IDENTIFICATION OF FREQUENCY OF BIASED HARMONIC SIGNAL

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Abstract: The paper is dedicated to problem of identification of unknown frequency of a biased sinusoidal signal $y(t) = \sigma_0 + \sigma \sin(\omega t + \phi)$. A new proposed approach to estimation of frequency of biased sinusoidal signal is robust with regard to unaccounted disturbances, presenting in measurement of effective signal. Unlike known analogs, this approach allows to regulate time of estimation of unknown frequency ω. The proposed approach also allows online amplitude and bias estimation. Dimension of the proposed identification algorithm is less than known analogs have.

Keywords: Identification, Harmonic signal, Robustness, Estimation, Disturbance

1. INTRODUCTION

This paper deals with problem of frequency identification of a sinusoidal signal $y(t) = \sigma_0 + \sigma \sin(\omega t + \phi)$ for any unknown constant values σ_0, σ, ϕ. Problem of frequency identification of a sinusoidal signal is a very important basic problem, which has different applications in theoretical and engineering disciplines, see (Clarke, 2001). Today we can mark out many different approaches dedicated to identification of unknown frequency of a sinusoidal function, see (Bodson and Douglas, 1997; Hsu, *et al.*, 1999; Mojiri and Bakhshai, 2004; Marino and Tomei, 2002; Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002; Hou, 2005). Let us note that today approaches to identification of parameter $\omega > 0$ are not limited with studying the case of a single sinusoid, see (Bodson and Douglas, 1997; Hsu, *et al.*, 1999; Mojiri and Bakhshai, 2004). In particular, paper (Hou M, (2005) considers problem of frequency identification of a biased sinusoidal signal, and papers (Marino and Tomei, 2002; Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002) present common case of a harmonic signal, which is a sum of *n* sinusoidal functions with different frequencies.

Algorithm proposed in this paper has dynamic order equal to three, and in its turn, that is better than the most known results, published in (Marino and Tomei, 2002; Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002; Hou, 2005). In (Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002; Hou, 2005) minimal dimension of dynamic order of the algorithm is four, and in (Marino and Tomei, 2002) dimension of the algorithm amounts to nine. Besides, algorithm of identification, proposed in the given paper allows to regulate rate of convergence of tuned parameter (estimation of frequency of signal $y(t) = \sigma_0 + \sigma \sin(\omega t + \phi)$) and has robust properties with regard to unaccounted disturbances.

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2. PROBLEM STATEMENT

Consider measured signal of the following view

$$y(t) = \sigma_0 + \sigma \sin(\omega t + \phi), \qquad (1)$$

which is a biased sinusoid with unknown bias σ_0 and amplitude σ, unknown frequency ω and unknown phase ϕ.

Let us formulate purpose of control as design of identification algorithm, which would ensure realization of condition

$$\lim_{t\to\infty} |\omega - \hat{\omega}(t)| = 0, \qquad (2)$$

where $\hat{\omega}(t)$ is a current estimation of parameter ω for any σ_0, σ, ϕ and $\omega > 0$.

3. MAIN RESULT

It is known that for generating of signal (1) it is possible to use differential equation of the view (3)

$$\dddot{y}(t) = -\omega^2 \dot{y}(t) = \theta\, \dot{y}(t), \qquad (3)$$

where $\theta = -\omega^2$ is a constant parameter.

Lemma. Consider an auxiliary second-order filter

$$\begin{cases} \dot{\varsigma}_1(t) = \varsigma_2(t), \\ \dot{\varsigma}_2(t) = -2\alpha\varsigma_2(t) - \alpha^2\varsigma_1(t) + y(t), \\ \varsigma(t) = \varsigma_1(t) \end{cases} \qquad (4)$$

or

$$\varsigma(t) = \frac{1}{(p+\alpha)^2} y(t), \qquad (5)$$

where p is differentiation operator and number $\alpha > 0$.
Then differential equation (3) can be rewritten in the form

$$\dot{y}(t) = 2\alpha\dddot{\varsigma}(t) + \alpha^2\ddot{\varsigma}(t) + \theta\dot{\varsigma}(t) + \varepsilon_y(t), \qquad (6)$$

where $\varepsilon_y(t)$ is exponentially decaying function of time caused by nonzero initial conditions.
Proof. After Laplace transform of equation (3) we obtain

$$sY(s) = \frac{s}{(s+\alpha)^2}\theta Y(s) + \frac{2\alpha s^2 + \alpha^2 s}{(s+\alpha)^2} Y(s) + \\ + \frac{D(s)}{(s+\alpha)^2}, \qquad (7)$$

where s is complex variable, $Y(s) = L\{y(t)\}$ is Laplace image of signal $y(t)$, and polynomial $D(s)$ denotes sum of all terms, containing nonzero initial conditions.
From equation (7) we find

$$\dot{y}(t) = \frac{p}{(p+\alpha)^2}\theta\, y(t) + \frac{2\alpha p^2 + \alpha^2 p}{(p+\alpha)^2} y(t) + \\ + \varepsilon_y(t), \qquad (8)$$

where exponentially decaying function of time $\varepsilon_y(t) = L^{-1}\{D(s)/(s+\alpha)^2\}$ is determined by nonzero initial conditions.
Substituting (5) into equation (8) we obtain

$$\dot{y}(t) = 2\alpha\dddot{\varsigma}(t) + \alpha^2\ddot{\varsigma}(t) + \theta\dot{\varsigma}(t) + \varepsilon_y(t),$$

which was to be proved.

Remark 1. As exponentially decaying function $\varepsilon_y(t) = L^{-1}\{D(s)/(s+\alpha)^2\}$ depends on parameter α, it is possible to accelerate convergence of $\varepsilon_y(t)$ to zero by increasing α.

Now, on base of lemma results one can formulate scheme of unknown parameter θ identification. First let us suppose that function $\dot{y}(t)$ is measured. Then, neglecting exponentially decaying item $\varepsilon_y(t)$, ideal identification algorithm can be written the following way

$$\dot{\hat{\theta}}(t) = k\dot{\varsigma}^2(t)(\theta - \hat{\theta}(t)) = k\dot{\varsigma}(t)\, z(t) - k\dot{\varsigma}^2(t)\hat{\theta}(t), \\ \hat{\omega}(t) = \sqrt{|\hat{\theta}(t)|}, \qquad (9)$$

where function $z(t) = \dot{y}(t) - 2\alpha\dddot{\varsigma}(t) - \alpha^2\ddot{\varsigma}(t)$ and number $k > 0$.
The following statement proves efficiency of ideal identification algorithm for achieving purpose (2).

Proposition. Let algorithm of identification of unknown parameter θ have the view

$$\dot{\hat{\theta}}(t) = k\dot{\varsigma}^2(t)(\theta - \hat{\theta}(t)),$$

where number $k > 0$, and function $\varsigma(t)$ is solution of differential equation (4).
Then purpose of the view (2) is achieved.

Proof of the proposition. Consider estimation error of parameter θ of the following form

$$\tilde{\theta}(t) = \theta - \hat{\theta}(t). \qquad (10)$$

After differentiation of equation (10) we have

$$\dot{\tilde{\theta}} = \dot{\theta} - \dot{\hat{\theta}}(t) = 0 - k\dot{\varsigma}^2(t)\tilde{\theta}(t) = -k\dot{\varsigma}^2(t)\tilde{\theta}(t)\,. \quad (11)$$

Solving differential equation (11) we obtain

$$\tilde{\theta}(t) = \tilde{\theta}(t_0)e^{-k\gamma(t,t_0)}\,, \quad (12)$$

where function

$$\gamma(t,t_0) = \int_{t_0}^{t} \dot{\varsigma}^2(\tau)d\tau\,. \quad (13)$$

It is obvious that as polynomial $(p+\alpha)^2$ is Hurwitz, function $\varsigma(t)$ takes the view

$$\varsigma(t) = \bar{\sigma}_0 + \bar{\sigma}\sin(\omega t + \bar{\phi}) + \Delta\,, \quad (14)$$

where $\bar{\sigma}_0$, $\bar{\sigma}$ and $\bar{\phi}$ are constant coefficients depending on parameters of signal $y(t) = \sigma_0 + \sigma\sin(\omega t + \phi)$ and number α, and Δ is an exponentially decaying item, caused by transients. Neglecting Δ and differentiating (14) we obtain

$$\dot{\varsigma}(t) = \bar{\sigma}\omega\cos(\omega t + \bar{\phi})\,.$$

Substituting $\dot{\varsigma}(t) = \bar{\sigma}\omega\cos(\omega t + \bar{\phi})$ into (13) we have

$$\gamma(t,t_0) = \int_{t_0}^{t} \dot{\varsigma}^2(\tau)d\tau = \bar{\sigma}^2\omega^2\int_{t_0}^{t}(\cos(\omega\tau + \bar{\phi}))^2 d\tau =$$
$$= \frac{\bar{\sigma}^2\omega^2 t}{2} - \frac{\bar{\sigma}^2\omega^2 t_0}{2} + \frac{\bar{\sigma}^2\omega^2\sin(2\omega t + 2\bar{\phi})}{4\omega} -$$
$$- \frac{\bar{\sigma}^2\omega^2\sin(2\omega t_0 + 2\bar{\phi})}{4\omega} = \gamma_0 t + \gamma_1(t,t_0)\,, \quad (15)$$

where function

$$\gamma_1(t,t_0) = -\frac{\bar{\sigma}^2\omega^2 t_0}{2} + \frac{\bar{\sigma}^2\omega^2\sin(2\omega t + 2\bar{\phi})}{4\omega} - \frac{\bar{\sigma}^2\omega^2\sin(2\omega t_0 + 2\bar{\phi})}{4\omega}$$

is bounded for any *t*, and number $\gamma_0 = \dfrac{\bar{\sigma}^2\omega^2}{2}$.

Let us substitute (15) into (12)

$$\tilde{\theta}(t) = \tilde{\theta}(t_0)e^{-k\gamma_0 t}e^{-k\gamma_1(t,t_0)}\,. \quad (16)$$

It follows from equation (16) that $\lim\limits_{t\to\infty}\tilde{\theta} = 0$, and hence $\hat{\omega}(t) = \sqrt{\left|\hat{\theta}(t)\right|} \to \omega(t)$ for $t \to \infty$. Proposition is proven.

Remark 2. It follows from equation (16) that function $\hat{\theta}(t)$ converges faster to parameter θ by increasing coefficient *k*. It means that it is possible to reduce or increase rate of convergence of the tuned parameter to its real value in identification algorithm (9) changing coefficient *k*.

Remark 3. It follows from equation (16) that system (11) is exponentially stable. In its turn it ensures robustness of identification algorithm with respect to external disturbances.

However, in our case signal *y*(*t*) is only measured but not its derivatives. To derive realizable scheme of identification algorithm let us consider the following variable

$$\chi(t) = \hat{\theta}(t) - k\dot{\varsigma}(t)y(t)\,. \quad (17)$$

Differentiating equation (17) we obtain

$$\dot{\chi}(t) = \dot{\hat{\theta}}(t) - k\ddot{\varsigma}(t)y(t) - k\dot{\varsigma}(t)\dot{y}(t) =$$
$$= k\dot{\varsigma}(t)(\dot{y}(t) - 2\alpha\ddot{\varsigma}(t) - \alpha^2\dot{\varsigma}(t)) - k\dot{\varsigma}^2(t)\hat{\theta}(t) -$$
$$-k\ddot{\varsigma}(t)y(t) - k\dot{\varsigma}(t)\dot{y}(t) =$$
$$= k\dot{\varsigma}(t)(-2\alpha\ddot{\varsigma}(t) - \alpha^2\dot{\varsigma}(t)) - k\dot{\varsigma}^2(t)\hat{\theta}(t) -$$
$$-k\ddot{\varsigma}(t)y(t)\,. \quad (18)$$

From equations (17), (18) we receive realizable identification algorithm of the following view

$$\dot{\chi}(t) = k\dot{\varsigma}(t)(-2\alpha\ddot{\varsigma}(t) - \alpha^2\dot{\varsigma}(t)) - k\dot{\varsigma}^2(t)\hat{\theta}(t) -$$
$$-k\ddot{\varsigma}(t)y(t)\,, \quad (19)$$

$$\hat{\theta}(t) = \chi(t) + k\dot{\varsigma}(t)y(t)\,, \quad (20)$$

$$\begin{cases} \dot{\varsigma}_1(t) = \varsigma_2(t), \\ \dot{\varsigma}_2(t) = -2\alpha\varsigma_2(t) - \alpha^2\varsigma_1(t) + y(t), \\ \varsigma(t) = \varsigma_1(t). \end{cases} \quad (21)$$

Remark 4. Let us notice that proposed approach also allows online amplitude and bias estimation. From equation (6) we obtain

$$\hat{\dot{y}}(t) = \hat{\theta}(t)\varsigma_1(t) + 2\alpha\dot{\varsigma}_2(t) + \alpha^2\dot{\varsigma}_1(t)\,, \quad (22)$$

$$\hat{\ddot{y}}(t) = \hat{\theta}(t)\dot{\varsigma}_2(t) + 2\alpha\hat{\theta}(t)\dot{\varsigma}_1(t) + \alpha^2\dot{\varsigma}_2(t)\,, \quad (23)$$

where $\hat{\dot{y}}(t)$ and $\hat{\ddot{y}}(t)$ are current estimations of $\dot{y}(t)$ and $\ddot{y}(t)$ correspondingly.

Let us consider the following variables

$$\beta(t) = \frac{\ddot{y}(t)}{\theta} = \sigma \sin(\omega t + \varphi),$$

$$\hat{\beta}(t) = \frac{\hat{\ddot{y}}(t)}{\hat{\theta}(t)}. \tag{24}$$

From equations (22), (24) we obtain realizable online amplitude and bias estimator

$$\hat{\sigma}(t) = \sqrt{\frac{-\hat{\dot{y}}^2(t)}{\hat{\theta}(t)} + \hat{\beta}^2(t)}, \tag{25}$$

$$\hat{\sigma}_0(t) = y(t) - \hat{\beta}(t). \tag{26}$$

4. EXAMPLE

Let us consider problem of frequency identification and bias and amplitude estimation of a biased sinusoidal signal without disturbances and in presence of disturbances to illustrate efficiency of identification algorithm (19)-(21). Figures 1-6 show graphs of tuning parameter $\hat{\theta}(t)$ and estimation of parameters $\hat{\sigma}(t)$ and $\hat{\sigma}_0(t)$ (see remark 4) for the biased sinusoidal signal $y(t) = 2 + \sin 2t$. Results of computer simulation show that tuned parameter $\hat{\theta}(t)$ converges faster to real value θ because of increasing coefficient *k* (see remark 2).

Figures 7-12 show graphs of tuning parameter $\hat{\theta}(t)$ and estimation of parameters $\hat{\sigma}(t)$ и $\hat{\sigma}_0(t)$ for the biased sinusoidal signal $y(t) = -3 + 2\sin 4t$. Like in previous example rate of convergence of estimate $\hat{\theta}(t)$ grows by increasing parameter *k*.

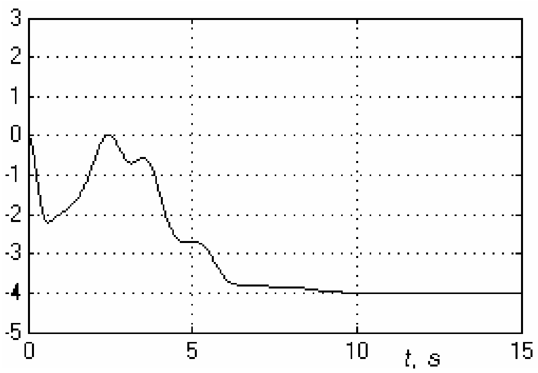

Fig. 1. Function $\hat{\theta}(t)$ for $\alpha = 1$ and $k = 10$

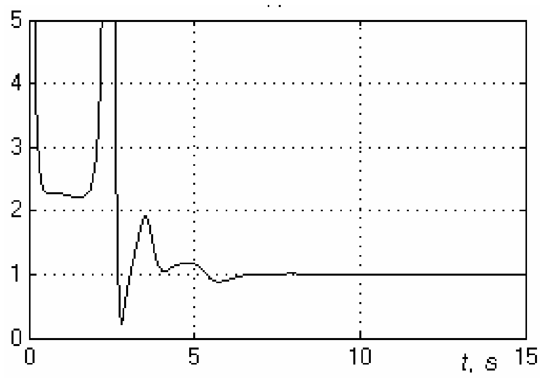

Fig. 2. Function $\hat{\sigma}(t)$ for $\alpha = 1$ and $k = 10$

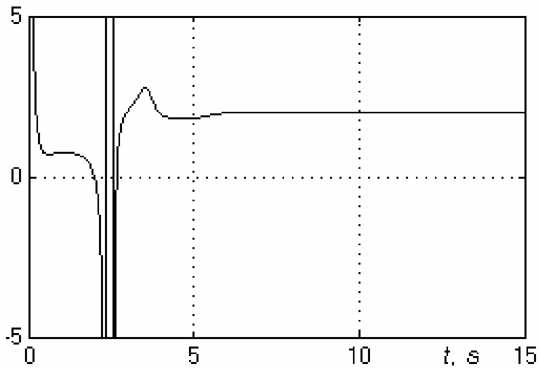

Fig. 3. Function $\hat{\sigma}_0(t)$ for $\alpha = 1$ and $k = 10$

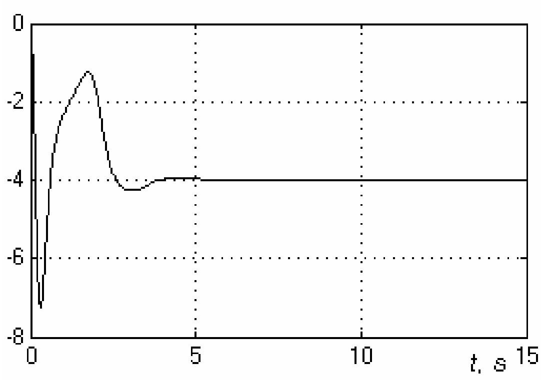

Fig. 4. Function $\hat{\theta}(t)$ for $\alpha = 2$ and $k = 50$

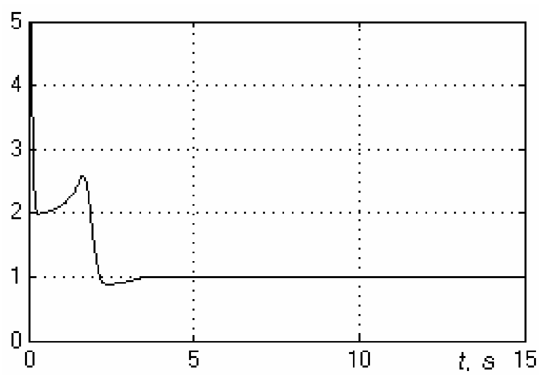

Fig. 5. Function $\hat{\sigma}(t)$ for $\alpha = 2$ and $k = 50$

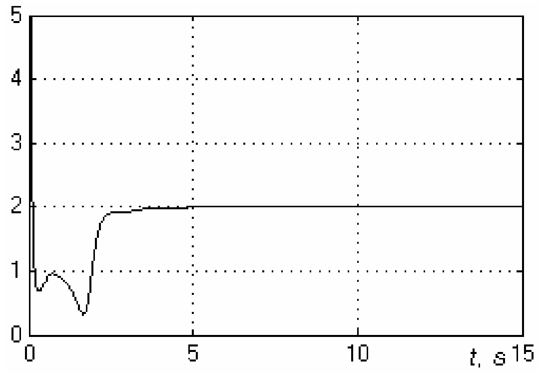

Fig. 6. Function $\hat{\sigma}_0(t)$ for $\alpha = 2$ and $k = 50$

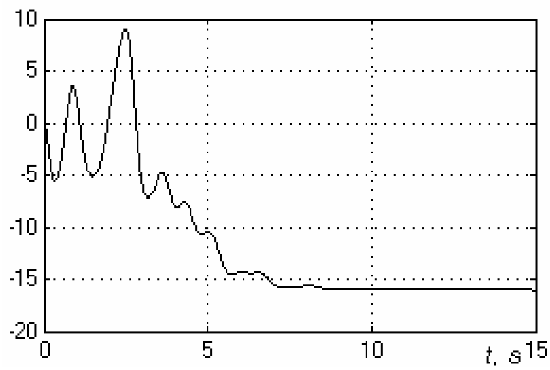

Fig. 7. Function $\hat{\theta}(t)$ for $\alpha = 1$ and $k = 10$

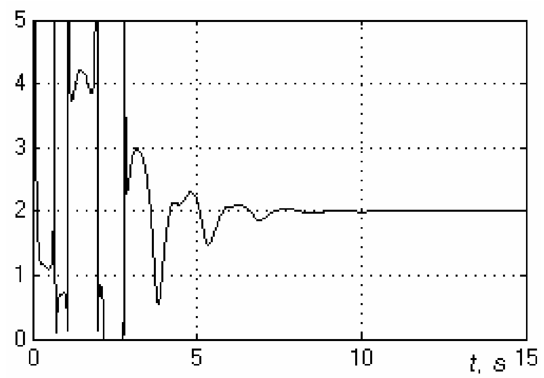

Fig. 8. Function $\hat{\sigma}(t)$ for $\alpha = 1$ and $k = 10$

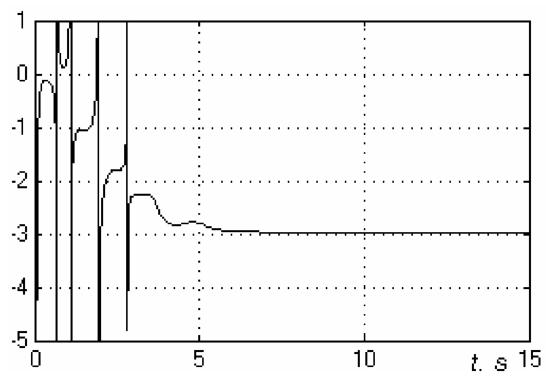

Fig. 9. Function $\hat{\sigma}_0(t)$ for $\alpha = 1$ and $k = 10$

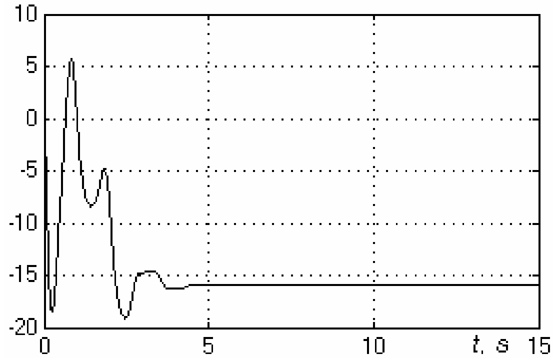

Fig. 10. Function $\hat{\theta}(t)$ for $\alpha = 2$ and $k = 50$

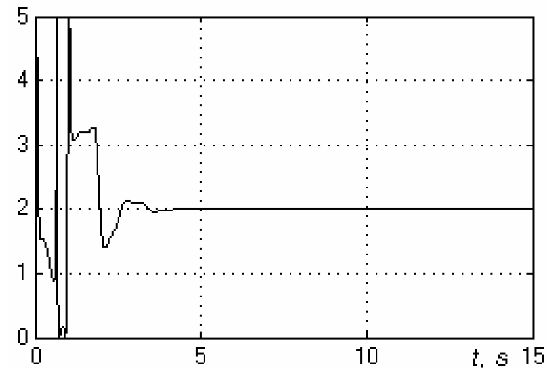

Fig. 11. Function $\hat{\sigma}(t)$ for $\alpha = 2$ and $k = 50$

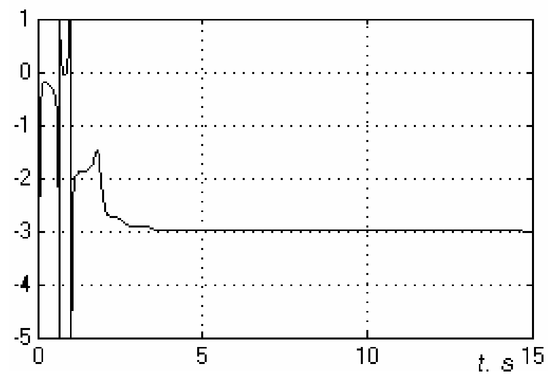

Fig. 12. Function $\hat{\sigma}_0(t)$ for $\alpha = 2$ and $k = 50$

Figures 13-15 show graphs of parameter $\hat{\theta}(t)$ tuning and estimation of parameters $\hat{\sigma}(t)$ и $\hat{\sigma}_0(t)$ for the biased sinusoidal signal $y(t) = -3 + 2\sin 4t$ disturbed by Gaussian noise of zero mean and variance 0.1^2. Computer simulation illustrates that robustness properties keep safe with respect to unaccounted disturbances (see remark 3).

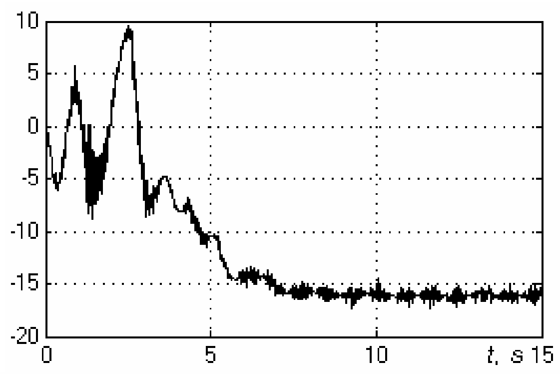

Fig. 13. Function $\hat{\theta}(t)$ for $\alpha = 1$ and $k = 10$

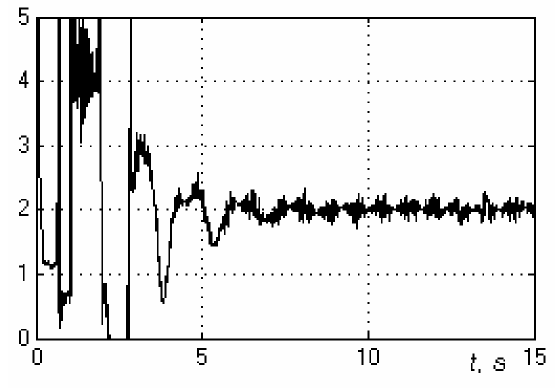

Fig. 14. Function $\hat{\sigma}(t)$ for $\alpha = 1$ and $k = 10$

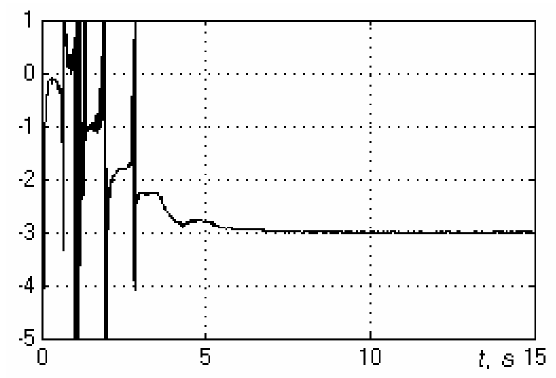

Fig. 15. Function $\hat{\sigma}_0(t)$ for $\alpha = 1$ and $k = 10$

5. CONCLUSION

Problem of identification of frequency of a sinusoidal signal $y(t) = \sigma_0 + \sigma \sin(\omega t + \phi)$ has been considered for any unknown constant values σ_0, σ, ϕ, $\omega > 0$. Designed algorithm of identification (19)-(21):

– is stable with regard to unaccounted disturbances presenting in measurement of effective biased sinusoidal signal;
– has been shown to allow accelerating rate of convergence of estimate $\hat{\theta}(t)$ to θ because of increasing coefficient k (see remarks 1 and 2 and example);
– has been shown to have ability of extension (22)-(26) for online estimation of amplitude and bias with no increase of dynamic order (see remark 4 and example);
– has also been shown to have the least dynamic order in comparison with works (Marino and Tomei, 2002; Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002; Hou, 2005).

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