INSTABILITIES OF SPIKING IN CLASS B LASER WITH VIBRATING EXTERNAL MIRROR

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Abstract

Finite dimensional maps are analytically derived for class B laser systems with an incoherent external feedback. On the base of such maps, a hierarchy of periodic solutions of various structures are demonstrated in an infinite-dimensional delayed system. Sets of initial conditions and parameters are determined for every type of the solutions. Bifurcations induced by an oscillating delay are described.

1. Introduction Compact lasers such as microchip solid-state or semiconductor lasers have great technological importance for a wide variety of applications. The high-sensitivity response to external optical feedback (FB) light because of extremely short photon lifetimes compared with the population lifetimes that are inherent in microchip lasers has resulted in such significant applications as self-aligned laser Doppler velocimetry, vibrometry, and imaging [1,2]. It stimulates investigations of instabilities induced by the delay itself as well as instabilities induced by combined factors such as periodic modulation of the time delay. Among other applications we note the possibility of using high-dimensional chaos for signal encryption in communicative systems [3-6]. Pulse-to-pulse jitter in laser diodes has been shown to be very sensitive to small variations in the cavity length [7,8].

External delayed FB in semiconductor laser systems can spontaneously arise due to reflecting of radiation from external objects or can be organized especially to control the generation [9-14]. In particular, an incoherent feedback scheme has been proposed to generate high-speed pulses [11]. Numerical simulations have shown that in this scheme sustained periodic and chaotic relaxation oscillations coexist with regenerative spiking oscillations [11]. The normal form analysis [15,16] has explained quasiperiodic relaxation oscillations near codimension two bifurcations. However, regular and chaotic spiking regimes cannot be described in the framework of the local analysis. It connects to general difficulties in theoretical description of nonlinear periodic orbits. Additionally, instabilities induced by the delay FB appear and develop in an infinite-dimensional phase space.

The aim of this work is to develop a nonlinear asymptotic theory of spiking oscillations in FB laser systems with constant and periodically varying delay time. Section 2 contains the mathematical model and brief description of the method of investigation. In Section 3 the results are given on the existence and the structure of the spiking oscillations of the period longer than the delay time. They are called slowly oscillating (SO) solutions in contrast with that of the period shorter than the delay and which are called fast oscillating (FO) solutions. It appears that SO solution can bifurcate to the attractor which is an alternation between SO and FO solutions. In Section 4 we apply the method to the case of the periodically varying delay and derive two-dimensional map responsible for the dynamics of SO solutions. On the base of such a map, bifurcations are described in dependence on the amplitude and the frequency of the delay modulation.

2. Model In the FB scheme proposed in [6], polarization of delayed feedback light from a reflecting mirror is orthogonal to that of the laser light. It allows to avoid coherent phase-amplitude interaction in the dynamics of the electric field. The situation is, therefore, modelled by the single-mode rate equations with a delayed argument

$$\frac{du}{dt} = vu(y-1), \tag{1}$$

$$\frac{dy}{dt} = q - y - y[u + \gamma u(t-\tau)], \tag{2}$$

where u and y - are proportional to the photon density and carriers inversion respectively, v is the ratio of the photon damping rate in the cavity to the damping rate of the inversion of population, q is the pumping rate normalized to the cavity loss, γ is the feedback coefficient, the delay time τ is equal to the round trip time in the external cavity. If the external cavity length is periodically varied the time delay reads

$$\tau = \tau_0 + B\cos(\omega t + \phi)$$

where τ_0 is a constant part of the delay, B and ω are the amplitude and the frequency of mirror oscillations, respectively, ϕ is the phase of modulation at the initial moment.

The phase space of system (1),(2) is the direct product of the Banach space $C_{[-\tau,0]}$ of continues functions by the number line R^1, i.e., the values of the functions from $C_{[-\tau,0]}$ and the value $y(0) \in R^1$ are given as initial conditions. In this space, we shall distinguish a (fairly wide) set $S(\Psi)$ dependent on the vector parameter Ψ and consider the solutions with initial conditions from this set. It is possible to construct uniform asymptotic approximations of all such solutions and show that after a certain time these solutions again fall within $S(\Psi)$. Thus, the operator of the shifting along the trajectories, which makes a function from S corresponds to a function also from S, is naturally determined. The properties of this operator are mainly determined by the finite-dimensional map $\bar{\Psi} = g(\Psi)$. To a fixed point of the map there corresponds the fixed point of the operator, and to the later point there corresponds a periodic solution of system (1),(2) of the same stability.

A way for asymptotic build-up of solutions of the spiking type is opened by the fact that for class B lasers including semiconductor, CO_2- and some solid-state lasers the parameters q, γ, τ are of the order of ~ 1 while the damping rate of photons is very high, $v \cong 10^2 \div 10^4$. Note that the method of asymptotic integration permits to obtain uniform asymptotic formulae for steady-state regimes with any degree of accuracy. We shall restrict ourselves to the leading terms of the solutions. Examples of the investigation of similar systems were given in our previous works [9,10].

3. SO solution in the case of constant delay Let's term a solution slow oscillating (SO) if it has no more than one pulse on the interval of the delay. Solutions of such a structure are the most simple. The set of initial conditions $S(c)$ is given by

$$y(0) = c > 1,\ u(s) = h(s),\ s \in [-\tau, 0), \tag{3}$$

where the functions $h(s) \in C_{[-\tau,0]}$ have the properties:

$$0 \le h(s) \ll 1,\ h(0) = 1,\ \int_{-\tau}^{0} h(s)ds \le v^{-1/2}. \tag{4}$$

Physically, such initial conditions correspond to radiation intensity of the noise level in the interval $s \in [-\tau_0, 0]$ and to the choice of $t = t_0 = 0$ at the onset of the emission pulse when $u(0) = 1$ and $\dot{u}(0) > 0$. Also, we denote the sequential positive time moments when $u(t, c) = 1$ by $t_1, t_2, ...$ so that $t_0, t_2 ... t_{2k} ...$ mean time moments of the onset of spikes while $t_1, t_3 ... t_{2k+1} ...$ - moments of the cessation of spikes. Note, that the pulse duration $(t_1 - t_0), ... \to 0$ with $v \to \infty$.

Under the initial conditions chosen a short pulse of radiation intensity $u(t) \gg 1$ takes place during the interval $t \in (0, t_1)$. Note, the pulse duration $t_1 \to 0$ with $v \to \infty$. Integrating the system asymptotically step by step we find :

$$y(t_1) = c - p,\ y(\tau_0) = q + (c - p - q)e^{-\tau_0},\ y(\tau_0 + t_1) = y(\tau_0)e^{-\gamma p}, \tag{5}$$

where $p = \int_0^{t_1} u(t)dt$ means the value of the pulse energy which determined as a positive root of the equation (with an accuracy $O(v^{-1})$)

$$c - p = ce^{-p}. \tag{6}$$

SO solution is realized if the function

$$a(t, c) = (q - 1)t + (c - p - q)(1 - e^{-t}) \tag{7}$$

fulfills the inequality

$$a(\tau_0, c) < 0. \tag{8}$$

This condition provides $u(t) \ll 1$ up to the moment $t = t_2 = T, u(t_2) = 1$, which is determined as the positive root of the equation $A(T, c) = 0$, where $A(t, c) = a(\tau_0, c) + (q-1)(t - \tau_0) + +[[(c - p - q)e^{-\tau_0} + q]e^{-\gamma p} - q](1 - e^{\tau_0 - t})$. At this moment we come to the problem which is similar to the original one. Actually, let us introduce the operator of the shifting along the trajectories $\Pi(h(s), c) = (\bar{h}(s), \bar{c})$ where $\bar{h}(s) = u(s + t_2), \bar{c} = y(t_2)$. If requirement (8) is fulfilled then $\bar{h}(s) \in S_0$ and

$$\bar{c} = q(1 - e^{\tau_0 - T} + e^{-\gamma p + \tau_0 - T}) + (c - p - q)e^{-\gamma p - T} \tag{9}$$

with $p = p(c), T = T(c)$. Thus, the problem of further constructing the solution again has returned to the original problem with c replaced by $\bar{c}$. Iterations of one-dimensional map (9) determine the evolution of the solution. In particular, fixed points of this map correspond to the periodic pulsations of the same stability in the original system. SO structure of such solutions is provided by condition (8) which has to be fulfilled for every iteration.

Such a positive external FB cannot be provided by purely optical means. Thus FO solutions cannot exist in system (1),(2). Instead of them special attractors arise due to an alternation between SO and FO solutions. Let us describe for example one of such structures in which radiation spikes follow one after another through time intervals that are longer and shorter than the delay τ_0. We denote this pattern by FO^1+SO. Starting with initial conditions (3),(4) and integrating system (1),(2) step by step we get sequentially

$$y(t_1) = ce^{-p},\ y(t_2) = q + (y(t_1) - q)e^{-\xi},\ y(t_3) = y(t_2)e^{-p_1},$$

where the value of the pulse energy p_1 is obtained as the root of the equation $y(t_2) - p_1 = y(t_2)\exp(-p_1)$ and the moment $t_3 = \xi$ is the root of the equation $a(\xi, c) = 0$. This moment determines the short interval between pulses, $\xi < \tau_0$. At the moment $t = t_4 = T$ which is equal to the positive root $T > \tau_0 + \xi$ of the equation

$$(T-\xi)(q-1) + (y(t_3) - q)(1 - e^{\xi - \tau_0}) + (y(\tau_0) - q)(1 - e^{-\xi}) + \\ (y(\tau_0 + \xi)e^{-\gamma p_1} - q)(1 - e^{\tau_0 + \xi - T}) = 0$$

one has returned to the original problem with c replaced by $\bar{c}$

$$\bar{c} = q + (y(\tau_0 + \xi)e^{-\gamma p_1} - q)e^{\tau_0 + \xi - T}. \tag{10}$$

Thus the evolution of the solution is determined by the dynamics of the one-dimensional map (10) if inequalities $\xi < \tau_0$, $T > \tau_0 + \xi$ are fulfilled for every iteration. Direct calculations prove an existence of a stable fixed point in this map. It undergoes, in turn, a saddle-node bifurcation that results in another alternative structure, FO^m+SO.

4. SO solution for an oscillating delay Here we construct SO solutions of system (1) in the case of the periodically varied time delay $\tau = \tau_0 + B\cos(\omega t + \phi)$. We assume that the amplitude of the modulation is limited with condition $B < \tau_0$ providing a positive value of the delay. The frequency of the modulation is assumed to be $|\omega| < 1/B$ providing a positive variation of the delayed argument $g(t, \phi) = t - \tau_0 - B\cos(\omega t + \phi)$, $g'(t, \phi) > 0$. As above we start with initial conditions (3),(4)for SO solutions and denote the sequential positive time moments when $u(t) = 1$ by $t_0, t_1, t_2, ...$ Also, we introduce a corresponding "delayed" sequence $\bar{t}_0(\phi), \bar{t}_1(\phi), \bar{t}_2(\phi), ...$ which can be obtained from the equation $g(\bar{t}_i, \phi) = t_i,\ i = 0, 1, 2...$ SO solution can realized if the inequality

$$a(\bar{t}_0(\phi), c) < 0 \tag{11}$$

is fulfilled. In the interval $t \in [\bar{t}_0, \bar{t}_1]$, the radiation intensity is still small, $u(t) \ll 1$, due to the above condition but the delay term is large, $u(t - \tau) \gg 1$. That is why one can obtain

$$y(\bar{t}_1) = y(\bar{t}_0)\exp\left(-\gamma \int_{\bar{t}_0}^{\bar{t}_1} u(g(s))ds\right).$$

The last integral can be rewritten with the new variable $\bar{s} = g(s)$ as follows

$$\int_{\bar{t}_0}^{\bar{t}_1} u(g(s))ds = \int_{t_0}^{t_1} \frac{u(\bar{s})}{g'(s)} d\bar{s}.$$

Finally, taking into account that $g'(s) = g'(\bar{t}_0) + O(v^{-1})$ we get

$$y(\bar{t}_1) = y(\bar{t}_0)\exp\left(-\gamma \frac{p}{g'(\bar{t}_0)}\right). \tag{12}$$

Since the moment $t = \bar{t}_1$ and up to the moment $t = t_2$ both the current and the delayed intensities are asymptotically small, $u(t) \ll 1, u(t - \tau_0) \ll 1$. The solution takes the form

$$\begin{aligned} y(t) &= q + [y(\bar{t}_1) - q]e^{\bar{t}_0 - t}, \\ u(t) &= \exp[v\bar{A}(t, c, \phi)], \end{aligned} \tag{13}$$

where

$$\bar{A}(t,c,\phi) = a(\bar{t}_0,c) + (q-1)(t-\bar{t}_0) + +[[(c-p-q)e^{-\bar{t}_0} + q]e^{-\gamma p/g'(\bar{t}_0)} - q](1-e^{\bar{t}_0 - t}).$$

At the moment $t = t_2 = T$ which is equal to the positive root $T > \tau$ of the equation

$$\bar{A}(T,c,\phi) = 0 \tag{14}$$

the obtained solution becomes analogous to the initial conditions with $u(s+t_2,c,\phi) \in S$ and with c,ϕ replaced by $\bar{c},\bar{\phi}$:

$$\begin{aligned} \bar{c} &= q(1 - e^{\bar{t}_0 - T} + e^{-\gamma p/g'(\bar{t}_0) + \bar{t}_0 - T}) + (c-p-q)e^{-\gamma p/g'(\bar{t}_0) - T} \\ \bar{\phi} &= \phi + \omega T, \ mod\ 2\pi, \end{aligned} \tag{15}$$

with $p = p(c),\ \bar{t}_0 = \bar{t}_0(\phi),\ T = T(c,\phi)$.

Thus the evolution of SO solutions of the periodically driven system is determined by the dynamics of two-dimensional map (15). The second equation from system (15) has a form of well-known circle map. Hence one can expect for quasiperiodic and locking phenomena. Actually, with increasing the frequency of the external driving ω we observe an interesting version of "devil's staircase" in 2-D map dynamics. Another route to chaos is realized when the amplitude B of the delay driving increases. Period tripling and period doubling bifurcations are possible in this case.

In conclusion, we have analytically obtained periodic solutions of various structure in the solid state laser with vibrating delayed FB. Doing so, the problem on dynamics of the original infinite-dimensional system has been reduced to the problem on the dynamics of nonlinear finite-dimensional maps. It opens a way to determine a hierarchy of spiking regimes on the base of pattern complexity, i.e. the order of inhomogeneity, on the delay interval. We distinguish between slowly oscillating solutions describing by the one-dimensional map and fast oscillating solutions describing by the $(2m+1)$-dimensional maps. In the case of the constant delay they undergoes typically a saddle-node bifurcation which gives rise to periodic alternative (or irregular intermittent) solutions. In the case of the periodically driven delay, we have derived two-dimensional map responsible for the dynamics and found quasiperiodic, period doubling, and period tripling routes to chaos.

The developed method has advantages in the problems on nonlocal organization of phase space and can be generalized to the system of coupling laser diodes with the external FB.

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