EXPERIMENTAL EVALUATION OF SYNCHRONIZATION-BASED DATA TRANSMISSION SCHEME FOR MULTIPENDULUM SETUP*

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Abstract

Application of the synchronization-based low-bitrate data transmission technology is demonstrated for computer-controlled mechanical system – the multipendulum setup. Experimental results show efficiency of the proposed method in the case of corrupted measurements and plant model uncertainty.

Key words

Data transmission, state observation, multipendulum setup, experimental study.

1 Introduction

During the last decade substantial interest has been shown in networked control systems (NCS). The idea is to use serial communication networks to exchange system information and control signals between various physical components of the systems that may be physically distributed. NCS are real-time systems where sensor and actuator data are transmitted through shared or switched communication networks, see e.g. [Ishii and Francis, Ishii and Francis, 2002, Goodwin et al., 2004, Abdallah and Tanner, 2007]. The introduction of a communication network can degrade overall control system performance due to quantization errors, transmission time delays and dropped measurements. Due to the digital nature of the communication channel, every transmitted signal is quantized to a finite set [Ishii and Francis, 2002]. Therefore, the finite set nature of the data should be explicitly taken into account in the design of NCS. The important problem is reducing the data-flow rate over the digital communication channel. An algorithmic solution to this problem attracted attention of many researches, see the surveys [Nair et al., 2007, Andrievsky et al., 2010], the monograph [Matveev and Savkin, 2009] and the references therein. Particularly, it has been shown that the control/observation of linear systems is possible if and only if the capacity of the communication channel exceeds the entropy production of the system at the equilibrium (the Data Rate Theorem) [Nair and Evans, 1997, Nair and Evans, 2000a, Nair and Evans, 2002, Nair and Evans, 2000b, Nair and Evans, 2003].

The coding-decoding schemes were proposed giving an opportunity to get closer to the minimum possible data rate. Two ideas are applied to this purpose: employing smart sensors, which incorporate the model of a plant dynamics, and using the zooming strategy, i.e. updating the coder range during the control or observation process [Liberzon, 2000, Brockett and Liberzon, 2000, Liberzon, 2003, Tatikonda and Mitter, 2004].

Synchronization of nonlinear systems under information constraints was studied in [Fradkov et al., 2006, Andrievskii and Fradkov, 2006, Fradkov et al., 2015]. It was shown that for the first-order coder-decoder scheme the upper bound of limit synchronization error is proportional to the maximum rate of the coupling signal and inversely proportional to the information transmission rate (the channel capacity). It was also shown that in the case of an ideal channel and non-corrupted measurements, the synchronization strategy with the full order encoder/decoder pair ensures exponentially vanishing synchronization error if the channel capacity exceeds a certain threshold. Further on, this approach has been applied to the observation of nonlinear systems over the limited-band communication channel in [Fradkov et al., 2010b].

The present paper is devoted to implementation and experimental evaluation of the data transmission

*Preliminary presented at PhysCon 2015, Istanbul, Turkey.

schemes of [Fradkov et al., 2010b] for a complex mechanical system – the diffusively coupled chain of pendulums as a part of the Multipendulum Mechatronic Setup of the Institute for Problems of Mechanical Engineering of Russian Academy of Sciences (MMS IPME) [Fradkov et al., 2012].

2 The Multipendulum Mechatronic Setup

The setup consists of the set of interconnected pendulum sections, representing a complex oscillatory dynamical system, characterized by nonlinearity and high number of degrees of freedom. Such a mechanical system can serve as a basis for numerous educational and research experiments related to dynamics, control and synchronization in the networks of multidimensional nonlinear dynamical systems.

The MMS IPME includes a modular multi-section mechanical oscillating system, electrical equipment (with computer interface facilities), the electric computer-controlled motor, the personal computer for data processing, representation of the results the real-time control. The setup consists of a number of identical pendulum sections diffusively connected by the springs. The foundation of the section is a hollow rectangular body. Inside the body an electrical magnet and electronic controller board are mounted. On the foundation the support containing the platform for placing the sensors in its middle part is mounted. The pendulum itself possesses a permanent magnet tip in the bottom part. The working ends of the permanent magnet and the electrical magnet are posed exactly opposite each other and separated with a non-magnetic plate in a window of the body. On the rotation axis of the pendulum the optical encoder disk for measuring the angle (phase) of the pendulum is mounted. It has 90 slits. The peripheral part of the disk is posed into the slit of the sensor support. The sensor consists of a radiator (emitting diode) and a receiver (photo-diode). The obtained sequences of signals allow to measure angle (phase) and angular velocity of the pendulum; evaluate amplitude and crossing times and other variables related to the pendulum dynamics. Axes of the neighboring sections are connected with the torsion springs, arranging force interaction and allowing exchanging energy between neighbor sections. In principle, any number of sections can be connected. At the moment mechanical parts of 50 sections are manufactured. The photo of 12 sections is presented in Fig. 1.

The Data Exchange System of the setup is intended to transfer data and control commands from the Control Computer to the interface board of the pendulum sections. Each interface board is an intelligent measuring/controlling electronic device, assigned for unloading processor of the Control Computer from chore of forming the control signal and preventing the Control Computer from a wasteful waiting the sensor replies.

The bus of the multipendulum setup works in a bidirectional mode. It consists of the bidirectional data link, the control lines, the confirmation line and the power supply lines. The pendulum sections and the electric motor are equipped with the electronic modules. The electronic modules have following functions: data exchange with the data bus; generation of control signals for executive devices (the pendulum actuating coils and the electric motors); processing the sensor signals.

Figure 1. Multipendulum mechatronic set-up MMS IPME.

The data, obtained from the setup were used for experimental study of low-rate data transmission technology of [Fradkov et al., 2010b] for the real-world complex mechanical system.

3 Synchronization-Based Data Transmission Scheme

The approach of [Fradkov et al., 2006] has been applied to the observation of nonlinear systems over the limited-band communication channel. The experimental results on application of this approach to feedback control and state estimation over the limited capacity communication channel are presented in [Fradkov et al., 2010b, Fradkov et al., 2012]. Let us briefly recall the mentioned observation schemes.

Consider the following nonlinear plant model:

$$\dot{x}(t) = Ax(t) + B\psi(y),\ y(t) = Cx(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the plant state variables vector; $y(t)$ is a scalar output variable; A is an $(n \times n)$-matrix; B is an $(n \times 1)$-matrix; C is an $(1 \times n)$-matrix, $\psi(y)$ is a continuous nonlinearity.

The problem is to obtain the state estimation of (1) over the digital communication channel with the limited bandwidth. The measured data are sampled with a certain sampling rate T_s and are represented by finite-length codewords to be transmitted over the channel. The following uniform memoryless (static) quantizer is employed:

$$q_{\nu,M}(y) = \begin{cases} \delta \cdot \langle \delta^{-1} y \rangle, & \text{if } |y| \le M, \\ M \operatorname{sign}(y), & \text{otherwise,} \end{cases} \quad (2)$$

where $M > 0$ is a real number (the *quantizer range*), $\nu \in \mathbb{Z}$ is the positive integer, $\delta = 2^{1-\nu}M$; $\langle\cdot\rangle$ denotes the round-up to the nearest integer, $\operatorname{sign}(\cdot)$ is the signum function. The quantization interval $[-M, M]$ is equally split into 2^ν parts. Therefore, the cardinality of the mapping $q_{\nu,M}$ image is equal to $2^\nu + 1$ and each codeword contains $R = \log_2(2^\nu + 1) = \log_2(2M/\delta + 1)$ bits.

The *one-step memory* coder uses the *central number* $c[k]$, $k = 0, 1, \ldots$ with the initial condition $c[0] = 0$ [Tatikonda et al., 1998, Tatikonda et al., 1999]. At step k, the coder compares the current measured output $y[k]$ with the number $c[k]$, forming the deviation signal $\partial y[k] = y[k] - c[k]$. Then $\partial y[k]$ is discretized with a given ν and $M = M[k]$ according to (2). The quantized output signal

$$\bar{\partial} y[k] = q_{\nu,M[k]}(\partial y[k]) \quad (3)$$

is represented as an R-bit codeword and transmitted over the communication channel to the decoder. At the next step, the central number $c[k+1]$ and the quantizer range $M[k]$ are renewed by the following update algorithms:

$$c[k+1] = c[k] + \bar{\partial} y[k], \quad c[0] = 0, \quad (4)$$

$$M[k] = (M_0 - M_\infty)\rho^k + M_\infty, \quad (5)$$

where $0 < \rho \le 1$ is the decay parameter, M_∞ stands for the limit value of $M[k]$, $k = 0, 1, \ldots$ denotes the discrete time instant. The initial value M_0 should be large enough to capture all the region of possible values of $y[0]$.

The coder of the *full order* embeds the observer. In [Fradkov et al., 2010b], the *observation error* (*innovation signal*) is transmitted over the channel rather than a measured plant output. For describing a such kind of the coders, let us introduce the error between the the plant and observer outputs as $\varepsilon(t) = y(t) - \hat{y}(t) = Ce(t)$. This signal is subjected to the coding procedure (2) – (5) instead of $y(t)$, forming the quantized signal $\bar{\varepsilon}[k]$. The following state estimation algorithm is implemented at the coder:

$$\begin{aligned} &\dot{\hat{x}}(t) = A\hat{x}(t) + B\psi(\hat{y}) + L\bar{\varepsilon}(t),\ \hat{y}(t) = C\hat{x}(t), \\ &\bar{\varepsilon}(t) = \bar{\varepsilon}[k] \ \text{ as } \ t \in [t_k, t_{k+1}),\ \ t_k = kT_s, \end{aligned} \quad (6)$$

where $\hat{x} \in \mathbb{R}^n$ stands for the estimate of the plant state vector $x(t)$, $(n \times 1)$-matrix L is the observer gain (the design parameter).

4 Experimental Results

In our experiments, the chain of four pendulum sections and the motor, attached via the spring to pendulum #1 were used. In the course of the experiments, the voltages in the form of harmonic or irregular oscillating signals were applied to the motor. The rotary angles of the drive shaft and the pendulums were measured with the sampling rate of 500 Hz and 2 degrees precision by means of the optical sensors. Then the measured signals were processed by the above coding algorithms for transferring over the channel. The leftmost rotary angle (the angle of the drive shaft) may be referred to as exogenous action, applied to the plant (the chain of the pendulums), it was coded by means of the first-order coder.

The state estimation scheme of Section 3 (Eqs. (2)–(6)) has been modified for taking into account presence of the external excitation signal $u(t)$, applied to the chain. In our experiments, the signal $u(t)$ has been measured by the optical encoder with the accuracy of 2 degrees and 100 Hz sampling frequency. The measured data were transmitted over the communication link without any restrictions on the data rate. Therefore, for the considered system, the autonomous plant model (1) was replaced by the following exogenous model:

$$\dot{x}(t) = Ax(t) + B\psi(y) + Du(t),\ y(t) = Cx(t), \quad (7)$$

where $u(t) \in \mathbb{R}^m$ stands for the external input, D is an $(n \times m)$ matrix, the other notations are the same as in (1). Respectively, equation of the modified observer (6) reads as

$$\begin{aligned} &\dot{\hat{x}}(t) = A\hat{x}(t) + B\psi(\hat{y}) + D\tilde{u}(t) + L\bar{\varepsilon}(t),\ \hat{y}(t) = C\hat{x}(t), \\ &\bar{\varepsilon}(t) = \bar{\varepsilon}[k] \ \text{ as } \ t \in [t_k, t_{k+1}),\ \ t_k = kT_s, \end{aligned} \quad (8)$$

where $\tilde{u}(t)$ denotes the measured value of the exogenous input signal $u(t)$. The errors of the plant input $u(t)$ and output $y(t)$ measurements introduce imperfections into the data transmission procedure and impose limitation on the estimation accuracy, which is achievable in the real-world systems.

The state estimation procedure (2)–(5), (8) has been tested as applied to the chain of four pendulum sections, excited by the motor, which was connected with the pendulum #1 via the torsion spring.

The chain of pendulums model was taken in the form of [Fradkov et al., 2012]:

$$\begin{cases} \ddot{\varphi}_1 + \mu\dot{\varphi}_1 + \omega_0^2 \sin\varphi_1 - k(\varphi_2 - 2\varphi_1) = k\varphi_m(t), \\ \ddot{\varphi}_i + \mu\dot{\varphi}_i + \omega_0^2 \sin\varphi_i - k(\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}) = 0, \\ \qquad\qquad (i = 2, 3, \ldots, N-1), \\ \ddot{\varphi}_N + \mu\dot{\varphi}_N + \omega_0^2 \sin\varphi_N - k(\varphi_N - \varphi_{N-1}) = 0, \end{cases}$$

where $\varphi_i = \varphi_i(t)$ $(i = 1,\ 2,\ \ldots, N)$ are the pendulum deflection angles; $\varphi_m(t)$ is the rotation angle of the drive shaft. The values ρ, ω_0, k are parameters of the system: ρ is the viscous friction parameter; ω_0 is the natural frequency of small oscillations of isolated pendulums; k is the stiffness parameter of the spring, connecting the pendulums.

This model is used for designing the remote state estimator as a part of the coder of full order. The following model parameters were preliminary found by means of identification procedure as $\mu = 0.95$ 1/s, $\omega_0 = 5.5$ 1/s, k = 5.8 1/s^2; the sampling times for each channel (one motor and four pendulums) were taken from the interval $[10, 100$ ms for different data transferring operations. The number of binary digits nu was changed from $\nu = 1$ (the binary coder) to $\nu = 9$.

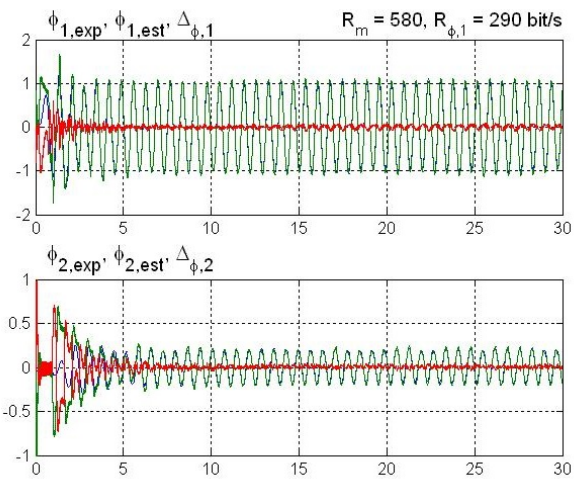

Figure 2. Time histories of pendulums #1,2 rotary angles and their transmitted values for the binary coder. $R_m = 580$ bit/s, $R_\varphi = 290$ bit/s. Measured data – blue line, transmitted – green line, transmission error – red line.

Two coding schemes have been implemented for transmission and estimation of the pendulum angles: the first-order coder with the observer at the receiver's side (for transferring the motor rotary angle), and the full-order coder at the coder's side and the observer, embedded to the decoder (for transferring pendulum rotary angle). The harmonic excitation voltage with the period 0.7 s has been applied to the motor. The measured data have been processed by means of the mentioned data transmission procedures and the estimation errors have been calculated. It should be noticed, that the "exact" values of the pendulums rotary angles are not known due to the measuring error of the optical sensors, effecting on the experimental accuracy evaluation. The data bit-per-second rate R for corresponding channel was calculated as $R = \nu/T_s$.

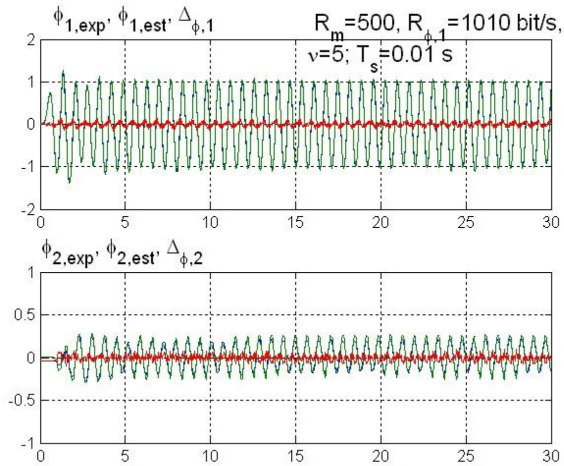

Figure 3. Time histories of pendulums #1,2 rotary angles and their transmitted values for the case of $\nu = 5$. $R_m = 500$ bit/s, $R_\varphi = 1010$ bit/s. Measured data – blue line, transmitted – green line, transmission error – red line.

The experimental results are depicted in Figs. 2–5. Examples of time histories of pendulums #1,2 rotary angles and their transmitted values for different rates R are plotted in Figs. 2, 3. Dependence of the generalized accuracy index – the mean-square relative transmission error in the steady-state mode from the coder parameters is demonstrated in Figs. 4, 5. The results obtained confirm with the theoretical statements of [Fradkov et al., 2010b] that the binary coder is optimal in the sense of the bit-per-second rate and that for the coder of full order there exists a certain threshold which limits the minimal bit-rate, making data transmission possible.

5 Conclusions

Application of the low-rate data transmission technology of [Fradkov et al., 2010b] is demonstrated for complex mechanical system – the multipendulum setup. It is shown that the data transmission rate may be taken about 200 bit/s, ensuring the appropriate accuracy of the state transmission over the digital communication channel. It is demonstrated that for the case of the full-order coder there exists a threshold, bounding the secure data transmission rate, which confirms the results of [Fradkov et al., 2010b].

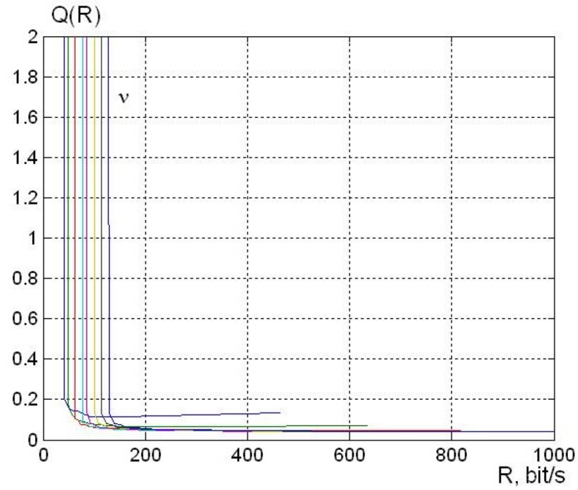

Figure 4. Mean-square relative transmission error Q v.s. transmission bitrate R (bit/s) for different word length $\nu = 1, \ldots, 9$.

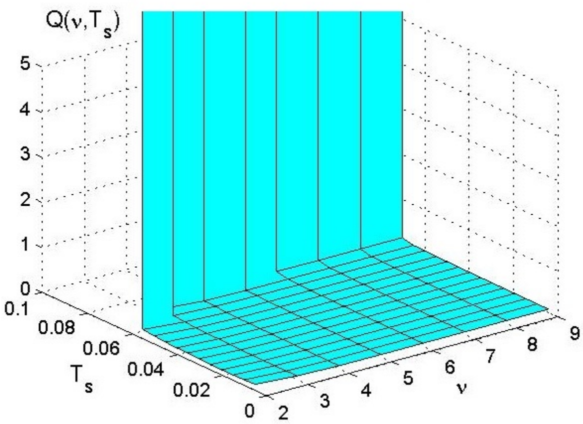

Figure 5. Mean-square relative transmission error Q v.s. sampling time T_s (s) and the word length $\nu = 1, \ldots, 9$.

Acknowledgment

This work was performed in the IPME RAS and supported by the Russian Science Foundation (project 14-29-00142).

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