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MODELING OF PLASMA BREAKDOWN CONDITIONS IN ITER

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Abstract
In this paper the mathematical model of optimization of control parameters of the initial plasma stage in ITER is considered. To create conditions for breakdown of plasma and raising its current (in accordance with a specified scenario) control system is required, which allows control parameters to influence the electromagnetic conditions in the chamber of the tokamak. The voltages in the active poloidal coils, the initial currents in the coil and ballast resistance in the coil with a resistor are considered as the control parameters. The control is performed in accordance with technological limitations and physical requirements.

Key words
Tokamak, breakdown, ITER, initial plasma stage.

1 Introduction

Currently, the leading international thermonuclear research centers in the field of controlled thermonuclear fusion participate in the international project ITER. It should be noted that in the ITER project plan to use many technical and scientific decisions based on existing tokamaks. One of them is experimental spherical tokamak Globus-M (Ioffe Institute, St. Petersburg) [Gusev, Aminov, Berezutskiy, 2010; Gusev, Aminov, Berezutskiy, 2011; Gusev, Bakharev, Ovsyannikov, 2013]. Also worth mentioning is the program of experiments on small tokamaks under the auspices of the IAEA [Ovsyannikov, Zhabko, Veremey, 2005; Vorobyov, Ovsyannikov, Suhov, 2006; Gryaznevich, Dejarnac, Ovsyannikov, 2009; Gryaznevich, Dejarnac, Ovsyannikov, 2007].

In this paper the model of the dynamics of the currents in the ITER tokamak at the initial plasma stage is considered. The problems associated with the position and shape of the plasma at later stages of the discharge is considered in [Ovsyannikov, Zhabko, Veremey, 2005; Belyakov, Kavin, Rumyantsev, 1999; Ovsyannikov, Veremey, Zhabko, 2006; Zavadsky, Ovsyannikov, Sakamoto, 2010; Ovsyannikov, Suhov, Vorobev, 2008]. The initial stage includes the preparation conditions for the breakdown of plasma (achieving low scattered field and the required loop voltage), followed by the rise of current in a given scenario.

When modeling the dynamics of the currents are taken into account geometrical and physical parameters of the vacuum chamber and the electromagnetic system of the ITER tokamak, including active and passive superconducting poloidal coils located outside the vacuum chamber [Gluhih, Belyakov, Mineev, 2006; Belyakov, Kavin, Lepihov, 2014].

2 Problem definition

To create an electromagnetic environment that holds the plasma in a tokamak, there is a system of coils, the current in which you can create different configurations of magnetic fields in the vacuum chamber. This paper will be considered poloidal system of conductive circuits of the installation, which will imply coils using electromagnetic system and a vacuum chamber. To build the model elements of the vacuum chamber are broken down into smaller parts (filaments).

The objective is to describe the behavior of the dynamics of poloidal currents in the coils of a tokamak with given initial data. Initial data includes parameters such as the resistance of all circuits, the evolution of the voltages on the coils, the initial currents in the active coils, the inductance (self and mutual) of all contours of the system and the scenario of raising the plasma current.

3 Modeling

Consider the matrix equation describing the dynamics of the currents in the poloidal coils in ITER[Ovsyannikov, Zhabko, Veremey, 2005; Vorobyov, Ovsyannikov, Suhov, 2006; Ovsyannikov, Veremey, Zhabko, 2006; Aminov, Ovsyannikov, 2014]:

$$\frac{d}{dt}I = L^{-1}U(t) - L^{-1}RI - L^{-1}M_pI_p'(t), \qquad (1)$$

where L is the matrix of the inductances (self- and mutual ones) for circuits (electromagnetic coil system and circuits, which can be used as a partition of the vacuum chamber); R is the diagonal matrix of circuit resistance; I is the vector of circuits; U is the vector of the voltages on the sources of circuit supplies whose components are not zero only for depicted in Fig. 1 coils CS3U, CS2U, CS1, CS2L, CS3L, PF1–PF6; I_p is the current in plasma; M_p are mutual inductances of plasma and coils; L_p — the plasma inductance; $R_p(t)$ — the plasma resistance. In this case, the current plasma occurs after completion of preparation for a breakdown and is considered a given, and its resistance is zero.

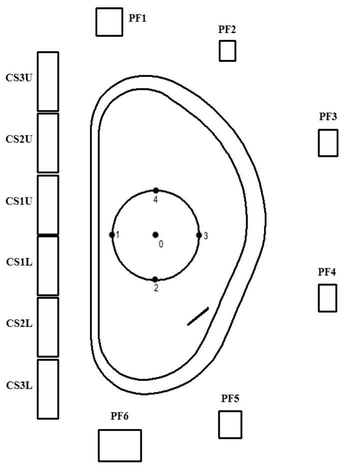

Figure 1. System poloidal coils and configuration of the passive structures in ITER. Point 0–4 are the characteristic points of the breakdown area.

4 Limitations and requirements

For the implementation in tokamak start of the discharge and the subsequent raising of the plasma current should ensure that certain requirements are met.

By the time of breakdown should achieve the required loop voltage $U_{\texttt{loop}}$ passing through the centre of the breakdown area and determined by ratio

$$U_{\texttt{loop}}(t) = 2\pi R_0 E_0 = \\ = \sum_{k=1}^{K} \frac{dI_k}{dt} L_k = \frac{d\Psi(R_0, Z_0)}{dt}, \qquad (2)$$

where K is number of circuits; I_k is current in the k-th circuit; L_k is its mutual inductance with a circular loop that goes through the point (R_0, Z_0); Ψ is magnetic flux; (R_0, Z_0) is the coordinates of the center of the breakdown area; E_0 is the field intensity at the point (R_0, Z_0).

There are some limitations due to physical processes during the start of the discharge, the magnetic field in the area of the breakdown.

Components of the magnetic field at the point (R, Z) from a single ring current, which is located at the point (R_1, Z_1), can be calculated according to the formulas

$$b_r(R, Z, R_1, Z_1) = \frac{\mu_0}{2\pi R} \frac{Z - Z_1}{\sqrt{(R+R_1)^2 + (Z-Z_1)^2}} \times \\ \times \left(\frac{R^2 + R_1^2 + (Z-Z_1)^2}{(R-R_1)^2 + (Z-Z_1)^2} E(k) - K(k) \right), \qquad (3)$$

$$b_z(R, Z, R_1, Z_1) = \frac{\mu_0}{2\pi} \frac{1}{\sqrt{(R+R_1)^2 + (Z-Z_1)^2}} \times \\ \times \left(K(k) - \frac{R^2 - R_1^2 + (Z-Z_1)^2}{(R-R_1)^2 + (Z-Z_1)^2} E(k) \right), \qquad (4)$$

$$k^2 = \frac{RR_1}{(R+R_1)^2 + (Z-Z_1)^2}, \qquad (5)$$

where b_r and b_z is radial and vertical components of the magnetic field and $K(K)$ and $E(k)$ is complete elliptic integrals of the first and second kind.

Radial and vertical magnetic field from the circuit at the point (R, Z) are determined by the ratios

$$B_r(R, Z, t) = \sum_{k \in K} (b_{rk} I_k(t)) = b_r I(t), \qquad (6)$$

$$B_z(R, Z, t) = \sum_{k \in K} (b_{zk} I_k(t)) = b_z I(t), \qquad (7)$$

in which K is the set of indices of the considered circuit, $I_k(t)$ is the current in each k-th circuit. And the total magnetic field from the circuit at the point (R, Z) is equal to

$$B(R, Z, t) = \\ = \sqrt{(B_z(R, Z, t))^2 + (B_r(R, Z, t))^2}. \qquad (8)$$

By the time of breakdown is required to ensure the fulfilment of $B_z(R, Z, t) \leq 2\ mT, B_r(R, Z, t) \leq 1\ mT$ in control points of the breakdown area.

After breakdown ($I_p > 0$) to avoid large displacements of the plasma from the equilibrium position in the vertical direction, the radial magnetic field must still be maintained close to zero. Vertical magnetic field must match the value of the equilibrium magnetic field, defined by Shafranov's formula, and satisfy the balance of the cord over a large radius:

$$B_s(t) = -\frac{\mu_0 I(t)}{4\pi R}\left(ln\left(\frac{8R}{a}\right) + \beta_p + \frac{l_i}{2} - \frac{3}{2}\right). \quad (9)$$

The demand to provide the maximum amount of magnetic flux at the point of breakdown at the moment of breakdown of plasma. Vertical magnetic flux at the point with coordinates (R, Z) from a single ring current with coordinates (R_1, Z_1) is computed according to the ratio

$$\begin{gathered} G(R, Z, R_1, Z_1) = \frac{2\mu_0}{k}\sqrt{RR_1}\times \\ \times\left[\left(1 - \frac{k^2}{2}\right)K(k) - E(k)\right], \end{gathered} \quad (10)$$

where $K(K)$ and $E(k)$ are complete elliptic integrals of the first and second kind, k is calculated according to the formula (5).

Thus, the magnetic flux from the set of circuits can be defined by the following expression:

$$\begin{gathered} \Psi(R_0, Z_0, t) = \sum_{k\in K} I_k(t)\times \\ \times \sum_{m\in M} G(R_0, Z_0, R_m, Z_m), \end{gathered} \quad (11)$$

where K is the set of indices of circuits taken into account in the calculation; M is the set of indices of circuit splits; R_m and Z_m are the coordinates of each such decomposition.

To avoid premature breakdown the aim is to provide up to the moment of breakdown condition

$$\frac{dU_{\mathtt{loop}}(t)}{dt} > 0,\ t \in [t_0, t_{\mathtt{bd}}], \quad (12)$$

where t_0 is the start of the preparation to the breakdown; $t_{\mathtt{bd}}$ is breakdown time.

Also on the entire trajectory is subject to the restrictions on the currents in the circuits, they must not exceed the maximum allowable to set values:

$$|I_k(t)| < I_k^{\mathtt{max}}, t \in [t_0, t_{\mathtt{end}}]. \quad (13)$$

Need not go beyond the limits on voltage power sources, due to their capabilities:

$$|U_k(t)| < U_k^{\mathtt{max}}, t \in [t_0, t_{\mathtt{end}}]. \quad (14)$$

here U_k is voltage on the power supply, $U_k^{\mathtt{max}}$ is maximum voltage at the power supply, K is the set of indices of coils having power supply.

5 Formulation of the optimization problem

To create conditions for breakdown of plasma and raising its current (in accordance with a specified scenario) control system is required, which allows control parameters to influence the electromagnetic conditions in the chamber of the tokamak [Ovsyannikov, Zhabko, Veremey, 2005; Belyakov, Kavin, Rumyantsev, 1999; Gluhih, Belyakov, Mineev, 2006; Zavadsky, Ovsyannikov, Chung, 2009; Belyakov, Kavin, Ovsyannikov, 2010; Aminov, Ovsyannikov, 2014]. The voltages in the active poloidal coils, the initial currents in the coil and ballast resistance in the coil with a resistor are considered as the control parameters.

To solve the problem of ensuring the fulfillment of the conditions for the breakdown on the basis of the model (1), described earlier, which is a system of linear differential equations, it is possible to construct a mathematical model of the optimization program control start discharge in tokamak ITER [Ovsyannikov, Zhabko, Veremey, 2005; Ovsyannikov, Veremey, Zhabko, 2006; Belyakov, Kavin, Ovsyannikov, 2010; Mizintseva, Ovsyannikov, Suhov, 2010]. It should be noted that similar optimization techniques are applied to optimize the dynamics of charged particle beams in the accelerator [Ovsyannikov, Altsybeyev, Durkin, 2014].

The programmed control should solve the problem of creation in the active contours tokamak currents that would ensure the creation of conditions for the breakdown of the plasma, as well as the implementation of all technological constraints and physical requirements.

The functional describing the requirement of maximizing the poloidal magnetic flux in the center of the breakdown area by the time of start of the discharge is:

$$g_1(t) = \frac{1}{(\alpha_\psi^* I(t))^2 + 1}, \quad (15)$$

here α_ψ^* is the pre-calculated vector, such that $\alpha_\psi^*(I(t)$ is a poloidal magnetic flux in the center of the zone breakdown at time t.

The requirement to achieve a given loop voltage by the time of start of discharge in the center of the zone of the breakdown is described by following functional

$$\begin{gathered} g_2(I(t), U(t), R) = (\alpha_\psi^* \dot{I}(t) + 2\pi R_0 E_0)^2 = \\ = (\alpha_\psi^*(L^{-1}U(t) - L^{-1}RI) + 2\pi R_0 E_0)^2, \end{gathered} \quad (16)$$

where E_0 is field intensity at the point (R_0, Z_0), which is the center area of the breakdown.

To account the constraints on the components of the magnetic field in each of a set of control points within the area of breakout at the start of the discharge functionals are used

$$g_3(I(t)) = \sum_{i=1}^{N} g_{3_i}(I(t)), \quad (17)$$

$$g_4(I(t)) = \sum_{i=1}^{N} g_{4_i}(I(t)). \quad (18)$$

Here

$$g_{3_i}(I(t)) = \\ = \begin{cases} 0, -B_r^{\max} < b^*_{r_i} I(t) < B_r^{\max}, \\ (b^*_{r_i} I(t) - B_r^{\max})^2, b^*_{r_i} I(t) \geq B_r^{\max}, \\ (b^*_{r_i} I(t) + B_r^{\max})^2, b^*_{r_i} I(t) \leq -B_r^{\max}, \end{cases} \quad (19)$$

$$g_{4_i}(I(t)) = \\ = \begin{cases} 0, -B_z^{\max} < b^*_{z_i} I(t) < B_z^{\max}, \\ (b^*_{z_i} I(t) - B_z^{\max})^2, b^*_{z_i} I(t) \geq B_z^{\max}, \\ (b^*_{z_i} I(t) + B_z^{\max})^2, b^*_{z_i} I(t) \leq -B_z^{\max}, \end{cases} \quad (20)$$

where $b^*_{r_i} I(t)$ and $b^*_{z_i} I(t)$ is respectively the radial and vertical components of the poloidal magnetic field in the i-th control point; $B_r^{\max}$ $B_z^{\max}$ is maximum value.

The functional responsible for a limit on the maximum current in the coils, has the form

$$\varphi_1(t, I(t), U(t), R) = \\ = \sum_{i=0}^{11} \varphi_{1_i}(t, I(t), U(t), R), \quad (21)$$

in which

$$\varphi_{1_i}(t, I(t), U(t), R) = \\ = \begin{cases} 0, -I_i^{\max} < I_i(t) < I_i^{\max}, \\ (I_i(t) - I_i^{\max})^2, I_i(t) \geq I_i^{\max}, \\ (I_i(t) + I_i^{\max})^2, I_i(t) \leq -I_i^{\max}, \end{cases} \quad (22)$$

and i is the number of the coil.

Restrictions on the maximum voltage on ballast resistance are described as follows

$$\varphi_2(t, I(t), U(t), R) = \\ = \sum_{i=0}^{7} \varphi_{2_i}(t, I(t), U(t), R), \quad (23)$$

where

$$\varphi_{2_i}(t, I(t), U(t), R) = \\ = \begin{cases} 0, -U_i^{\max} < I_i(t)R < U_i^{\max}, \\ (I_i(t)R - U_i^{\max})^2, I_i(t)R \geq U_i^{\max}, \\ (I_i(t)R + U_i^{\max})^2, I_i(t)R \leq -U_i^{\max}, \end{cases} \quad (24)$$

and i is the number of coil; restrictions on the maximum field in the coils are described by the following functional

$$\varphi_3(t, I(t), U(t), R) = \\ = \sum_{i=0}^{11} \varphi_{3_i}(t, I(t), U(t), R), \quad (25)$$

here

$$\varphi_{3_i}(t, I(t), U(t), R) = \\ = \begin{cases} (B_i - B_i^{\max})^2, B_i > B_i^{\max}, \\ 0, B_i \leq B_i^{\max}, \end{cases} \quad (26)$$

i is the number of the coil, $B_i(t) = \sqrt{(b^*_{r_i} I(t))^2 + (b^*_{z_i} I(t))^2}$.

After the breakdown, at the stage of raising plasma current, additionally one introduces the following functional:

$$\varphi_4(t, I(t), U(t), R) = \\ = \sum_{1}^{5} \int_{t_{\text{bd}}}^{t_{\text{end}}} (B_{z_i}(t) - B_s(t))^2 dt, \quad (27)$$

where $B_s(t)$ corresponds to (9).

Thus, accounting for all constraints used functional

$$J = g(I(t), U(t), R) + \\ + \int_0^{t_{\text{end}}} \varphi(t, I(t), U(t), R) dt = \\ = \sum_1^4 c_k g_k(I(t), U(t), R) + \\ + \sum_1^5 \int_0^{t_{\text{end}}} c_k \varphi(t, I(t), U(t), R) dt. \quad (28)$$

Minimization of the functional (28) will be called the problem of optimization programmed (estimated, allocated) control.

6 Conclusion

Based on the above physico-mathematical model for the dynamics of poloidal currents in the circuits of the camera tokamak with all the restrictions and requirements imposed on the system we created a special software in the programming language C++, which allows to simulate the behavior of the currents in the electromagnetic system of the ITER tokamak in the initial stage of the discharge, and with the current technological limitations and the physical requirements to analyze the achievement of conditions of breakdown and regularity of the rise of the plasma current in accordance with a given scenario.

As input data for simulation we take the voltage on the control coils (as an example Fig. 2 present the evolution of the voltage in the coil PF6), the initial currents and resistance in the control coils and electrical parameters, namely, inductance and resistance, which are calculated in advance based on the geometrical conditions of installation and materials that will be used for its production. Also specified scenario of plasma current at the stage of his recovery.

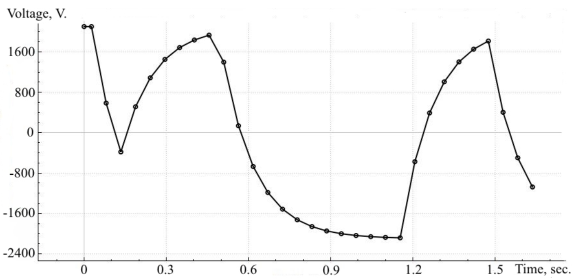

Figure 2. The voltage in the control coil PF6

Fig. 3, Fig. 4, Fig. 5 present some results of calculations of the program. Selected mode of operation of the tokamak ensures conditions for the breakdown of the plasma, the subsequent rise of plasma current for a given scenario, the line after the breakdown of vertical equilibrium and magnetic fields, as well as the implementation of all technological constraints and physical requirements.

Developed software currently allows to simulate the dynamics of the currents in the electromagnetic system of the tokamak, and analyze the results. The initial data are chosen from the set of valid values, and given that the variety is large and the selection is carried out "manually", then the task of searching such a mode of operation of the tokamak which would run all imposed on the system requirements is non-trivial and time-consuming. It's also worth noting that whenever you make any changes to the system (whether the changing scenario of the rise of the plasma current, any technological change, etc.), initial data will have to be regenerated. Thus, there is the problem of automated search (optimization) of the initial data and offices.

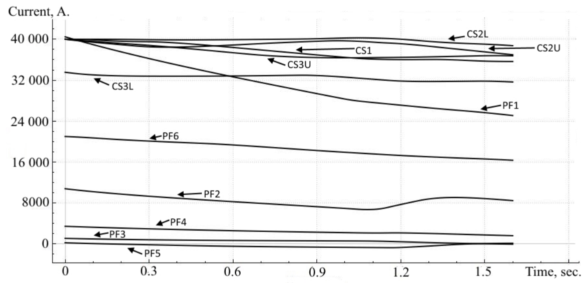

Figure 3. The currents in the control coils

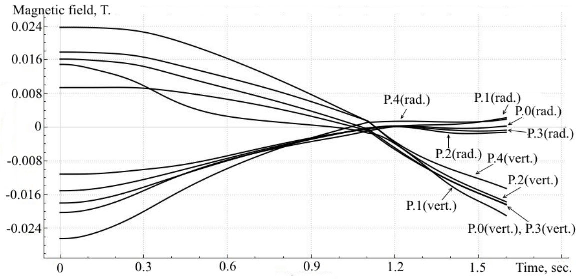

Figure 4. Vertical (vert.) and radial (rad.) the magnetic field in 5 control points

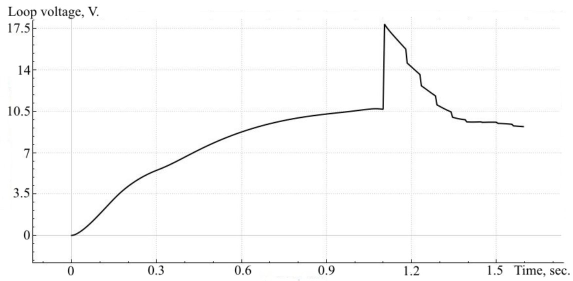

Figure 5. The loop voltage in the centre of the breakdown area

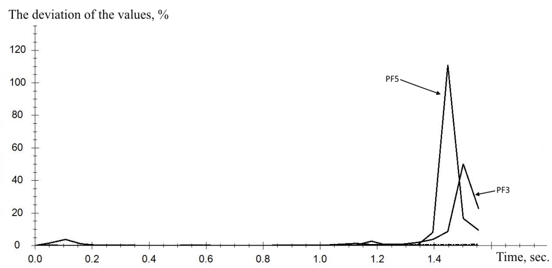

Figure 6. The deviation of the values of the currents in the coils CS3U, CS2U, CS1, CS2L, CS3L, PF1—PF6

To check the produced program the results of calculations were compared with the given AO "D. V. Efremov Institute of Electrophysical Apparatus". In Fig. 6 depicted deflection currents (obtained in the calculations with those that were given as a test) in the control coils CS3U, CS2U, CS1, CS2L, CS3L, PF1—PF6. "Jump" for coils PF3 and PF5 are valid as at the moments when they occur, the currents in them are close to zero and therefore even a small difference in the values gives this effect.

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