Stabilizing a wave amplified by a beam of particles with test-waves

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Abstract

The intensity of an electromagnetic wave interacting self-consistently with a beam of charged particles as in a free electron laser, displays large oscillations due to an aggregate of particles, called the macro-particle. In this article, we propose a strategy to stabilize the intensity by destabilizing the macro-particle. This strategy involves the study of the linear stability (using the residue method) of a specific periodic orbit of a mean-field model. As a control parameter - the amplitude of an external wave - is varied, a bifurcation occur in the system which have drastic effect on the modification of the self-consistent dynamics, and in particular, of the macro-particle. We show how to obtain an appropriate tuning of the control parameter which is able to strongly decrease the oscillations of the intensity without reducing its mean-value.

Extensive abstract

The self-consistent interaction between an electromagnetic wave and a beam of charged particles is ubiquitous in many branches of physics, e.g. accelerator and plasma physics. For instance, it plays a crucial role in the Free Electron Laser, which is used to generate a tunable, coherent, high power radiations. Such devices differ from conventional lasers in using a relativistic electron beam as its lasing medium. The physical mechanism responsible for the light emission and amplification is the interaction between the beam and a wave, which occurs in presence of a magnetostatic periodic field generated in an undulator. Due to the effect of the magnetic field, the electrons are forced to follow sinusoidal trajectories, thus emitting synchrotron radiation. This initial seed, termed *spontaneous emission*, acts as a trap for the electrons which in turn amplify it by emitting coherently, until the laser effect is reached.

The coupled evolution of radiation field and N particles can be modeled within the framework of a simplified Hamiltonian picture [1]. The $N+1$ degree of freedom Hamiltonian displays a kinetic contribution, associated with the particles, and a potential term accounting for the self-consistent coupling between the particles and the field. Hence, direct inter-particle interactions are neglected, even though an effective coupling is indirectly provided because of the interaction with the wave.

The linear theory predicts [1], for the amplitude of the radiation field, a linear exponential instability and a late oscillating saturation. Inspection of the asymptotic phase-space suggests that a bunch of particles gets trapped in the resonance and forms a clump that evolves as a single *macro-particle* localized in phase space. The untrapped particles are almost uniformly distributed between two oscillating boundaries, and populate the so-called *chaotic sea*.

Furthermore, the macro-particle rotates around a well defined center and this peculiar dynamics is shown to be responsible for the macroscopic oscillations observed for the intensity [2, 3]. It can be therefore hypothesized that a significant reduction in the intensity fluctuations can be gained by implementing a dedicated control strategy, aimed at destroying the macro-particle in space. As a side remark, note that the size of the macro-particle is directly related to the bunching parameter, a quantity of paramount importance in FEL context[3].

For example, a static electric field [4] can be used to increase the average wave power. While the chaotic particles are simply accelerated by the external field, the trapped ones transmit the extra energy to the radiation field, thus being responsible for the amplification of the latter. Furthermore, the experiment by Dimonte and Malmberg [5] suggests that a strategy based on the destruction of the macro-particle may reduce the oscillations of the intensity of the wave.

The dynamics can be investigated from a topological point of view, by looking at the phase space structures. In the framework of a simplified mean field description, i.e. the so-called *test-particle* picture where the particles passively interact with a given electromagnetic wave,

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the trajectories of trapped particles correspond to invariant tori, whereas unbounded particles evolve in a chaotic region of phase-space. Thus, the macro-particle corresponds to a dense set of invariant tori. Our strategy is to modify the macro-particle dynamics by restoring or destroying invariant tori in selected regions of phase space.

A technique of Hamiltonian control can be used [6, 7] to reconstruct additional invariant tori around the macro-particle, in order to enhance the trapping. A specific perturbation is computed, which guarantees the confinement of trajectories characterized by a specific energy, on invariant tori of the dynamics.

We propose a strategy [8] to stabilize the intensity of the wave, by chaotizing the part of phase-space covered by the macro-particle. An additionnal test-wave is introduced into the system, whose amplitude is used as a control parameter to modify the topology of phase-space. The residue method [9, 11, 10, 12] is implemented to identify the important local bifurcations happening in the system when the parameter is varied, by an analysis of linear stability of specific periodic orbits : then the system is driven through the relevant bifurcations, until the prescribed behaviour (optimal chaoticity in the present) is obtained. Although first developed in a mean-field approach, our strategy proves to be robust in the self-consistent framework.

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