resistance (f=k=0) and try to answer the following question: Is it feasible to choose the parameters B and α such that the absolute stability is provided independent on the characteristic of the valve q(ξ)? This question relates to the classical Lurie problem of the absolute stability for control systems with servomotors [Lurie, 1951]. The Lurie problem in the canonical form for a system of third order

$$z_1' = \lambda_1 z_1 + q(\xi),$$
$$z_1' = \lambda_2 z_2 + q(\xi),$$
$$\xi' = e_1 z_1 + e_2 z_2 - rq(\xi) \quad (4)$$

is considered detailed in [Merkin, 1976]
Here e_1, e_2, λ_1, λ_2, r (Re λ_1>0, Re λ_2>0) are parameters to be chosen so that the system has absolute stability independent on the allowed function q(ξ).
Solution of this problem is representable as the following condition

$$\Theta > -\Psi^2/4 \tag{5}$$

where

$$\Theta = \frac{r(\lambda_2-\lambda_1)+(e_2-e_1)(\lambda_2-\lambda_1)}{4\lambda_2\lambda_1} \tag{6}$$

$$\Psi = \sqrt{r+e_1/\lambda_1+e_2/\lambda_2}+\sqrt{r} \tag{7}$$

In order to apply this result to the system (1), (2) it is converted to (4) with using the following transformation

$$\xi' = (z_1-z_2)/(\lambda_2-\lambda_1)$$
$$\xi'' = (\lambda_1 z_1+\lambda_2 z_2)/(\lambda_2-\lambda_1), \tag{8}$$

where λ_1 and λ_2 are roots of the equation

$$\lambda^2 + B\lambda + 1 + \alpha = 0 \tag{9}$$

Here

$$\Theta = -\frac{1}{2\lambda_2\lambda_1}, \ \Psi = \sqrt{\frac{1}{\lambda_2\lambda_1}} \tag{10}$$

Thus $\Theta < -\Psi^2/4$ and the condition of absolute stability (5) is not fulfilled. It means that the parameters B and α can not be chosen such that the stability takes place at any characteristic of valve.

Thus, only a combination of damping, stiffness and characteristic of the valve can provide the absolute stability.

3. Linear characteristic of valve

The simplest analysis of the absolute stability of the system (1),(2) (f=k=0) is possible at the linear characteristic of a valve:

$$q(\xi) = q'_\xi \xi, \tag{11}$$

where q'_ξ is a constant.

In this case the following characteristic equation of the system is valid

$$\lambda^3 + B\lambda^2 + (1+\alpha)\lambda + q'_\xi = 0 \tag{12}$$

Because of physical feasibility only B>0 and 1+α >0 are possible. Thus the Routh-Hurwitz criterion [Merkin, 1976] leads to the inequality

$$q'_\xi < B(1+\alpha) \tag{13}$$

This simple condition gives in many practical cases a good estimation of parameters that provide stability. In dimensional form it looks as follows

$$D_v > D_{cr} = \frac{Q'_x m}{A(1+cc_h/A^2)} \tag{14}$$

There is some damping threshold D_{cr} that must be exceeded to achieve stability. It is necessary either to increase the actual damping D_v or to decrease the damping threshold D_{cr} through the following measures:
- decreasing the piston mass,
- increasing the spring rate and the hydraulic capacity of the working cell,
- decreasing the slope of the valve characteristic Q´(x).

4. Non-linear characteristic of valve

In view of the previous result a usage of triangular notches as in Fig.2 seems to be attractive. In this case q´(0)=0 and according to the condition (9) the stability in small is guaranteed at any damping. But the absolute stability in non-linear cases can differ from the stability in small.
With the aid of the Lapunov's functions it can be shown [Barbashin, 1970] that for the absolute stability an allowed function q(ξ) has to satisfy the following condition *for all* ξ.

$$q'_\xi(\xi) < B(1+\alpha) \tag{15}$$

Thus, the absolute stability in the case of non-linear characteristic of valve is determined by the maximum of the slope q´(ξ) and can not be provided at sufficiently small damping even when q′(0)=0.

5. Non-linear dissipative forces

Let us consider the case of non-linear dissipative forces that consist of Coulomb friction, square resistance and viscous damping. The technique of harmonic linearization [Panovko and Gubanova, 1979; Tondl, 1976] is here most suitable. We use it in the following interpretation. Let the system (1),(2) be disturbed from the equilibrium. Amplitude of its free oscillations can increase or decrease, what can be characterized as local seemingly stability or instability (Fig.3). Independent on this the oscillations are assumed to be close to some harmonic oscillations

$$\begin{aligned} &\xi = \xi_0 + X\sin\Omega\tau, \\ &P = P_0 + P_s\sin\Omega\tau + P_c\cos\Omega\tau \end{aligned} \qquad (16)$$

and the behavior of the system is similar to behavior of some effective linear system

$$\begin{aligned} &\xi'' + B_{eff}\xi' + \xi = -P \\ &P' = q'_{\xi eff}\xi + q_0 + \alpha\xi' \end{aligned} \qquad (17)$$

that depends on Ω and X. The closer the system to the stability boundary, the more precisely are these approximations.
This effective system is constructed so that its root-mean-square
deviations

$$\frac{1}{2\pi}\int_0^{2\pi}[q(\xi) - (q'_{\xi eff}\xi + q_0)]^2 d\theta \;\Big|_{\xi = \xi_0 + X\sin\theta} \qquad (19)$$

$$\frac{1}{2\pi}\int_0^{2\pi}[B_{eff}\xi' - (B\xi' + f\,\mathrm{sign}\,\xi' + k\xi'^2\,\mathrm{sign}\,\xi')]^2 d\theta \;\Big|_{\xi' = X\Omega\sin\theta} \qquad (18)$$

from the non-linear system (1),(2) are minimal. The minimizing expressions (18),(19) with respect to values $q'_{\xi eff}, q_0$ and B_{eff} leads to the following equations

$$\int_0^{2\pi}[q(\xi) - (q'_{\xi eff}\xi + q_0)]\xi\, d\theta = 0 \qquad (20)$$

$$\int_0^{2\pi}[q(\xi) - (q'_{\xi eff}\xi + q_0)]\, d\theta = 0 \qquad (21)$$

$$\int_0^{2\pi}[B_{eff}\xi' - (B\xi' + f\,\mathrm{sign}\,\xi' + k\xi'^2\,\mathrm{sign}\,\xi')]\xi' d\theta = 0 \qquad (22)$$

It gives

$$q'_{\xi eff} = \frac{1}{\pi X}\int_0^{2\pi} q(\xi_0 + X\sin\theta)\sin\theta\, d\theta \qquad (23)$$

$$q_0 = \frac{1}{2\pi}\int_0^{2\pi} q(\xi_0 + X\sin\theta)\, d\theta - q'_{\xi eff}\xi_0 \qquad (24)$$

$$B_{eff} = \frac{8}{3\pi}kX\Omega + \frac{4f}{\pi X\Omega} + B \qquad (25)$$

The effective equations (17) could also be interpreted from standpoint of the concept of vibrational mechanics [Blekhman, 2000]

In order to find the boundary of stability we substitute the expressions (16),(23),(24) and (25) into equations (17) and balance the terms with sinus, cosines and the constant terms. It gives a system of 6 equations for 6 unknown values $X, \Omega, P_s, P_c, P_0, \xi_0$:

$$\begin{aligned} &-X\Omega^2 + X + P_s = 0 \\ &B_{eff}\Omega X + P_c = 0 \\ &\xi_0 = -P_0 \\ &-\Omega P_c = Xq'_{\xi eff} \\ &P_s = \alpha X \\ &q_0 + \xi_0 q'_{\xi eff} = 0 \end{aligned} \qquad (26)$$

It follows that

$$\Omega^2 = 1 + \alpha \qquad (27)$$

$$\bar{q} = \frac{1}{2\pi}\int_0^{2\pi} q(\xi_0 + X\sin\theta)\, d\theta = 0 \qquad (28)$$

$$q'_{\xi eff} = B_{eff}(1+\alpha) \qquad (29)$$

It worth noting, that according to (27) the frequency has a constant value (1+α) and does not depend on amplitude and damping. The expression (28) determines a relationship between the mean ξ_0 and the amplitude X. The equation (29) gives a boundary of stability that coincides formally with the boundary of stability (13) for the linear system according to the Routh-Hurwitz criterion. However, here the left and the right parts are functions of X. The condition

$$q'_{xeff}(X) < B_{eff}(X)(1+a) \qquad (30)$$

defines a region of amplitudes X of free oscillations that demonstrates the seemingly stability of the equilibrium point (Fig4).

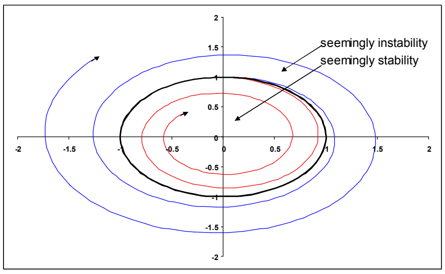

Figure 4. Seemingly stability (red) and instability (blue) of the equilibrium point

If the equation (29) has no solutions regarding X, the system does not change the character of stability with X that it has at arbitrarily small X. Thus, if the system is stable in small, it is desirable that the equation (29) has no roots. Then the system is absolutely stable. If the instability in small is inevitable, it is desirable that the equation (29) has the only solution, herewith as small as possible.

Let us consider on the base of the inequality (30) the character of stability for two typical characteristics of valves that we will call linear and square characteristics. These two characteristics lead through the harmonic linearization corresponding to constant and linear functions q'_{xeff} (X):

$$q'_{xeff} = c_1 \qquad (31)$$

$$q'_{xeff} = c_2 X \qquad (32)$$

Here c_1 and c_2 are some constants.

Such dependencies are typical for rectangular und triangular notches (Fig.2). It is of no importance whether the function q(ξ) is smooth or takes a breaking point at ξ=0

In Fig.5 the quality diagrams are presented, that explain the character of stability for these two cases with Coulomb friction, square resistance, viscous damping and with a mixed dissipation (all types of dissipative forces together).

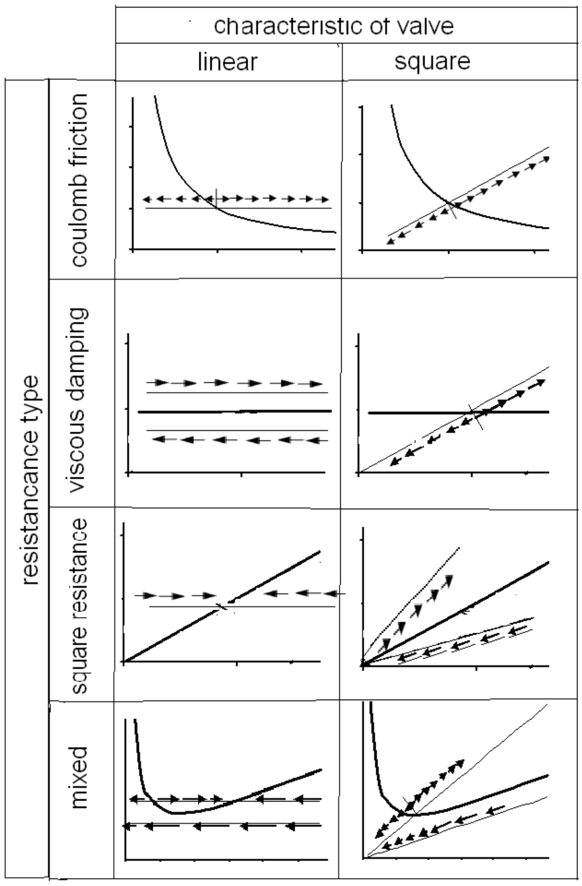

Figure 5. Character of stability for different valves

The effective damping $B_{eff}(X)$ according to (25) is shown in bold everywhere. The thin lines represent the dependency $q'_{\xi eff}(X)$ for the regarding cases (31) and (32). The arrows correspond to the local direction of the amplitude change. The diagrams illustrate the following quality tendencies.

1. With only Coulomb friction the system is stable in small but not absolute. Both for linear and for square characteristic of valve some rather big amplitudes of initial free oscillations increase with time unlimited.
2. In the case of viscous damping the system with the square characteristic of

valve behaves as in the case of Coulomb friction. For the linear characteristic of valve either absolute stability or absolute instability is possible. It depends on the fact whether the condition $c_1<B(1+\alpha)$ is fulfilled or not. Thus, in the case of viscous damping the linear characteristic of valve is preferable.

3. In the case of square resistance, in contrast, the square characteristic of valve makes it possible either absolute stability or absolute instability. It depends on the fact whether the condition

$c_2<8/(3\pi)k(1+\alpha)^2$

is fulfilled or not. In this case the system with linear characteristic of valve is always instable in small and has seemingly stability at rather big amplitudes. It can be practically acceptable, if the steady-state amplitude

$$X_* = \frac{3\pi c_1}{8k(1+\alpha)^2} \qquad (33)$$

is sufficiently small.

Nevertheless, in the case of square resistance the square characteristic of valve is preferable.

4. In the case of mixed dissipation when all types of damping take place the absolute stability is possible both for linear and for square characteristic of valve. It demands fulfilling the following conditions:

- for linear characteristic of valve:

$$c_1 < (8/\pi\sqrt{2/3fk} + B)(1+\alpha) \qquad (34)$$

-for square characteristic of valve:

$$c_2<8/(3\pi)k(1+\alpha)^2 \qquad (35)$$

If these conditions are not fulfilled, the system is stable in small but not absolute. From some level of initial amplitudes the oscillations increase. However in the case of linear characteristic of the valve amplitudes increase only till some steady-state level. The initial amplitudes that exceed this level demonstrate seemingly stability. Opposite, amplitudes for square characteristic of the valve can increase unlimited. Therefore, in the case of mixed dissipation the linear characteristic of valve should be preferred.

The described tendencies are confirmed through numerical simulation and serve as a good basis for preliminary estimation of parameters and for search of acceptable variants in the case of problems with stability.

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