THE MODELING OF FUZZY SYSTEMS BASED ON LEE-OSCILLATORY CHAOTIC FUZZY MODEL (LOCFM)

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Abstract

This paper introduces a new fuzzy membership function - LEE-oscillatory Chaotic Fuzzy Model (LoCFM). The development of this model is based on fuzzy logic and the incorporation of chaos theory - LEE Oscillator. Prototype systems are being developed for handling imprecise problems, typically involving linguistic expression and fuzzy semantic meaning. In addition, the paper also examines the mechanism of the LEE Oscillator through analyzing its structure and neural dynamics. It demonstrates the potential application of the model in future development.

Key words

Coupled Chaotic Oscillators, LEE Oscillator, Retrograde Signaling, Chaotic Fuzzy Model, Neural Dynamics

1 Introduction

Chaotic oscillator shows non-linear dynamic structures and hierarchy of a system. This paper introduces LEE Oscillator model and its enhancement LEE Oscillator (Retrograde Signalling) model. The structure and analysis of neural dynamics of LEE Oscillator are presented in section II, then LEE Oscillator (Retrograde Signalling) model are shown in section III. In section IV, we propose a new type of fuzzy membership function - Lee-Oscillatory Chaotic Fuzzy Model (LoCFM) by implementing chaotic oscillator in fuzzy logic. It is to supplement type-1 fuzzy set on modelling uncertainty, we anticipate that LoCFM is suitable for classifying complexity problems in real world.

2 The Background of LEE Oscillator

2.1 Structure of LEE Oscillator Model

Research on neuroscience and brain science in recent years has observed various chaotic phenomena in brain functions [Faure and Korn, 2001],[Korn and Faure, 2003] and behaviour of neurons are interactive triggering oscillation between excitatory and inhibitory neurons [Aihara and Matsumoto, 1987]. Based on these research findings, we have developed the LEE Oscillator and been able to simulated neural behaviour [Lee, 2004],[Lee, 2006]. We believe chaotic oscillators are capable of predicting and modelling highly complex problems in real world. LEE Oscillator has shown outstanding results in pattern recognitions and explained progressive memory recall scheme by Rubin-vase experiment [Lee, 2004],[Lee, 2006].

A LEE Oscillator consists of four neural dynamics of four constitutive neural elements: *u*, *v*, *w* and *z*, the neural dynamics of each of these constituent neurons are given by

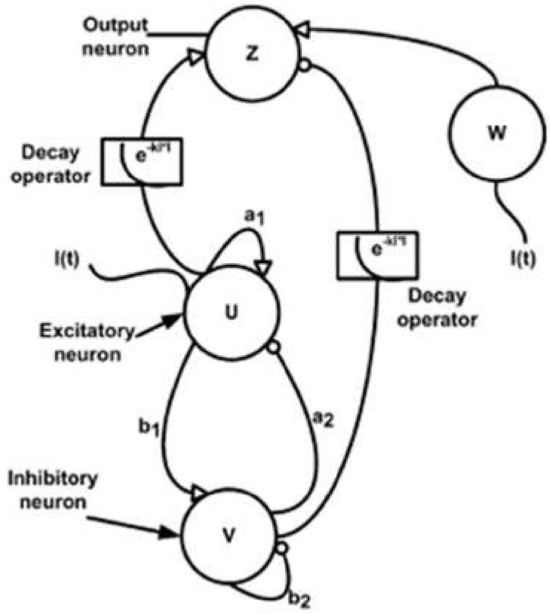

Figure 1: LEE Oscillator Model [Lee, 2004]

$$u' = tanh(a_1 \cdot \mathrm{u} - \mathrm{a}_2 \cdot \mathrm{v} + \mathrm{I}) \quad (1)$$

$$v' = tanh(b_1 \cdot \mathrm{u} - \mathrm{b}_2 \cdot \mathrm{v}) \quad (2)$$

$$w = tanh(I) \quad (3)$$

$$z = (u' - v') \cdot e^{-kI^2} + w \quad (4)$$

where *u*, *v*, *w* and *z* are the state variables of the excitatory, inhibitory, input, and output neurons, respectively; *tanh()* is a hyperbolic tangent function; a_1, a_2, b_1 and b_2 are the weight parameters for these constitutive neurons; *I* is the external input stimulus; and k is the decay constant. Fig. 1 illustrates the LEE Oscillator model and Fig. 2 shows its bifurcation behavior where regions A and C are in sigmoid shape and without chaotic activities. However, there exists a chaotic region B when weak external input stimulus is received.

2.2 Analysis of LEE Oscillator Model

The largest Lyapunov Exponent (λ) is calculated by Wolf's algorithm [Wolf et al., 1985],[Peitgen et al., 2004],[Sprott, 2003]

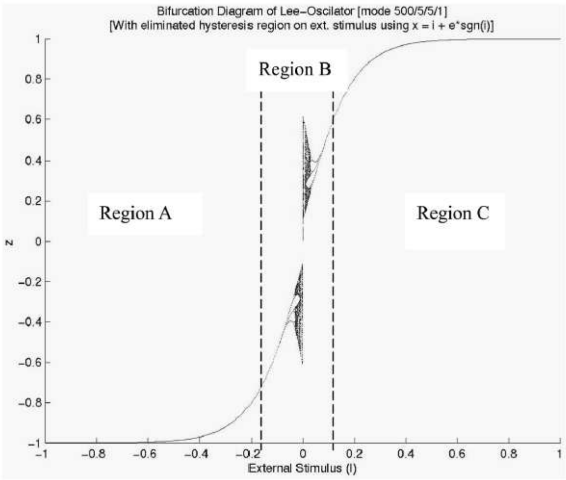

Figure 2: Bifurcation Diagram of LEE Oscillator [Lee, 2004]

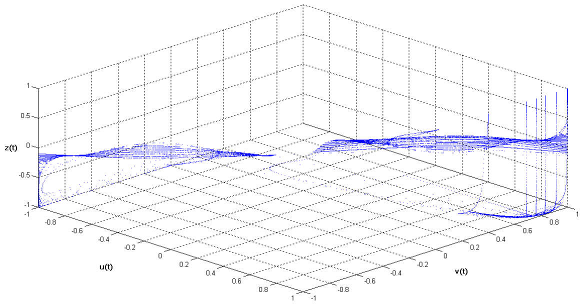

Figure 3: 3D Projection of LEE Oscillator (5/1/500) in *u, v* vs. *z*

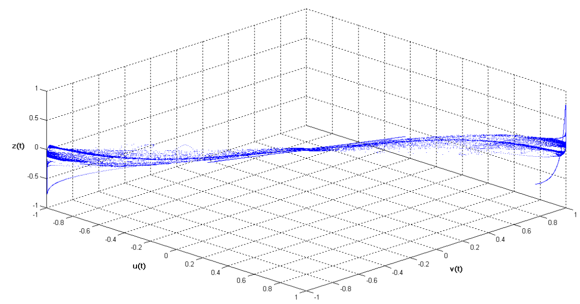

Figure 4: 3D Projection of LEE Oscillator (10/20/500) in *u, v* vs. *z*

and uses to analyse the dynamics of LEE Oscillator, we had tested using two fixed sets of parameters with varying I in [-1, +1], those parameters are i) a_1=5, a_2=5, b_1=1, b_2=1 and k=500 (Fig. 3); and ii) a_1=10, a_2=10, b_1=20, b_2=20 and k=500 (Fig. 4). The Lyapunov Exponents for the former one is $0.000001 \leq \lambda \leq 3.3661$ and the latter one is $0.1817 \leq \lambda \leq 2.3200$.

3 The Enhancement of LEE Oscillator - LEE Oscillator (Retrograde Signalling) Model

3.1 The Implementation of Retrograde Signalling Feature in Axonal Transport

The enhancement of the LEE Oscillator is derived from retrograde transport mechanism in axons, axonal transport (also named as axoplasmic flow) was discovered by Paul Weiss in 1948 [Nicholls et al., 2001]. Axons are not able to synthesize proteins; all synthesis of proteins takes places in the cell body. Receptors, signalling proteins and enzymes for synthesis of neu-

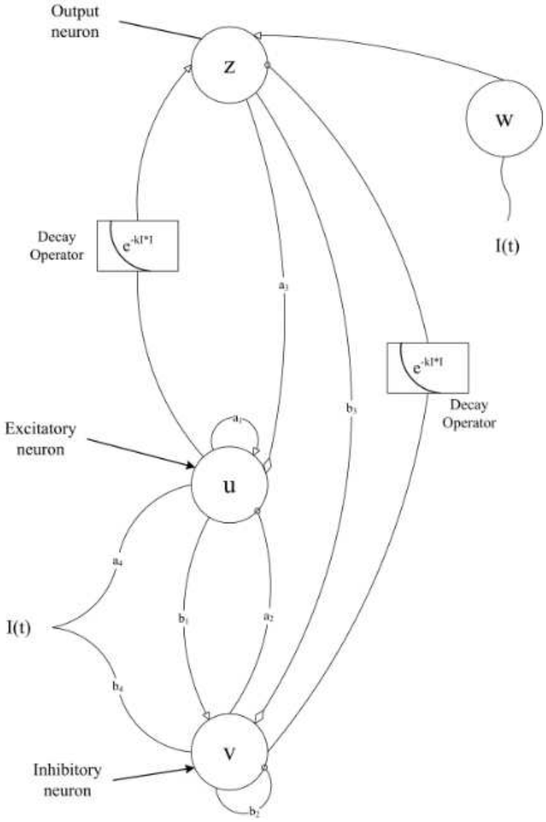

Figure 5: LEE Oscillator (Retrograde Signalling) Model

rotransmitter must be moved to distant axon terminals or dendrites. This movement of a cell body towards terminals or dendrites is called anterograde transport. On the other hand, retrograde transport is a backward transmission mechanism which transmits neurotrophin from axon terminals to the cell body [Freberg, 2005],[Levitan and Kaczmarek, 2001],[Neet and Campenot, 2001],[Nicholls et al., 2001].

Neuroscientists have made an important discovery in recent years, they found out the major functionality of retrograde transport is signalling. This functionality is also called Retrograde Signalling. Retrograde signals influence neuronal survival, differentiation, homeostasis and plasticity [Howe and Mobley, 2005],[Sanyal et al., 2004],[Zweifel et al., 2005]. The latest research on neuroscience pointed out that neurological diseases such as Alzheimer's disease and Down's syndrome are significantly related to malfunction of retrograde transport mechanism [Copper et al., 2001]. Fig. 5 is a depiction of LEE Oscillator (Retrograde Signalling) Model [Wong et al., 2008],[Kwong et al., 2008] and Eqs. 5 to 8 represent the neural dynamics of this model.

$$u' = tanh(a_1 \cdot \mathrm{u} - \mathrm{a}_2 \cdot \mathrm{v} + \mathrm{a}_3 \cdot \mathrm{z} + \mathrm{a}_4 \cdot \mathrm{I}) \tag{5}$$

$$v' = tanh(b_3 \cdot \mathrm{z} - \mathrm{b}_1 \cdot \mathrm{u} - \mathrm{b}_2 \cdot \mathrm{v} + \mathrm{b}_4 \cdot \mathrm{I}) \tag{6}$$

$$w = tanh(I) \tag{7}$$

$$z' = (v' - u') \cdot e^{-kI^2} + w \tag{8}$$

The meaning of all variables is the same as the previous model where *u*, *v*, *w* and *z* act as the excitatory, inhibitory, input, and output neurons respectively; *tanh()* is a hyperbolic tangent function; $a_{1\ to\ 4}$ and $b_{1\ to\ 4}$ are the weight parameters for the neurons; *I* and *k* depict external input stimulus and decay constant respectively.

Table 1: Different Parameter Settings used in LEE Oscillator (RS) Model

Parameters	Oscillator Type			
	A	B	C	D
a_1	0.6	1	0.55	1
a_2	0.6	1	0.55	1
a_3	-0.5	1	-0.5	1
a_4	0.5	1	0.5	1
b_1	-0.6	-1	-0.55	-1
b_2	-0.6	-1	-0.55	-1
b_3	-0.5	-1	0.5	-1
b_4	0.5	-1	-0.5	-1
k	50	50	50	300

3.2 Modification on Previous Model

There are three major modifications on previous model. Firstly, output value of z is re-imported into both the excitatory neuron (Eq. 5) and the inhibitory neuron (Eq. 6). This process simulated retrograde signalling mechanism to send signals back for neuronal plasticity. One more provision for assigning z into Eq. 6 is to simulate human learning behaviour, in which they usually learn through experiences, errors and pains. Secondly, I assigned in Eq. 6 is due to incoming signals which should be considered in inhibitory neurons, instead of excitatory neurons only. Thirdly, switching v-u (Eq.4) from u-v (Eq.8) in the original work [Lee, 2004],[Lee, 2006] is a different concept against the previous model where the concept of presynaptic inhibition is implemented in this modification. Presynaptic inhibition is a neurotransmitter release mechanism, a strong suppression of responses of an excitatory neuron before stimulus reaches synaptic terminals mediated by an inhibitory neuron [Levitan and Kaczmarek, 2001], [Nicholls et al., 2001]. Lastly, new parameters a_3, a_4, b_3 and b_4 were added in Eqs. 7 & 8 as every variable should have its own parameter to adjust the outcomes.

3.3 Enhancement

There are three improvements from previous model. First, the variability of chaotic state is increased because of new parameters being introduced in this model. Chaotic regions are easier to reshape and it is possible to generate either single or dual chaotic regions by tuning different parameters. Figs. 6 (Table 1, Type A) and 7 (Table 1, Type B) show bifurcation behaviour by using different parameter sets on Lee Oscillator (Retrograde Signalling) model, more details of these oscillators are shown in Appendix. The values in x-axis are represented by I and numbers in y-axis are results of z. Second, the chaotic region is wider than that of previous model, so the temporal information processing can be more effective. Lastly, the generation of chaotic region in the LEE Oscillator (Retrograde Signalling) model requires just one-tenth of the number of iterations of previous models, and for this reason, the computation time is reduced.

3.4 Analysis of LEE Oscillator (Retrograde Signalling) Model

We choose a set of constant parameters (shown in Table 1) and vary the value of I in [-1, +1], leading to the following Lyapunov Exponents:

Type A oscillator (Fig. 8), $0.2861 \leq \lambda \leq 1.0087$
Type B oscillator (Fig. 9), $0.1643 \leq \lambda \leq 1.0716$
Type C oscillator (Fig. 10), $0.2832 \leq \lambda \leq 1.0091$
Type D oscillator (Fig. 11), $0.1643 \leq \lambda \leq 1.8580$.

The above Lyapunov Exponent values are in positive, so LEE Oscillator (Retrograde Signalling) model is a chaotic system.

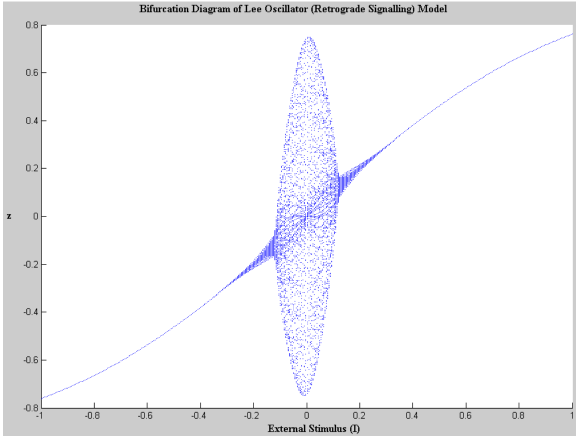

Figure 6: Bifurcation Diagram of LEE Oscillator (Retrograde Signalling) Model - Type A Oscillator

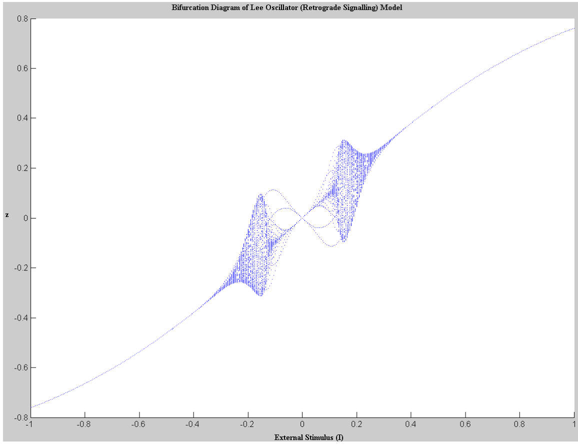

Figure 7: Bifurcation Diagram of LEE Oscillator (Retrograde Signalling) Model - Type B Oscillator

Technical details of type A to D oscillators are given in Appendix. Figs. 8 to 11 present different neural dynamics when the oscillator is with different parameters, ellipse orbits are shown in Figs. 8 and 10, spiral shell shape displayed in Fig. 9 and finally Fig. 11 demostrates similar behaviour as a Lissajous curve. The reasons why different parameters generate different shapes are still under investigation, but we assume that one of the possible reasons is that the oscillator contains characteristic of hyperchaos. Another research direction is why parameters changed in LEE Oscillator still keep its dynamics in two armed spiral pattern (Figs. 3 and 4), but retrograde signalling model changed its dynamics according to parameters.

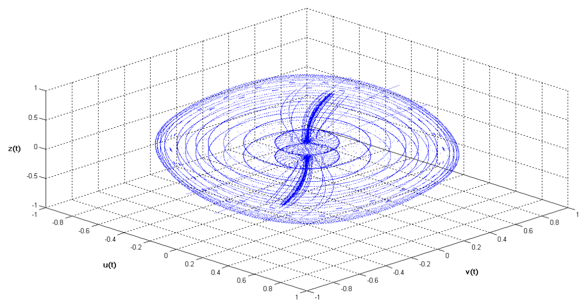

Figure 8: 3D Projection of Type A Oscillator in u, v vs. z

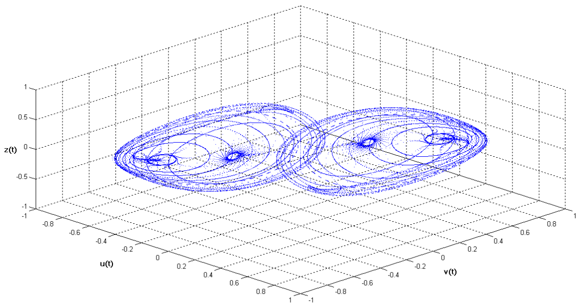

Figure 9: 3D Projection of Type B Oscillator in *u, v* vs. *z*

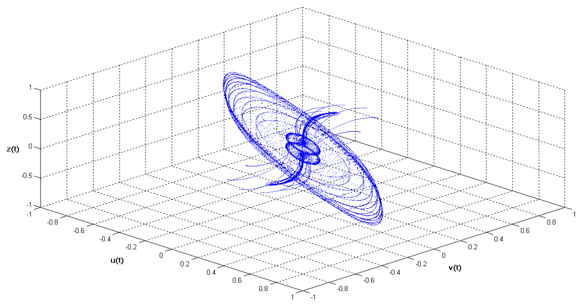

Figure 10: 3D Projection of Type C Oscillator in *u, v* vs. *z*

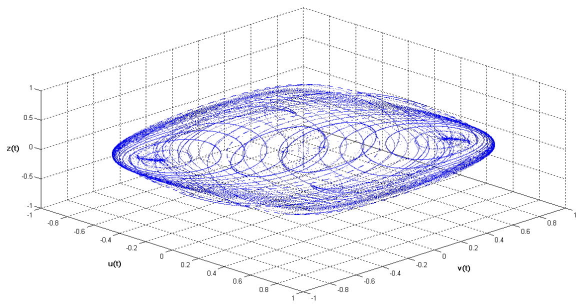

Figure 11: 3D Projection of Type D Oscillator in *u, v* vs. *z*

4 The Prototype of a New Fuzzy Membership Function - LEE-Oscillatory Chaotic Fuzzy Model (LoCFM)

As we all know human mind does not work in digital process, but computer does. Therefore, fuzzy logic is needed to link human's fuzzy feeling to computer through implementing probability and degree of membership associated with fuzzy set. It is doubt the possibility of modelling uncertainties, although Prof. ZADEH proposed the concept of type-2 fuzzy logic to strengthen the concept of uncertainty [Zadeh, 1975], [Zadeh, 2006], [Zadeh, 2008]. According to a number of research in brain science, scientists have found that our brain is in fact organized by chaos [Faure and Korn, 2001],[Korn and Faure, 2003]. We conjecture relationship between chaotic patterns and fuzzy concepts in brain as follows. Brain operates in the rules of chaos theory by generating different neural dynamics signals, then fuzzy feelings are the results of the combination of different signals. These feelings let us realize the temperature of water, emotion or even cognition. Therefore, chaos theory may be one of the more suitable tools in bridging human and computer. For the above reasons and the successfulness of LEE Oscillator model, we believe that there is another way to model uncertainties in computational intelligence. So, we proposed a theoretic work for a new series of fuzzy logic - LEE-Oscillatory Chaotic Fuzzy Model (LoCFM). In next section, we introduce two prototypes to show the integration between chaotic oscillator and fuzzy membership function to become the proposed models.

4.1 Prototype Structure

Fig. 12 and Eqs. 9-12 show the prototype of Chaotic Fuzzy Set - Number One (Prototype CF1):

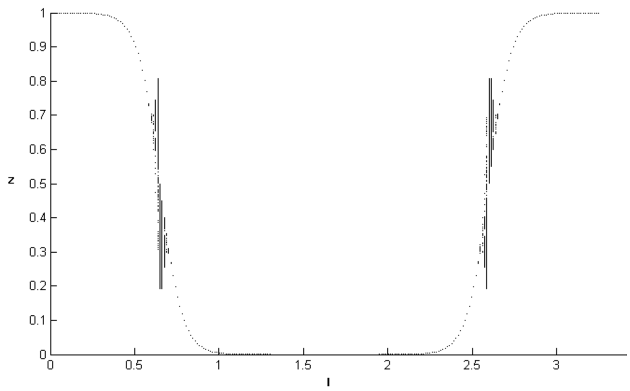

Figure 12: LEE-Oscillatory Chaotic Fuzzy Model - Prototype CF1

$$u' = tanh(a_1 \cdot \mathrm{u} - \mathrm{a}_2 \cdot \mathrm{v} + \mathrm{I}) \tag{9}$$
$$v' = tanh(b_1 \cdot \mathrm{u} - \mathrm{b}_2 \cdot \mathrm{v}) \tag{10}$$
$$w = tanh(I) \tag{11}$$
$$z = m \cdot ((u' - v') \cdot e^{-kI^2} + w) + n \tag{12}$$

Prototype CF1 had a minor modification based on Eq. 4, *m* and *n* are parameters of slope-intercept form, Fig. 12 shows the membership function of Prototype CF1. This prototype uses as similar as a shoulder membership function in type-1 fuzzy logic, but it is with chaotic regions.

Fig. 13 and Eqs. 13-15 show the prototype of Chaotic Fuzzy Set - Number Two (Prototype CF2):

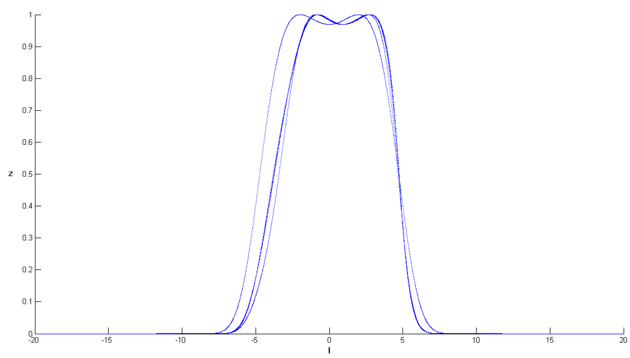

Figure 13: LEE-Oscillatory Chaotic Fuzzy Model - Prototype CF2

$$u' = a_1 \cdot \mathrm{u} - \mathrm{a}_2 \cdot \mathrm{v} + \mathrm{a}_3 \cdot \mathrm{z} + \mathrm{a}_4 \cdot \mathrm{I} \tag{13}$$

$$v' = b_3 \cdot \mathrm{z} - \mathrm{b}_1 \cdot \mathrm{u} - \mathrm{b}_2 \cdot \mathrm{v} + \mathrm{b}_4 \cdot \mathrm{I} \tag{14}$$

$$z' = e^{-\frac{[(u' \cdot v' \cdot k) - c]^2}{2\sigma^2}} \tag{15}$$

The above equations have been modified based on LEE Oscillator (Retrograde Signalling) model, and the variables and structures are more or less the same as those of oscillator, but with two major changes. First, the hyperbolic tangent function (*tanh*) is removed in order to compute input values which are smaller than -1 or larger than +1. Second, a Gaussian function is implemented in output neuron (*z*) for generating bell curves as membership function, *c* and σ are parameters of Gaussian function to control the position of the centre of the peak and the width of the *bell curve*. Fig. 13 illustrates prototype CF2, which is used to extend the normal Gaussian fuzzy set to a chaotic Gaussian fuzzy set.

4.2 Potential Advantages

LoCFM combined the characteristics of type-1 fuzzy membership functions and chaos theory. We assume that the mixture of fuzzy linguistic and chaotic phenomenon offered a fertile ground for fuzzy classification and prediction on data which contained chaotic patterns. It is because chaotic fuzzy set can provide one more dimension in the chaotic region to transform crisp into membership value through non-linear dynamics, that type-1 fuzzy logic cannot be simulated. On the other hand, chaotic region is generated by a small number of equations, comparing with complicated calculations used in type-2 fuzzy. LoCFM may be a possible solution for reducing computation time and performing the functionality of type-2 fuzzy logic as well.

5 Summary

In this paper, we have reported and analyzed LEE Oscillator model and its enhanced retrograde signalling model. Then, we have introduced a new concept of chaotic fuzzy set - LEE-Oscillatory Chaotic Fuzzy Model (LoCFM) which provides an advanced paradigm in future computational intelligence techniques. The above models are required for further investigations, our future developments are in four folds. First, the LEE Oscillator models are desired an in-depth study on chaotic dynamics. Second, Exploring different chaotic fuzzy sets, such as implementing chaotic features into triangular and trapezoid shaped fuzzy sets. Third, towards a comprehensive theory of LoCFM, there are two major problems which need to be solved. i. How chaotic fuzzy models process by different operators including union, intersection and complement operations. ii. Inference engine and defuzzification process are required to be modified in order to suit with the LoCFM. Forth, A lot of experiments are desired to compare the performance and accuracy of LoCFM with fuzzy features.

Acknowledgements

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Appendix Bifurcation Behavior of LEE Oscillator (Retrograde Signalling) Model and Technical Details of Figures

For the following Bifurcation diagrams, **X-axis** represents the values of ***I*** and **Y-axis** are the output values (***z***) of the LEE Oscillator (Retrograde Signalling) model .

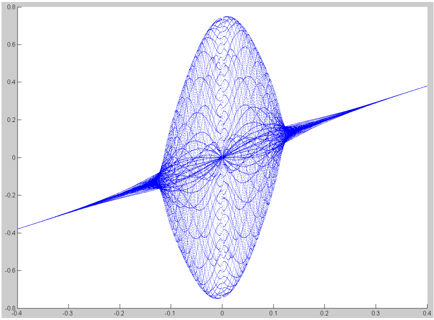

Figure 14: Bifurcation Behavior (detailed part) of Type A Oscillator

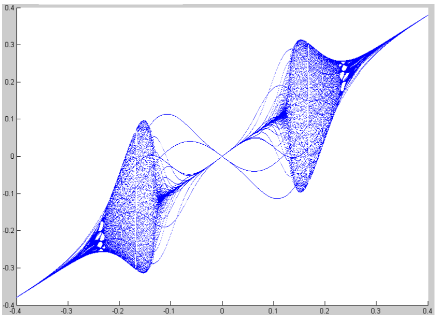

Figure 15: Bifurcation Behavior (detailed part) of Type B Oscillator

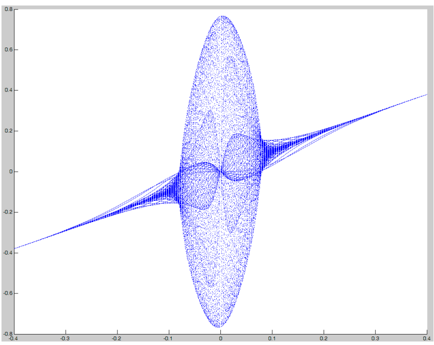

Figure 16: Bifurcation Behavior (detailed part) of Type C Oscillator

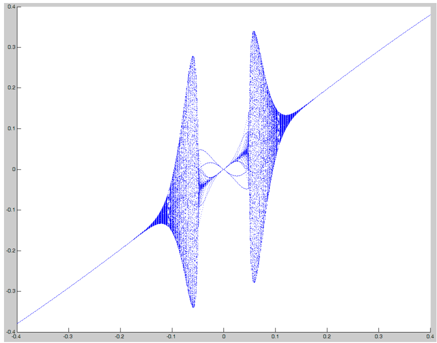

Figure 17: Bifurcation Behavior (detailed part) of Type D Oscillator

Table 2: Table of Technical Details of Figures for Generating LEE Oscillator's Neural Dynamic

Figure	Eqs. No.	Parameters	Initial Conditions
3	1-4	a_1=5 a_2=5 b_1=1 b_2=1 k=500	u=0 v=0 z=0 I=[-1,+1]
4	1-4	a_1=10 a_2=10 b_1=20 b_2=20 k=500	u=0 v=0 z=0 I=[-1,+1]
8	5-8	a_1=0.6 a_2=0.6 a_3=-0.5 a_4=0.5 b_1=-0.6 b_2=-0.6 b_3=-0.5 b_4=0.5 k=50	u=0 v=0 z=0 I=[-1,+1]
9	5-8	a_1=1 a_2=1 a_3=1 a_4=1 b_1=-1 b_2=-1 b_3=-1 b_4=-1 k=50	u=0 v=0 z=0 I=[-1,+1]
10	5-8	a_1=0.55 a_2=0.55 a_3=-0.5 a_4=0.5 b_1=-0.55 b_2=-0.55 b_3=0.5 b_4=-0.5 k=50	u=0 v=0 z=0 I=[-1,+1]
11	5-8	a_1=1 a_2=1 a_3=1 a_4=1 b_1=-1 b_2=-1 b_3=-1 b_4=-1 k=300	u=0 v=0 z=0 I=[-1,+1]