OUTPUT ADAPTIVE CONTROL FOR PLANTS USING TIME DELAY IN OUTPUT SIGNAL BASED ON THE MODIFIED ALGORITHM OF ADAPTATION OF THE HIGH ORDER

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In the paper the problem of adaptive control for plants with time delay using in output signal in conditions of prior uncertainty was solved. In such conditions, derivatives of input and output signals are not measured. Design of the closed-loop system of control is based on new modified high order adaptation algorithm and usage of the state filters. Filters permit to compensate the influence of a time delay to stability without usage of predicators.

Keywords: adaptive control, plant with time delay using in output signal, controlled filters, observer, modified algorithm of adaptation of the high order, Lyapunov-Krasovskiy functional.

1. INTRODUCTION

The synthesis of systems of control for plants using time delay requires taking into account the influence of delay on stability and quality of transients in closed-loop system (Huretskiy 1973; Kolmanovskiy and Nosov, 1981). In this connection, there is a necessity of delay account for fulfillment of target conditions. In (Smith, 1959) design of a predicator is considered on the base of supposition about complete and exact possession of the information about the plant. For control the plant in conditions of prior uncertainty adaptive predictors were designed, both at measurement of a state vector of the plant (Tsykunov, 2000), and in the case of measurement the scalar input and output of the plant (Furtat and Tsykunov, 2005). The special interest is paper (Niculescu and Annaswamy, 2003), where for design of closed-loop system predicators was not used, however, relative degree of the plant didn't exceed two. It considerably narrows down the class of investigated systems.

At present moment in a class of problems of adaptive systems were received many approaches and methods (for example Miroshnik et. al., 1999) for control of linear plant on an output in the conditions of priory uncertainty. These methods conditionally can be divided into two classes: adaptive control with the extended error (Nikiforov and Fradkov, 1994) and adaptive control with algorithm of adaptation of the high order (Feuer and Morse, 2000; Miroshnik et. al., 1999; Morse, 1992; Nikiforov, 1999). The concept of the extended error consists of reception of simple scheme of the extending signal, what it is the sum of the tracking error and the generator of extension.

Another approach is the method of algorithm adaptation of the high order. These algorithms also can be divided into two classes by the principle of realization: algorithms with an estimation of derivatives from a tracking error, for what different observers are used. The order of the closed-loop system in the second case is less than in the first case. In this paper it is proposed to synthesize closed-loop system for single input single output linear plant with the help of new modified algorithm of adaptation of the high order (Furtat and Tsykunov, 2006) with the purpose of exception of influencing of delay and reaching of a target conditions without a prognosis of values of a controlled variable in conditions of prior uncertainty.

2. PROBLEM STATEMENT

Consider single-input, single-output linear system which dynamics are described by the following differential equation

$$Q(p)y(t) = kR(p)u(t-h), \tag{1}$$

where $u(t)$, $y(t)$ are scalar input and output; $h > 0$ is the known constant delay; $Q(\lambda)$, $R(\lambda)$ are monic normalized Hurwitz polynomials with unknown constant coefficients dependent on some vector of unknown parameters $\Theta \in \Xi$, Ξ is known set of possible meanings of the vector Θ and λ is complex variable in Laplace transformation; $k > 0$; $\deg Q(P) = n$; $\deg R(P) = m$; systems (1) has relative degree $\gamma = n - m > 1$; $p = d/dt$ denotes differential operator.

And consider a reference model

$$Q_m(p)y_m(t) = k_m R_m(p)r(t), \tag{2}$$

where $r(t)$ is uniformly bounded reference input; $Q_m(p)$, $R_m(p)$ are monic normalized Hurwitz polynomials with unknown constant coefficients; $\deg Q_m(p) = n$, $\deg R_m(p) = m$. The target condition is formed of the following inequality

$$\overline{\lim_{t\to\infty}}|e(t)| = \overline{\lim_{t\to\infty}}|y(t) - y_m(t-h)| < \delta, \tag{3}$$

where δ is enough small number, which can be decreased by control law selection .

3. PARAMETERIZATION OF THE PLANT EQUATION

Decompose operators $Q(p)$ and $R(p)$ on addendums $Q(p) = Q_m(p) + \Delta Q(p)$ and $R(p) = R_m(p) + \Delta R(p)$, where $\Delta Q(p)$, $\Delta R(p)$ are operators with unknown coefficients of orders $n-1$ and $m-1$ respectively. The tracking error $e(t) = y(t) - y_m(t-h)$, using procedure of "operator's division", transforms to the form

$$e(t) = \frac{kR_m(p)}{Q_m(p)}\left[u(t-h) - \frac{\Delta Q(p)}{Q(p)}u(t-h) + \right.$$
$$\left. + \frac{\Delta R(p)Q_m(p)}{R_m(p)Q(p)}u(t-h) - \frac{k_m}{k}r(t-h)\right]. \quad (4)$$

It is necessary to form the vector

$$\theta_1(t) = \frac{1}{Q(p)}\left[u(t-h), pu(t-h), ..., p^{n-1}u(t-h)\right], \quad (5)$$

for the component $\frac{\Delta Q(p)}{Q(p)}u(t-h)$ in (4). However, $Q(\lambda)$ is unknown polynomial, therefore let's take the tuned filter

$$\dot\theta_1(t) = F_m\theta_1(t) + b_{01}\tau^T(t)\theta_1 + b_{01}u(t),$$
$$\theta_1(0) = 0, \quad (6)$$

where $\theta_1(t) \in R^{n\times 1}$ is the state vector; $F_m \in R^{n\times n}$ is matrix in Frobenius form with characteristic polynomial $Q_m(\lambda)$; $b_{01}^T = [0,...,0,1]$ is vector of corresponding dimension; $\tau(t) \in R^{n\times 1}$ is the vector of adjustable parameters. Transform the equation (6) to the form

$$\dot\theta_1(t) = (F_m + b_{01}\tau_0^T(t))\theta_1(t) +$$
$$+ b_{01}(\tau(t) - \tau_0)^T\theta_1(t) + b_{01}u(t), \quad (7)$$

where τ_0 is the constant vector of unknown parameters, such that the matrix $F_m + b_{01}\tau_0^T$ has the characteristic polynomial $Q(\lambda)$. Therefore, if the target condition $(\tau(t) - \tau_0)^T\theta_1(t) \xrightarrow[t\to\infty]{} 0$ is true, than, using filter (7) is possible to receive the demanded vector $\theta_1(t)$. If this target condition is true, let's take a controlling input by the form $u(t) = -c_1{}^T(t)\theta_1(t) + u_1(t)$, then the first equation of the filter (7) will be conversed to the form

$$\dot\theta_1(t) = F_m\theta_1(t) + b_{01}(\tau(t) - \tau_0)^T(t)\theta_1(t) +$$
$$+ b_{01}u_1(t) - b_{01}(c_1(t) - \tau_0)^T\theta_1. \quad (8)$$

So, the vector τ_0 depends on coefficients of a polynomial $Q(\lambda)$. Let some vector c_{01}. It depends on coefficients of the polynomial $\Delta Q(\lambda)$. Then from operator's division of the polynomial $Q(\lambda)$ follows that the vectors of unknowns parameters τ_0 и c_{01} are equal on modulo and are opposite on the value, that is $\tau_0 = -c_{01}$. Thus, the equation of a filter (8) can be rewritten as

$$\dot\theta_1(t) = F_m\theta_1(t) + b_{01}u(t). \quad (9)$$

Now, for the component $\frac{\Delta R(p)Q_m(p)}{R_m(p)Q(p)}u(t-h)$ in the equation (4) consider a shaping filter. Bring in the variable $\zeta(t) = \frac{Q_m(p)}{Q(p)}u(t-h)$ which will be formed as $q_m^T\theta_1(t)$, where q_m is the vector which dependents on coefficients of polynomial $Q_m(\lambda)$. Then, from the filter (8) we will receive second vector

$$\theta_2(t) = \frac{1}{R_m(p)}\left[\zeta(t), p\zeta(t), ..., p^{m-1}\zeta(t)\right]. \quad (10)$$

Therefore, the vector of regression can be formed in the form $w_1(t) = [\theta_1(t), \theta_2(t), r(t)]^T$. On the basis of vectors (5), (10) and vector of regression the equation (4) will be rewritten to

$$e(t) = \frac{kR_m(p)}{Q_m(p)}\left[u(t-h) - c_0^T w_1(t-h)\right]. \quad (11)$$

Here $c_0 \in R^{n\times 1}$ is a constant vector of unknown parameters depending on coefficients of polynomials $\Delta R(\lambda)$ and $\Delta Q(\lambda)$.

4. THE MODIFIED ALGORITHM OF ADAPTATION OF THE HIGH ORDER

Due to absence of information about derivatives of an input and output of control plant (1) and reference model, let's form the law of control as

$$u(t) = T(p)\bar v(t), \; v(t) = c^T(t)w(t), \quad (12)$$

where $v(t)$ is a new control influence; $c(t) \in R^{(2n+m+1)\times 1}$ is the vector of adjustable parameters; $\bar v(t)$ is the estimation of function $v(t)$; $T(\lambda)$ is any Hurwitz polynomial of $\gamma - 1$ degree.

For estimating a variable v and it's derivatives let's use any observers (Khalil, 1996; Mahmoud and Khalil, 1997; Morse, 1992; Nikiforov, 1999; Slotine et al., 1987;). Let's take an algorithm proposed in (Khalil, 1996; Mahmoud and Khalil, 1997)

$$\dot\xi(t) = G_0\xi(t) + D_0(\bar v(t) - v(t)), \; \bar v(t) = C\xi(t), \quad (13)$$

where $\xi \in R^{\gamma-1}$; $D_0 = [-\frac{d_1}{\mu}, -\frac{d_2}{\mu^2}, ..., -\frac{d_{\gamma-1}}{\mu^{\gamma-1}}]^T$, and $d_1, ..., d_{\gamma-1}$ are chosen from Hurwitz matrix $G = G_0 - DC$, where $G_0 = \begin{bmatrix} 0 & I_{\gamma-2} \\ 0 & 0 \end{bmatrix}$, $I_{\gamma-2}$ is identity; $D=[d_1,d_2,...,d_{\gamma-1}]$; $C=[1,0,...,0]$; μ is enough small value.

In this case in (12) variables from the observer (13) will be used. It means that in conditions of not measurability of derivatives of functions: $y(t)$, $u(t)$ and $r(t)$, the control law (12) is technically realizable as it has measurable and known functions.

Transform the tracking error (11) with using (12) and (13)

$$e(t)=kH(p)\Big[(c(t-h)-c_0)^T w(t-h)+ \\ +\bar{v}(t-h)-v(t-h)\Big], \quad (14)$$

where $H(p)=R_m(p)T(p)Q_m^{-1}(p)$, $w(t)=w_1(t)/T(p)$ is the new vector of regression. Let's take into consideration the vector of deviations $\bar{\eta}=\Gamma^{-1}(\xi-\theta)$, $\Gamma=diag\{\mu^{\gamma-2},\mu^{\gamma-1},...,\mu,1\}$, $\theta=[v,\dot{v},...,v^{\gamma-1}]$.

The equation of deviation, using the equation (13) and vector of deviation, will has the form $\varepsilon_1(t)=\bar{v}(t)-v(t)=\mu^{\gamma-2}C\bar{\eta}(t)$. The dynamic of the vector of deviation is described:

$$\dot{\bar{\eta}}(t)=\mu^{-1}G\bar{\eta}(t)+C^T v^{(\gamma)}(t).$$

Converse the equation of deviation and vector of deviation to the equivalent equation concerning the output $\varepsilon_1(t)$

$$\dot{\eta}(t)=\mu^{-1}G\eta(t)+b_{01}\dot{v}(t),\ \varepsilon_1(t)=\mu^{\gamma-2}C\eta(t), \quad (15)$$

Rewrite the equation (14), taking into consideration (15)

$$e(t)=kH(p)\Big[(c(t-h)-c_0)^T w(t-h)+ \\ +\mu^{\gamma-2}C\eta(t-h)\Big]. \quad (16)$$

Introduce the extended vector $X(t)=[x^T(t),w^T(t)]^T$, where $x(t)$ is the state vector of system (1). Compose the extended plant as

$$\dot{X}(t)=AX(t)+b\Big[(c(t-h)-c_0)^T w(t-h)+ \\ +\mu^{\gamma-2}h\eta(t-h)\Big],\ y(t)=LX(t). \quad (17)$$

Where $L(\lambda I-A)^{-1}b=R_m(\lambda)/Q_m(\lambda)$; matrixes (A,b,L) are realizations of the system (17) and have corresponding dimensions.

For extended plant (17) let's take extended reference model with a state vector X_m, such that the dynamic of the extended tracking error $\varepsilon(t)=X(t)-X_m(t)$ in the force (15) will be set by the equation

$$\dot{\varepsilon}(t)=A\varepsilon(t)+b\Big[(c(t-h)-c_0)^T w(t-h) \\ +\mu^{\gamma-2}C\eta(t-h)\Big],\ e(t)=L\varepsilon(t). \quad (18)$$

Since vectors $\theta_1(t)$ and $\theta_2(t)$ are subcomponents of a vector of regression $w(t)$, which one in turn depend on control law $u(t)$, then following estimates are:

$$|u(t)|\le k_1\|X(t)\|,\ k_1>0,\ \|w(t)\|\le\|X(t)\|. \quad (19)$$

Theorem. Let polynomial $T(\lambda)$ is Hurwitz of $\gamma-1$ degree. Then there is a number μ_0 and algorithm of adaptation

$$\dot{c}(t)=-\rho e(t)w(t-h), \quad (20)$$

and when $\mu\le\mu_0$ and $\rho>0$, the control systems (5), (10), (12), (13), (20) are dissipative and the target condition (3) will be satisfied if the system starts in some set Ω_0.

Proof of the theorem. Let's rewrite equations (15) (18) of the form

$$\dot{\varepsilon}(t)=A\varepsilon(t)+b[(c(t-h)-c_0)^T w(t-h)+ \\ +\mu_2{}^{\gamma-2}C\eta(t-h)],\ e(t)=L\varepsilon(t), \quad (21) \\ \mu_1\dot{\eta}(t)=\mu_2 C^T\dot{v}(t)+G\eta(t),$$

where $\mu_1=\mu_2=\mu$. Let's use a lemma (Brusin, 1995; Fradkov, 1990).

Lemma (Brusin, 1995; Fradkov, 1990)**.** If the any system is described by the equation $\dot{x}=f(x,\mu_1,\mu_2)$, $x\in R^{s_1}$, $\mu=\mathrm{col}(\mu_1,\mu_2)\in R^{s_2}$, where $f(x,\mu_1,\mu_2)$ is uniformly bounded continuous function on x. If $\mu_2=0$ this function has the bounded closed set of dissipative $\Omega_1=\{x\,|\,P(x)\le C_1\}$, where $P(x)$ is undermined piece-smooth positively determined function in R^{s_1}. If $\mu_2\le\mu_0$, $\mu_0>0$ and for some numbers $C_1>0$, $\bar{\mu}_1>0$ and $\mu_2=0$ the condition

$$\sup_{|\mu_1|\le\bar{\mu}_1}\left\langle\left[\frac{\partial P(x)}{\partial x}\right]^T, f(x,\mu_1,0)\right\rangle\le -C_2, \quad (22)$$

is satisfied for $P(x)=C_1$. Then the initial system will provides the set of dissipative Ω_1.

Following to the lemma, consider the system (26) with $\mu_2=0$, then

$$\dot{\varepsilon}=A\varepsilon+b\big(c(t-h)-c_0\big)^T w(t-h),\ e=L\varepsilon, \\ \mu_1\dot{\eta}=G\eta. \quad (23)$$

Solutions of the third equation of the system (23) are asymptotically stable because the matrix G is Hurwitz. Consider Lyapunov-Krasovskiy functional $V_1=V_1(e,c-c_0,c(t-h)-c_0)$ on the following form

$$V_1=\varepsilon^T(t)Q_1\varepsilon(t)+k\rho^{-1}(c(t)-c_0)^T(c(t)-c_0)+ \\ +\int_{-h}^{0}df\int_{f-h}^{f}\dot{c}^T(z)\dot{c}(z)dz,$$

where matrix Q_1 should satisfy to matrix equations $A^T Q_1+Q_1A=-Q_2$, $Q_1b=L^T$, $Q_1=Q_1^T>0$ and $Q_2=Q_2^T>0$.

Using the Leibniz-Newton formula for (20), we have

$$c(t-h)=c(t)+\rho\int_{t-h}^{t}e(s)w(s-h)ds\,. \qquad (24)$$

Difference functional V_1 in the force equations (21) and (24):

$$\dot V_1 \le -\varepsilon^T(t)(0.5Q_2-\rho^2h\|w(t-h)\|^2)\varepsilon(t)-$$
$$-0.25\varepsilon^T(t)Q_2\varepsilon(t)-\Big(0.25\varepsilon^T(t)Q_2\varepsilon(t)-$$
$$-2\varepsilon^T(t)Q_1b\rho\int_{t-h}^{t}e(s)w(s-h)dsw(t-h)+$$
$$\left.+\rho^2\int_{t-h}^{t}\|e(s)w(s-h)\|^2ds\right)\le$$
$$\le\left(0.5Q_2-\rho^2h\|w(t-h)\|^2\right)\varepsilon(t)-$$
$$-\left(\sqrt{\|Q_2\|}\|\varepsilon\|-2\|Q_1b\|\int_{t-h}^{t}\|e(s)w(s-h)\|ds\right)^2. \qquad (25)$$

For fulfilling a target condition $\lim_{t\to\infty}e(t)=0$ the inequalities $0.5Q_2-\rho^2h\|w(t-h)\|^2>0$ and $\|Q_2\|>4\|Q_1b\|^2$ should be fulfilled. However the inequality (25) has a vector of regression depending on reference influence $r(t)$, which is bounded function, and from vectors $\theta_1(t)$ and $\theta_2(t)$, which depend on law of control $u(t)$. The proof of boundedness of a vector $w(t)$ is based of works (Niculescu, 2001; Niculescu and Annaswamy 2003).

Consider vector $w(t)$ on the interval $[t_0-h,t_0]$, where t_0 is some initial time of counting. Evidently $\sup\limits_{\sigma_1\in[t_0-h,t_0]}\|u(\sigma_1)\|^2\le\gamma_{01}$, where $\gamma_{01}>0$. Then $\sup\limits_{\sigma_1\in[t_0-h,t_0]}\|w(\sigma_1)\|^2\le\gamma_1$, where $\gamma_1>0$. Thus, on the given interval of time there may be such positive real number $\rho=\rho_1$, that the inequality $0.5Q_2-\rho_1^2h\gamma_1>0$ will be satisfied. So $0.5Q_2-\rho_1^2h\|w(l-h)\|^2>0$ will be satisfied for $l\in[t_0-h,t_0]$. Consequently Lyapunov-Krasovskiy functional is non-increasing on the same time-interval. Thus:

$$\lambda_{\min}(Q_2)\|\varepsilon(l)\|^2\le V_{10},\ \text{for } l\in[t_0-h,t_0]\,,$$
$$V_{10}\lambda_{\min}^{-1}(Q_2)+X_{m0}\ge X_0^2\,, \qquad (26)$$

where $V_{10}=V_1(\varepsilon(t_0),c(t_0)-c_0,(dc(t)/dt)_{t=t_0})$, X_{m0} is a bounded quantity, that depending on initial conditions of the plant (1). As inequalities (26) are satisfied, than from inequalities (19) following conditions will also be fulfilled:

$$\|w(l)\|\le X_0 \ \text{for } l\in[t_0-h,t_0]\text{, and}$$
$$0.5Q_2-\rho_1^2hX_0^2>0\,.$$

Therefore, the inequality (25) will be satisfied on the interval $[t_0-h,t_0]$, and then $\dot V_1\le0$, i.e. functional V_1 is non-increasing on the same time-interval.

Now, we shall consider a vector $w(t)$ on the interval $[t_0,t_0+h]$. Obviously, that $\sup\limits_{\sigma_2\in[t_0,t_0+h]}\|u(\sigma_2)\|^2\le\gamma_{02}$, where $\gamma_{02}>0$, then $\sup\limits_{\sigma_2\in[t_0,t_0+h]}\|w(\sigma_2)\|^2\le\gamma_2$, where $\gamma_2>0$.

Thus, on the given interval of time there is such positive real number $\rho=\rho_2$, that the inequality $0.5Q_2-\rho_2^2h\gamma_2>0$ will be satisfied. So $0.5Q_2-\rho_2^2h\|w(l-h)\|>0$ will be satisfied for $l\in[t_0,t_0+h]$, and it follows that the Lyapunov-Krasovskiy functional is non-increasing on the same time-interval. Thus:

$$\lambda_{\min}(Q_2)\|\varepsilon(l)\|^2\le V_{10}\ \ \text{for } l\in[t_0,t_0+h]\,,$$
$$V_{10}\lambda_{\min}^{-1}(Q_2)+X_{m0}\ge X_0^2\,, \qquad (27)$$

Therefore, inequalities (27) had been satisfied, than conditions, which are followed from inequalities (19)

$$\|w(l)\|\le X_0 \ \text{for } l\in[t_0,t_0+h]\text{, and}$$
$$0.5Q_2-\rho_2^2hX_0^2>0\,,$$

will also be satisfied.

Consequently, the inequality (25) will be satisfied on the interval $[t_0,t_0+h]$. Then $\dot V_1\le0$, i.e. functional V_1 is non-increasing on the same time-interval.

Considering, subsequent time-interval for any t_0, it's possible to conclude, that for any $w(t_i)$, where i is chosen time-interval, the condition $\sup\limits_{\sigma_i\in[t_i-h,t_i]}\|u(\sigma_i)\|^2\le\gamma_{0i}$, $\gamma_{0i}>0$ should be satisfied, and consequently $\sup\limits_{\sigma_i\in[t_i-h,t_i]}\|w(\sigma_i)\|^2\le\gamma_i$, $\gamma_i>0$. Then, choosing parameter from the condition $\rho=\min\{\rho_i\}$ for $i=\overline{1,n}$ in the algorithm of adaptation (20) the Lyapunov-Krasovskiy functional is non-increasing for all $t>t_i$. Consequently, the vector of regressive $w(t)$ is bounded, so the inequality (25) is satisfied. The satisfy of inequality (27) provides boundedness of the tracking error $\varepsilon(t)$. Then, according to boundedness $\varepsilon(t)$ and equations (21) the function $(c(t-h)-c_0)^Tw(t-h)$ is bounded too. As a vector $w(t-h)$ is bounded, than from product $(c(t-h)-c_0)^Tw(t-h)$ the function

$c(t-h)-c_0$ is bounded. I.e. vector c_0 is constant therefore vector $c(t)$ is bounded. From the equation (12) and boundness of vectors $w(t)$ and $c(t)$ the boundedness of control law $v(t)$ is followed. It means, that in closed-loop system all signals are bounded and conditions $V_1 \geq 0$, $\dot{V}_1 \leq 0$ are satisfied, then $\lim_{t\to\infty} e(t)=0$ is satisfied too.

As a vector $w(t)$ is bounded and the functional $V_1(t)$ is non-increasing, than it means, that, if the system starts to work from some set of initial values Ω_0, that will be exist of set

$$\Omega = \{e(t), w(t), c(t), \dot{w}(t), \dot{c}(t):$$
$$|w(t)| < k_1, |c(t)| < k_2, |\dot{w}(t)| < k_3, |\dot{c}(t)| < k_4\}$$

with some set of attraction Ω_1, for which the condition $\lim_{t\to\infty} e(t)=0$ is satisfied.

Evidently, the condition of a lemma had been fulfilled. Consequently the system (21) has set of a dissipativity Ω. However, the set of attraction may be another. Therefore consider the functional $P(x)$ and take the Lyapunov-Krasovskiy functional

$$P_1 = \varepsilon^T Q_2 \varepsilon + \frac{k}{\rho}(c-c_0)^T(c-c_0) +$$
$$+\int_{-h}^{0} df \int_{t-h}^{t} \dot{c}^T(z)\dot{c}(z)dz + \theta_1^T R_1 \theta_1 + \theta_2^T R_2 \theta_2 +$$
$$+\theta_2^T R_2 \theta_2 + \eta^T N \eta + \int_{-h}^{0} \eta^T(t+s) N_1 \eta(t+s)ds,$$

where R_1, R_2, N, N_1 are symmetric positive definite matrixes. Choose a number C_1 so, that a bounded closed surface $P(x)=C_1$, where $x_1^T(t)=[\varepsilon, \eta^T, \theta_1, \theta_2]$, is in set Ω on variables $x_1(t)$. As set of attraction Ω_1 lays in open set $V(x)<C_1$ and the system is dissipative, than variables $x(t)$ will tend to the set of attraction Ω_1. Consequently there is a number C_2, for which the (22) is satisfied. Only variables $\eta(t)$ and their speed of convergence to zero depend on choice μ_1. Thus, according to a lemma, there is a number $\mu_0 > 0$. If $\mu < \mu_0$, than the set of dissipativity of systems (5), (10), (12), (13) and (20) there is a set Ω.

Let in (21) $\mu_1 = \mu_2 = \mu_0$. Let's consider, that the moution of the system starts in initial set of initial conditions Ω_0. Consequently all trajectories of the system will be in the field of dissipativity Ω.

Consider the Lyapunov-Krasovskiy functional $V = V_1(e, c-c_0) + V_2(\eta, \eta(t-h))$ of the form

$$V = \varepsilon^T Q_1 \varepsilon + k\rho^{-1}(c-c_0)^T(c-c_0) + \eta^T N \eta +$$
$$+\int_{-h}^{0} df \int_{t-h}^{t} \dot{c}^T(z)\dot{c}(z)dz + \int_{-h}^{0} \eta^T(t+s) N_1 \eta(t+s)ds.$$

Take a derivative from the functional $V(t)$ on trajectories of the system, using results (24)

$$\dot{V} \leq -\varepsilon^T\left(0.5Q_2 - \rho^2 h \|w(t-h)\|^2\right)\varepsilon -$$
$$-\left(0.25\varepsilon^T Q_2 \varepsilon - 2\varepsilon^T Q_1 b\rho \int_{t-h}^{t} e(s)w(s-h)ds \cdot w(t-h)\right.$$
$$\left.+\rho^2 \int_{t-h}^{t} \|e(s)w(s-h)\|^2 ds\right) + 2\eta^T NC(\dot{c}^T w + c^T \dot{w})$$
$$-\mu_0^{-1}\eta^T(Q_3 - \mu_0 N_1)\eta - \left(0.25\varepsilon^T Q_2 \varepsilon -\right.$$
$$\left.-2\varepsilon^T Q_1 b \mu_0^{\gamma-2} C\eta(t-h) + \eta^T(t-h) N_1 \eta(t-h)\right), \quad (28)$$

where $G^T N + NG = -Q_3$. As trajectories of the system are in the set Ω, than following estimations will be fair:

$2\eta^T NC(\dot{c}^T w + c^T \dot{w}) \leq 2|\eta(t)|K_0$, where $K_0 =$
$= |NC|(k_4 k_1 + k_2 k_3)$, $2|\eta|K_0 \leq \mu_0^{-1}|\eta|^T|\eta| + \mu_0 K_0^2$,

also take advantage of estimations

$$-\eta^T(t-h) N_1 \eta(t-h) +$$
$$+2\varepsilon^T H b \mu_0^{\gamma-2} C\eta(t-h) - 0.25\varepsilon^T Q_2 \varepsilon \leq$$
$$-\left(0.5\sqrt{\|Q_3 N_1\|}\|\eta(t-h)\| - \left\|Q_1 b \mu_0^{\gamma-2} C\right\|\|\varepsilon\|\right)^2 \leq 0$$

for $\left\|Q_1 b \mu_0^{\gamma-2} C\right\|^2 \leq 0.25\|Q_3 N_1\|$,

$Q_1 - \mu_0 N_1 - (1+\mu_0) I_{\gamma-1} = Q_4 \geq 0$.

To put estimations in the inequality (28), and using the result (25), we shall receive

$$\dot{V} \leq -\varepsilon^T(0.5Q_2 - \rho^2 h X_0^2)\varepsilon - \mu_0^{-1}\eta^T Q_4 \eta -$$
$$-\left(\sqrt{\|Q_2\|}\|\varepsilon\| - 2\|Q_1 b\| \int_{t-h}^{t} \|e(s)w(s-h)\|ds\right)^2 -$$
$$-\left(0.5\sqrt{\|Q_3 N_1\|}\|\eta(t-h)\| - \left\|Q_1 b \mu_0^{\gamma-2} C\right\|\|\varepsilon\|\right)^2 +$$
$$+\mu_0 K_0^2.$$

It means, that all signals in closed-loop system will be bounded, and the target condition (3) has been satisfied, where by decreasing of the parameter μ_0 the value of the function V can be as much as small.

5. SIMULATIONS

In this section the adaptive control algorithm is applied to a third order linear system described by the following differential equation

$$(p+q_1)(p+q_2)(p+q_3)y(t) = ku(t-h), \quad (29)$$

where q_1, q_2, q_3 and k are unknown parameters, h is known time delay. The reference model described of the next equation

$$(p+1)^3 y_m(t) = 0{,}5 + 2\sin 1{,}5t + 0{,}5\sin 0{,}05t \,. \quad (30)$$

Choose the control law according to equations (12). For relative degree $\gamma = 3$ the polynomial $T(\lambda)$ choose as $T(\lambda) = \lambda^2 + 0{,}1\lambda + 1$. Taking into account the equations of plant (29) and reference model (30), the filters of the state and the vector of regressive are formed according to (9) and (10). For estimating auxiliary control $v(t)$ and it's $\gamma - 1 = 2$ derivatives, the algorithm of estimation (13) choose in the form (13), where $d_1 = d_2 = 1$, $\mu = 10$.

Results of a computer simulation for the tracking error $e(t)$ and $\rho = 0{,}3$ in the algorithm of adaptation (20) and zero initial conditions in closed-loop system are presented at figures.

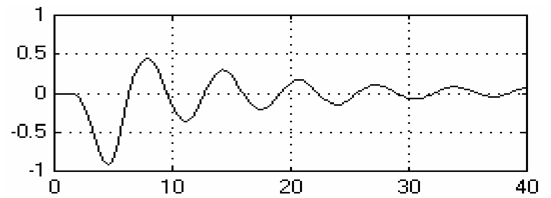

Fig. 1. Tracking error $e(t)$ during the first 40 seconds (the case $q_1 = 0.1$, $q_2 = 0.2$, $q_3 = 1$, $k = 1$ and $h = 1$).

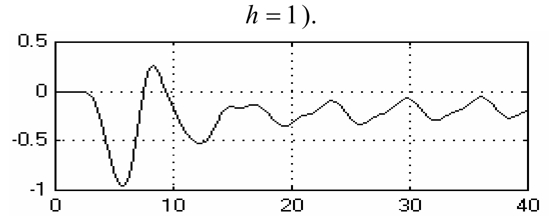

Fig. 2. Tracking error $e(t)$ during the first 40 seconds (the case $q_1 = 0.1$, $q_2 = 0.7$, $q_3 = 4$, $k = 2$ and $h = 2$).

6. CONCLUSION

The modified algorithm of adaptation of the high order was considered in the paper. It designed for plants with time delay using in output signals in conditions of prior uncertainty. Thus, the time delay doesn't affect on stability of closed-loop system. Also, for obtaining a convenient parameterized model the special filters of a state was used. It is allows to receive a strictly positive real transfer function with known parameters. The simulation on a computer demonstrates the small error δ in final periods of transients.

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