ON STABILIZATION OF LEVITRON

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Abstract
Consider a system of differential equations, which in the linear approximation describe the dynamics of a body with an electric charge in the electrostatic field. The problem of stability and stabilization is investigated.

Key words
differential equations, stability, stabilization

1 Statement of the problem and the method of Chetayev's bundles

Consider a system of differential equations, which in the linear approximation describes the dynamics of a body with an electric charge in an electrostatic field [1]:

$$\begin{array}{l} \alpha'' - (-\delta x - H\beta' + \chi\alpha) = 0, \\ \beta'' - (-\delta y + H\alpha' + \chi\beta) = 0, \\ x'' - (x - \delta\alpha) = 0,\ y'' - (y - \delta\beta) = 0 \end{array} \quad (1)$$

The system has been described in terms of dimensionless variables and parameters.It may also be applied for describing the motion of a rigid body on a special suspension [2] and for describing the motion of a movable magnet located in the field of a immovable magnet [3]. The description of generalized coordinates $\alpha,\ \beta,\ x,\ y$, design data $\delta,\ \chi$ and stabilizing parameter H is given in [1]. According to the problem's conditions, parameter $\chi > 0$, hence, the potential energy of system (1) in zero has the maximum. Without gyroscopic forces ($H = 0$) the potential system (1) is unstable with respect to all the variables. Now investigate the possibility of system's stabilization with respect to all the variables by introducing gyroscopic forces (rotation) with respect to some part of the variables [1]. These forces are defined by parameter H. In order to find the values of H, for which system (1) is stable, let us use known theorems of stability theory. The following two approaches may be applied: the method of Chetayev's bundles [4] and the investigation of the characteristic equation. System (1) assumes the four first integrals [2]:

$$V_1 = 1/(2\delta)(\alpha^2\delta^2 + \beta^2\delta^2 + x^2(1+\delta^2-\chi)$$
$$+y^2(1+\delta^2-\chi) - 2Hyx' + (-1+\chi)(x')^2$$
$$+\beta(-2y\delta + 2H\delta x') + 2Hxy' + (-1+\chi)(y')^2 +$$
$$\alpha(-2x\delta - 2H\delta y') + 2\delta x'\alpha' + 2\delta y'\beta'),$$
$$h = 1/2(-x^2 - y^2 + 2x\alpha\delta + 2y\beta\delta - \alpha^2\chi - \beta^2\chi$$
$$+ (x')^2 + (y')^2 + (\alpha')^2 + (\beta')^2),$$
$$V_2 = 1/2\left(H\alpha^2 + H\beta^2\right) + yx' - xy' + \beta\alpha' - \alpha\beta',$$
$$V_3 = 1/(2\delta)(Hx^2 + Hy^2 - H(x')^2 + \beta(-2Hy\delta$$
$$-2\delta x') - H(y')^2 + \alpha(-2Hx\delta + 2\delta y')$$
$$+y((2-2\chi)x' - 2\delta\alpha') + x((-2+2\chi)y' + 2\delta\beta'))$$

Unfortunately, we do not manage to construct a sign-definite bundle of two integrals. Let us now construct the bundle of all the four integrals $V = h + \mu_i V_i$. The integral constructed is decomposed into the two forms of four variables each one:

$$W_1 = 1/2\Big(\alpha^2(-\chi + H\mu_1 + \delta\mu_2) + 2x\alpha(\delta - \mu_2$$
$$-H\mu_3) + x^2(-\delta + \mu_2 + \delta^2\mu_2 - \chi\mu_2 + H\mu_3)/\delta$$
$$+(\delta - \mu_2 + \chi\mu_2 - H\mu_3)(y')^2/\delta + (-2\alpha\mu_1$$
$$+2x\mu_3)\beta' + (\beta')^2 + y'\Big(2\alpha(\mu_3 - H\mu_2)$$
$$-x(2\delta\mu_1 - 2H\mu_2 + 2\mu_3 - 2\chi\mu_3)/\delta + 2\mu_2\beta'\Big)\Big),$$
$$W_2 = \frac{1}{2}\Big(\beta^2(-\chi + \delta\mu_1 + H\mu_2) + 2\beta(H\mu_1 - \mu_3)x'$$
$$+y^2(-\delta + (1+\delta^2-\chi)\mu_1 + H\mu_3)/\delta$$

$$+(\delta+(\chi-1)\mu_1-H\mu_3)(x')^2/\delta+2y\Big(\beta(\delta-\mu_1$$
$$-H\mu_3)+(-H\mu_1+\delta\mu_2-(-1+\chi)\mu_3)x'/\delta\Big)$$
$$+2(\beta\mu_2-y\mu_3+\mu_1x')\alpha'+(\alpha')^2\Big)$$

The quadratic forms $W_1,\ W_2$ are definite positive under Sylvester conditions, which are identical to both of the forms:

$$\begin{aligned}
&\big\{f_1(\mu_1,\mu_3)=(\delta-\mu_1+\chi\mu_1-\delta\mu_1^2-H\mu_3)/\delta>0,\\
&f_2(\mu_1,\mu_2,\mu_3)=-\delta^2+2\delta\mu_1+\delta^3\mu_1-2\delta\chi\mu_1\\
&-\mu_1^2-H^2\mu_1^2+2\chi\mu_1^2+\delta^2\chi\mu_1^2-\chi^2\mu_1^2-\delta\mu_1^3\\
&-\delta^3\mu_1^3+\delta\chi\mu_1^3-\delta^2\mu_2^2+2H\delta\mu_3-H\delta^2\mu_1\mu_3+\\
&H\delta\mu_1^2\mu_3-\mu_3^2-H^2\mu_3^2-\delta^2\mu_3^2+2\chi\mu_3^2-\chi^2\mu_3^2\\
&-\delta\mu_1\mu_3^2+\delta\chi\mu_1\mu_3^2+H\delta\mu_3^3+\mu_2\big(2H\delta\mu_1-2\delta\mu_3\\
&+2\delta\chi\mu_3-2\delta^2\mu_1\mu_3\big)>0,\ f_3(\mu_1,\mu_2,\mu_3)=-\delta^4\\
&+\delta^2\chi+\big(-\delta^4+\delta^2\chi\big)\mu_1^4-H\delta^2\mu_2+\delta^2\mu_2^2\\
&+\delta^2\chi\mu_2^2-H\delta^2\mu_2^3+\delta^2\mu_2^4+3H\delta^3\mu_3-2H\delta\chi\mu_3\\
&+2H^2\delta\mu_2\mu_3-4\delta^3\mu_2\mu_3+2\delta\chi\mu_2\mu_3-2\delta\chi^2\mu_2\mu_3\\
&-4H\delta\mu_2^2\mu_3+2H\delta\chi\mu_2^2\mu_3+2\delta\mu_2^3\mu_3-2\delta\chi\mu_2^3\mu_3\\
&-\delta^2\mu_3^2-3H^2\delta^2\mu_3^2+\chi\mu_3^2+H^2\chi\mu_3^2+3\delta^2\chi\mu_3^2\\
&-2\chi^2\mu_3^2+\chi^3\mu_3^2-H\mu_2\mu_3^2-H^3\mu_2\mu_3^2\\
&+5H\delta^2\mu_2\mu_3^2+2H\chi\mu_2\mu_3^2-H\chi^2\mu_2\mu_3^2+\mu_2^2\mu_3^2+\\
&H^2\mu_2^2\mu_3^2-2\delta^2\mu_2^2\mu_3^2-2\chi\mu_2^2\mu_3^2+\chi^2\mu_2^2\mu_3^2\\
&+H\delta\mu_3^3+H^3\delta\mu_3^3-3H\delta\chi\mu_3^3-2\delta\mu_2\mu_3^3\\
&-H^2\delta\mu_2\mu_3^3+2\delta\chi\mu_2\mu_3^3+\delta^2\mu_3^4-\delta\mu_1^3\big(2\delta^2\\
&+H^2\delta^2-2\chi-H^2\chi-2\delta^2\chi+2\chi^2+H\mu_2\\
&-H\delta^2\mu_2+H\chi\mu_2-H\delta\mu_3\big)+\mu_1^2\big(-\delta^2-H^2\delta^2\\
&+2\delta^4+\chi+H^2\chi-2\chi^2-\delta^2\chi^2+\chi^3-H\mu_2\\
&-H^3\mu_2+2H\chi\mu_2+H\delta^2\chi\mu_2-H\chi^2\mu_2+\mu_2^2\\
&+H^2\mu_2^2+3\delta^2\mu_2^2-2\chi\mu_2^2-\delta^2\chi\mu_2^2+\chi^2\mu_2^2\\
&+H\delta\mu_3+H^3\delta\mu_3+H\delta^3\mu_3-3H\delta\chi\mu_3-2\delta\mu_2\mu_3\\
&-H^2\delta\mu_2\mu_3-4\delta^3\mu_2\mu_3+2\delta\chi\mu_2\mu_3+\delta^2\mu_3^2\\
&+\delta^2\chi\mu_3^2\big)+\mu_1\big(2\delta^3-2\delta\chi-2\delta^3\chi+2\delta\chi^2\\
&+2H\delta\mu_2+3H\delta^3\mu_2-4H\delta\chi\mu_2-2\delta\mu_2^2\\
&2H^2\delta\mu_2^2-4\delta^3\mu_2^2+2\delta\chi\mu_2^2-2H\delta\mu_2^3\\
&-2H\delta^2\mu_3+H\delta^2\chi\mu_3+4\delta^2\mu_2\mu_3-3H^2\delta^2\mu_2\mu_3\\
&+4\delta^2\chi\mu_2\mu_3+3H\delta^2\mu_2^2\mu_3-4\delta^3\mu_3^2+\\
&2\delta\chi\mu_3^2+H^2\delta\chi\mu_3^2-2\delta\chi^2\mu_3^2-H\delta\mu_2\mu_3^2\\
&-H\delta\chi\mu_2\mu_3^2+H\delta^2\mu_3^3\big)>0\big\}
\end{aligned}\tag{2}$$

If there exist $\mu_1,\ \mu_2,\ \mu_3$ such that conditions (2) hold for some values of parameters $H,\ \delta,\ \chi$, then the first integral V satisfies Lyapunov's theorem on stability. Systems (1) is stable with respect to all the variables under such values of parameters. It is hardly ever possible to obtain an analytical solution of inequalities (2). It is possible write down - likewise in [2] - some relations obtained from the necessary conditions of solvability for inequalities (2). It follows from $f_1(\mu_1,\mu_3)>0$ that $(1+4\delta^2-2\chi+\chi^2-4H\delta\mu_3)>0$. When considering $f_2(\mu_1,\mu_2,\mu_3)$ as a polynomial with respect to μ_2, we obtain the necessary condition of satisfaction of the second inequality (2): $\big(-\delta^2\big(-\delta+\mu_1-\chi\mu_1+\delta\mu_1^2+H\mu_3\big)\big(-\delta+\mu_1+\delta^2\mu_1-\chi\mu_1+H\mu_3-\delta\mu_3^2\big)\big)>0$. Hence, since $f_1(\mu_1,\mu_3)>0$, we have $\big(\delta\big(-\delta+\mu_1+\delta^2\mu_1-\chi\mu_1+H\mu_3-\delta\mu_3^2\big)\big)>0$. Note the following important property: the discriminant's sign of the 4th degree polynomial $f_3(\mu_1,\mu_2,\mu_3)$ (2) with respect to any μ_i is defined by the sign of the expression

$$\begin{aligned}
D&=4H^2+8H^4+4H^6+16\delta^2+60H^2\delta^2\\
&+12H^4\delta^2+128\delta^4+48H^2\delta^4+27H^4\delta^4\\
&+256\delta^6-16\chi-48H^2\chi-32H^4\chi-192\delta^2\chi\\
&-124H^2\delta^2\chi-36H^4\delta^2\chi-512\delta^4\chi\\
&-144H^2\delta^4\chi+64\chi^2+88H^2\chi^2+8H^4\chi^2\\
&+352\delta^2\chi^2+196H^2\delta^2\chi^2+128\delta^4\chi^2\\
&-96\chi^3-48H^2\chi^3-192\delta^2\chi^3-4H^2\delta^2\chi^3\\
&+64\chi^4+4H^2\chi^4+16\delta^2\chi^4-16\chi^5.
\end{aligned}\tag{3}$$

2 Investigation of the characteristic equation

Consider the characteristic equation of the linear system (1):

$$\begin{aligned}
&q_0\lambda^8+\lambda^6q_1+\lambda^4q_2+\lambda^2q_3+q_4=\\
&q_0z^4+z^3q_1+z^2q_2+zq_3+q_4=0
\end{aligned}\tag{4}$$

where $z=\lambda^2$, $q_0=1$, $q_1=-2+h-2\chi$, $q_4=\sigma^2-2\sigma\chi+\chi^2$, $q_3=h+2\sigma-2\chi+2\sigma\chi-2\chi^2$, $q_2=1-2h-2\sigma+4\chi+\chi^2$.

Unlike that in [1], we are choosing a different method for our investigation of the system's (1)characteristic equation. System (1) is stable only when all the roots λ of equation (4) are various purely imaginary. The polynomial of even degrees has various purely imaginary roots if and only if [5] the Routh-Hurwitz conditions hold for an auxiliary polynomial

$$\begin{aligned}
&\lambda^{2n}q_0+n\lambda^{-1+2n}q_0+(-1+n)\lambda^{-3+2n}q_1\\
&+\lambda^{-2+2n}q_1+(-2+n)\lambda^{-5+2n}q_2+\lambda^{-4+2n}q_2\\
&+(-3+n)\lambda^{-7+2n}q_3+\lambda^{-6+2n}q_3+\ldots\\
&+\lambda q_{-1+n}+\lambda^2q_{-1+n}+q_n
\end{aligned}$$

The main diagonal minors of the matrix

$$\begin{pmatrix}
4q_0 & 3q_1 & 2q_2 & q_3 & 0 & 0 & 0 & 0\\
q_0 & q_1 & q_2 & q_3 & q_4 & 0 & 0 & 0\\
0 & 4q_0 & 3q_1 & 2q_2 & q_3 & 0 & 0 & 0\\
0 & q_0 & q_1 & q_2 & q_3 & q_4 & 0 & 0\\
0 & 0 & 4q_0 & 3q_1 & 2q_2 & q_3 & 0 & 0\\
0 & 0 & q_0 & q_1 & q_2 & q_3 & q_4 & 0\\
0 & 0 & 0 & 4q_0 & 3q_1 & 2q_2 & q_3 & 0\\
0 & 0 & 0 & q_0 & q_1 & q_2 & q_3 & q_4
\end{pmatrix}$$

must be positive for the equation (4). The resulting system of inequalities writes:

$q_0 > 0,\, q_1 > 0,\, q_2 > 0,\, q_3 > 0,\, q_4 > 0,\, 3q_1^2 - 8q_0q_2 > 0,\, (q_1^2q_2 - 4q_0q_2^2 + 3q_0q_1q_3 > 0,\, (q_1^2q_2^2 - 4q_0q_2^3 - 3q_1^3q_3 + 14q_0q_1q_2q_3 - 18q_0^2q_3^2 - 6q_0q_1^2q_4 + 16q_0^2q_2q_4 > 0,\; q_1^2q_2^2q_3 - 4q_0q_2^3q_3 - 4q_1^3q_3^2 + 18q_0q_1q_2q_3^2 - 27q_0^2q_3^3 + 3q_1^3q_2q_4 - 12q_0q_1q_2^2q_4 - 7q_0q_1^2q_3q_4 + 48q_0^2q_2q_3q_4 - 16q_0^2q_1q_4^2 > 0,\; (q_1^2q_2^2q_3^2 - 4q_0q_2^3q_3^2 - 4q_1^3q_3^3 + 18q_0q_1q_2q_3^3 - 27q_0^2q_3^4 - 4q_1^2q_2^3q_4 + 16q_0q_2^4q_4 + 18q_1^3q_2q_3q_4 - 80q_0q_1q_2^2q_3q_4 - 6q_0q_1^2q_3^2q_4 + 144q_0^2q_2q_3^2q_4 - 27q_1^4q_4^2 + 144q_0q_1^2q_2q_4^2 - 128q_0^2q_2^2q_4^2 - 192q_0^2q_1q_3q_4^2 + 256q_0^3q_4^3) = -D_1 > 0,$

where D_1 is the discriminant of equation (4) with respect to $z = \lambda^2$ (according to the Lienard-Chipart criterion, the 7th and the 9th inequalities may be excluded). In terms of parameters of system (1) these inequalities have the form:

$$
\begin{aligned}
&\big\{ -2 + G - 2\chi > 0, 1 - 2G - 2\sigma + 4\chi + \chi^2 > 0,\\
&G + 2\sigma - 2\chi + 2\sigma\chi - 2\chi^2 > 0,\ (\sigma - \chi)^2 > 0,\\
&4 + 4G + 3G^2 + 16\sigma - 8\chi - 12G\chi + 4\chi^2 > 0,\\
&4\chi - 2G - 4G^2 - 2G^3 - 4\sigma - 18G\sigma - 2G^2\sigma\\
&-16\sigma^2 + 16G\chi + 12G^2\chi + 24\sigma\chi + 14G\sigma\chi - 8\chi^2\\
&-18G\chi^2 + G^2\chi^2 - 4\sigma\chi^2 + 4\chi^3 - 4G\chi^3 > 0,\\
&G + 2G^2 + G^3 + 4\sigma + 12G\sigma + 2G^2\sigma + 16\sigma^2\\
&-2G\sigma^2 - 4\chi - 12G\chi - 8G^2\chi - 36\sigma\chi\\
&-20G\sigma\chi - 6G^2\sigma\chi - 48\sigma^2\chi + 16\chi^2 + 22G\chi^2\\
&+2G^2\chi^2 + 60\sigma\chi^2 + 32G\sigma\chi^2 - 24\chi^3 - 12G\chi^3\\
&-28\sigma\chi^3 + 16\chi^4 + G\chi^4 - 4\chi^5 > 0,\\
&-G\big(6G\sigma^2 + 12G^2\sigma^2 + 6G^3\sigma^2 + 16\sigma^3 + 66G\sigma^3\\
&+6G^2\sigma^3 + 64\sigma^4 - 4G\sigma\chi - 8G^2\sigma\chi - 4G^3\sigma\chi\\
&-32\sigma^2\chi - 100G\sigma^2\chi - 44G^2\sigma^2\chi - 160\sigma^3\chi\\
&-30G\sigma^3\chi + 2G\chi^2 + 4G^2\chi^2 + 2G^3\chi^2 + 24\sigma\chi^2\\
&+70G\sigma\chi^2 + 34G^2\sigma\chi^2 + 192\sigma^2\chi^2 + 112G\sigma^2\chi^2\\
&+13G^2\sigma^2\chi^2 + 144\sigma^3\chi^2 - 8\chi^3 - 24G\chi^3\\
&-16G^2\chi^3 - 128\sigma\chi^3 - 114G\sigma\chi^3 - 18G^2\sigma\chi^3\\
&-288\sigma^2\chi^3 - 70G\sigma^2\chi^3 + 32\chi^4 + 44G\chi^4\\
&+4G^2\chi^4 + 192\sigma\chi^4 + 98G\sigma\chi^4 + 64\sigma^2\chi^4\\
&-48\chi^5 - 24G\chi^5 - 96\sigma\chi^5 - 2G\sigma\chi^5 + 32\chi^6\\
&+2G\chi^6 + 8\sigma\chi^6 - 8\chi^7\big) > 0,\\
&D_1 = G^2\sigma^2\big(4G + 8G^2 + 4G^3 + 16\sigma + 60G\sigma\\
&+12G^2\sigma + 128\sigma^2 + 48G\sigma^2 + 27G^2\sigma^2 + 256\sigma^3\\
&-16\chi - 48G\chi - 32G^2\chi - 192\sigma\chi - 124G\sigma\chi\\
&-36G^2\sigma\chi - 512\sigma^2\chi - 144G\sigma^2\chi + 64\chi^2\\
&+88G\chi^2 + 8G^2\chi^2 + 352\sigma\chi^2 + 196G\sigma\chi^2\\
&+128\sigma^2\chi^2 - 96\chi^3 - 48G\chi^3 - 192\sigma\chi^3 - 4G\sigma\chi^3\\
&+64\chi^4 + 4G\chi^4 + 16\sigma\chi^4 - 16\chi^5\big) < 0\big\}
\end{aligned}
\tag{5}
$$

where $\sigma = \delta^2$, $G = H^2$. Note, D_1 in the latter inequality (5) coincides with D (3) with the accuracy up to the positive multiplier. Hence, $D < 0$ when system (1) is stable.

A numerical experiment allows one to define lower boundaries for the constructive system's parameters $\delta^2 > 4, \chi > 5$. Upon assigning one of the constructive parameters, one can determine possible values of the other two parameters under which the system (1) is stable. When $\delta^2 = 5$, the domain of values of the parameters χ, H is very narrow. Fig.1 shows a 3-D domain of values of the parameters in the vicinity of the boundary values of δ^2, χ.

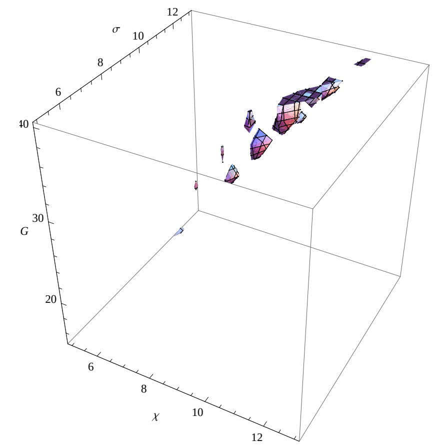

Figure 1. The domain $(\chi,\ \sigma,\ H^2)$ in the vicinity of the boundary values of $\delta^2,\ \chi$.

Under $\chi = 10$ the domain of parameters σ, H^2 is shown in Fig.2.

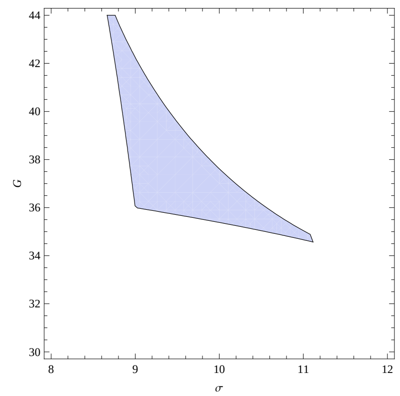

Figure 2. The domain $(\sigma,\ H^2)$, $\chi = 10$

The 3-D domain of parameters in the intervals of δ^2, χ, distant from boundary values, is shown in Fig.3.

3 On asymptotic stability

We have to remind the reader of the fact that so far we have been discussing stabilization of the linear system in the critical case in the sense of Lyapunov. The

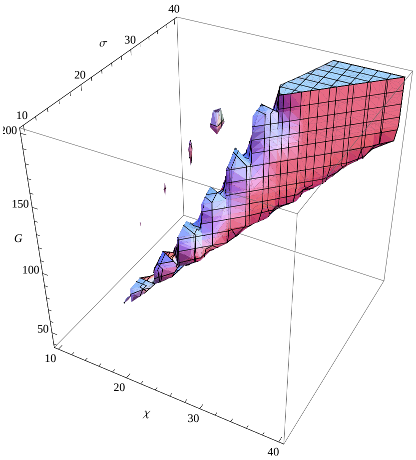

Figure 3. The domain $(\chi,\ \sigma,\ H^2)$

result cannot be extended onto a nonlinear system, for which system (1) is the one of the first approximation. Any attempts to stabilize system (1) up to asymptotic stability by adding dissipative and accelerating forces linear with respect to velocities failed. In [1], the possibility of stabilization by adding linear dissipative forces with matrix $-B$ and linear nonconservative forces with matrix $-P$

$$B = \begin{pmatrix} d_{11} & 0 & 0 & 0 \\ 0 & d_{22} & 0 & 0 \\ 0 & 0 & d_{33} & 0 \\ 0 & 0 & 0 & d_{44}, \end{pmatrix},\quad P = \begin{pmatrix} 0 & p_1 & 0 & 0 \\ -p_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_2 \\ 0 & -p_2 & 0 & 0, \end{pmatrix}$$

was investigated.

We have managed to show that stabilization is impossible in case of incomplete dissipation (one of the elements $d_{ii} = 0$) or when the element $p_1 = 0$.

Let $d_{11} = d_{22} = d_1,\ d_{33} = d_{44} = d_2$, likewise in [1]. The characteristic equation of system (1) with the addition of dissipative and conservative forces has the form:

$$\begin{aligned} &\lambda^8 + \lambda^7 a_1 + \lambda^6 a_2 + \lambda^5 a_3 + \lambda^4 a_4 + \lambda^3 a_5 \\ &+\lambda^2 a_6 + \lambda a_7 + a_8 = 0, \end{aligned} \tag{6}$$

where $a_1 = 2(d_1 + d_2),\ a_2 = \big(-2 + H^2 - 2\chi + d_1^2 + 4d_1d_2 + d_2^2\big)$, $a_3 = \big(-4d_1 - 2\chi d_1 - 2d_2 + 2H^2d_2 - 4\chi d_2 + 2d_1^2d_2 + 2d_1d_2^2 + 2Hp_1\big),\ a_4 = \big(1 - 2H^2 - 2\delta^2 + 4\chi + \chi^2 - 2d_1^2 - 4d_1d_2 - 4\chi d_1d_2 + H^2d_2^2 - 2\chi d_2^2 + d_1^2d_2^2 + 4Hd_2p_1 + p_1^2 + p_2^2\big),\ a_5 = \big(2d_1 - 2\delta^2d_1 + 4\chi d_1 - 2H^2d_2 - 2\delta^2d_2 + 4\chi d_2 + 2\chi^2d_2 - 2d_1^2d_2 - 2\chi d_1d_2^2 - 4Hp_1 + 2Hd_2^2p_1 + 2d_2p_1^2 + 2d_1p_2^2\big),\ a_6 = \big(H^2 + 2\delta^2 - 2\chi + 2\delta^2\chi - 2\chi^2 + d_1^2 - 2\delta^2d_1d_2 + 4\chi d_1d_2 + \chi^2d_2^2 - 4Hd_2p_1 - 2p_1^2 + d_2^2p_1^2 + H^2p_2^2 - 2\chi p_2^2 + d_1^2p_2^2\big),\ a_7 = \big(2\delta^2d_1 - 2\chi d_1 + 2\delta^2\chi d_2 - 2\chi^2d_2 + 2Hp_1 - 2d_2p_1^2 + 2H\delta^2p_2 - 2\chi d_1p_2^2 + 2Hp_1p_2^2\big),\ a_8 = \delta^4 - 2\delta^2\chi + \chi^2 + p_1^2 + 2\delta^2p_1p_2 + \chi^2p_2^2 + p_1^2p_2^2$.

In [1], this characteristic equation was investigated by the method of D-decomposition under the assumption that $d_1 \ll 1,\ d_2 \ll 1, p_1 \ll 1,\ p_2 \ll 1$.

Let us conduct the investigation without an assumption that parameters of the stabilizing forces are small. The system with equation (6) is asymptotically stable when the Lienard-Chipart conditions hold:

$$a_i > 0\ (i = 1, \ldots, 8),\ \Delta_3 > 0,\ \Delta_5 > 0,\ \Delta_7 > 0, \tag{7}$$

where the Hurwitz determinant is

$\Delta_3 = a_1a_2a_3 - a_3^2 - a_1^2a_4 + a_1a_5$,

$\Delta_5 = a_1a_2a_3a_4a_5 - a_3^2a_4a_5 - a_1^2a_4^2a_5 - a_1a_2^2a_5^2 + a_2a_3a_5^2 + 2a_1a_4a_5^2 - a_5^3 - a_1a_2a_3^2a_6 + a_3^3a_6 + a_1^2a_3a_4a_6 + 2a_1^2a_2a_5a_6 - 3a_1a_3a_5a_6 - a_1^3a_6^2 + a_1a_2^2a_3a_7 - a_2a_3^2a_7 - a_1^2a_2a_4a_7 - a_1a_2a_5a_7 + 2a_3a_5a_7 + 2a_1^2a_6a_7 - a_1a_7^2 - a_1^2a_2a_3a_8 + a_1a_3^2a_8 + a_1^3a_4a_8 - a_1^2a_5a_8$,

$\Delta_7 = \big(a_1a_2a_3a_4a_5a_6a_7 - a_3^2a_4a_5a_6a_7 - a_1^2a_4^2a_5a_6a_7 - a_1a_2^2a_5^2a_6a_7 + a_2a_3a_5^2a_6a_7 + 2a_1a_4a_5^2a_6a_7 - a_5^3a_6a_7 - a_1a_2a_3^2a_6^2a_7 + a_3^3a_6^2a_7 + a_1^2a_3a_4a_6^2a_7 + 2a_1^2a_2a_5a_6^2a_7 - 3a_1a_3a_5a_6^2a_7 - a_1^3a_6^3a_7 - a_1a_2a_3a_4^2a_7^2 + a_3^2a_4^2a_7^2 + a_1^2a_4^3a_7^2 + a_1a_2^2a_4a_5a_7^2 - a_2a_3a_4a_5a_7^2 - 2a_1a_4^2a_5a_7^2 + a_4a_5^2a_7^2 + 2a_1a_2^2a_3a_6a_7^2 - 2a_2a_3^2a_6a_7^2 - 3a_1^2a_2a_4a_6a_7^2 + a_1a_3a_4a_6a_7^2 - a_1a_2a_5a_6a_7^2 + 3a_3a_5a_6a_7^2 + 3a_1^2a_6^2a_7^2 - a_1a_2^3a_7^3 + a_2^2a_3a_7^3 + 3a_1a_2a_4a_7^3 - 2a_3a_4a_7^3 - a_2a_5a_7^3 - 3a_1a_6a_7^3 + a_7^4 - a_1a_2a_3a_4a_5^2a_8 + a_3^2a_4a_5^2a_8 + a_1^2a_4^2a_5^2a_8 + a_1a_2^2a_5^3a_8 - a_2a_3a_5^3a_8 - 2a_1a_4a_5^3a_8 + a_5^4a_8 + a_1a_2a_3^2a_5a_6a_8 - a_3^3a_5a_6a_8 - a_1^2a_3a_4a_5a_6a_8 - 2a_1^2a_2a_5^2a_6a_8 + 3a_1a_3a_5^2a_6a_8 + a_1^3a_5a_6^2a_8 + 2a_1a_2a_3^2a_4a_7a_8 - 2a_3^3a_4a_7a_8 - 2a_1^2a_3a_4^2a_7a_8 - 3a_1a_2^2a_3a_5a_7a_8 + 3a_2a_3^2a_5a_7a_8 + a_1^2a_2a_4a_5a_7a_8 + 4a_1a_3a_4a_5a_7a_8 + a_1a_2a_5^2a_7a_8 - 4a_3a_5^2a_7a_8 - a_1^2a_2a_3a_6a_7a_8 + a_1a_3^2a_6a_7a_8 + 3a_1^3a_4a_6a_7a_8 - 5a_1^2a_5a_6a_7a_8 + 3a_1^2a_2^2a_7^2a_8 - 5a_1a_2a_3a_7^2a_8 + 2a_3^2a_7^2a_8 - 3a_1^2a_4a_7^2a_8 + 4a_1a_5a_7^2a_8 - a_1a_2a_3^3a_8^2 + a_3^4a_8^2 + a_1^2a_3^2a_4a_8^2 + 3a_1^2a_2a_3a_5a_8^2 - 4a_1a_3^2a_5a_8^2 - 2a_1^3a_4a_5a_8^2 + 2a_1^2a_5^2a_8^2 - a_1^3a_3a_6a_8^2 - 3a_1^3a_2a_7a_8^2 + 4a_1^2a_3a_7a_8^2 + a_1^4a_8^3\big)$.

Note, under the values of a_i (6), the expression for Δ_7 is factorized.

Let $d_1 = d_2 = d$ and $p_2 = 0$. Having solved the system of inequalities (7) with respect to d and p_1 ($\delta = 3, \chi = 10, H = 6.23$), we obtain the domain of values of parameters for the asymptotic stability for the system with potential, gyroscopic, non-potential and dissipative forces (Fig.4), (Fig.5). For the purpose reaching asymptotic stability of the levitron it is necessary to choose parameter d of dissipative forces within the interval of $0 < d < 0.319366$, and parameter p_1 of nonconservative forces – depending on the configuration of the domain – varies within the interval of $0 < p_1 < 0.950483$. Let $d = 0.1$. Hence the system of inequalities (7) gives the following solution $d = 0.1,\ -0.0178833 < p_2 < 0.0181985,\ 0.260134 < p_1 < 0.384563$.

All the computations have been executed with the aid of the computer algebra system MATHEMATICA.

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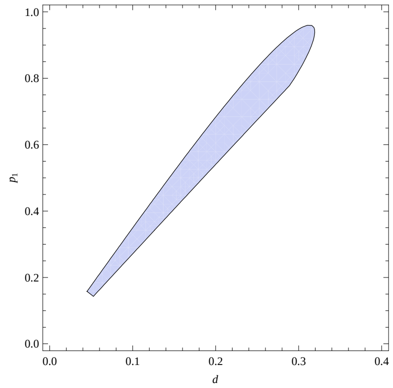

Figure 4. The domain $(d,\ p_1)$

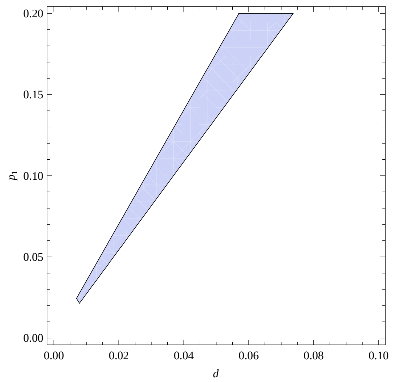

Figure 5. The domain $(d,\ p_1)$

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