PHYSCON 2009, Catania, Italy, September, 1–September, 4 2009

PARAMETRIC OPTIMIZATION FOR TOKAMAK PLASMA CONTROL SYSTEM

Sergey Zavadsky
Faculty of Applied Mathematics and Control Processes
Saint-Petersburg State University
Russia
sergzav@pochtamt.ru

Alexander Ovsyannikov
Faculty of Applied Mathematics and Control Processes
Saint-Petersburg State University
Russia
ovs74@mail.ru

Noburu Sakamoto
University of Nagoya
Nagoya
Japan

Abstract

In order to design the control system for plasma current, shape and position the structural parametric optimization of transient processes is suggested. Optimization approach to plasma dynamic is based on the consideration of transient processes of the full-sized control object that is closed by a regulator of a decreased dimension. It is suggested to use an integral performance criterion as a functional that allows optimizing the transient process, perturbed at the initial point set and the set of external disturbances. In the framework of this approach the optimization of plasma dynamics of the ITER tokamak is given.

Key words

Tokamak, plasma, control system, optimization.

1 Introduction

The problem of control plasma in nuclear reactor tokamak occupies the leading place in controlled thermonuclear fusion. The main task in this area is a plasma feedback control system design. Problems of analysis and synthesis of stabilizing regulators of current, position and plasma shape in tokamak are of great importance.

The mathematical model of ITER tokamak plasma control system is a very complex object which includes various subsystems and differential equations that define plasma behavior. The structural schema of ITER control system links external disturbances, plasma state equations, filters system, vertical controller, current and shape controller, power system and set of diagnostic signals. Mentioned schema can be represented using the structural diagram in Fig. 1. Matrices of these subsystems are known with constant components.

The plasma state equations are done based on the linearization of differential equations that define plasma behavior in deviation of equilibrium position, where $x_{st} \in E^{67}$ is the state space vector, $u_p \in E^{11}$ is the control voltages vector, $y_s \in E^{18}$ is the diagnostic signals vector, $e \in E^7$ is the measurement variables vector. Measurement vector includes deviation of plasma current and six checked clearances between plasma and tokamak chamber, which signed as $g_1, ...g_6$ and called *gaps*. The function $f(t)$ is external plasma disturbance, which is called $l_i, \beta - drops$ disturbance and is defined in the following form

$$f(t) = \begin{pmatrix} w_1(t) \\ w_2(t) \end{pmatrix}, \quad w_1(t) = d_\beta e^{-(t/t_\beta)}, \ \ w_2(t) = d_l e^{-(t/t_l)}, \tag{1}$$

where d_β, d_l, t_β, t_l are known real numbers. Power and filters system are given and defined by construction features of tokamak. To plasma shape stabilize the control object is closed with a shape controller of a decreased dimension with the following structure:

$$\begin{aligned} \dot{x}_c &= A_c\, x_c + B_c\, y_f \\ u &= C_c\, x_c \end{aligned} \tag{2}$$

where vectors $u \in E^{11}$, $y_f \in E^{18}$ are the control voltages and the diagnostic signals of the tokamak control system respectively, matricesA_c, B_c, C_c are the

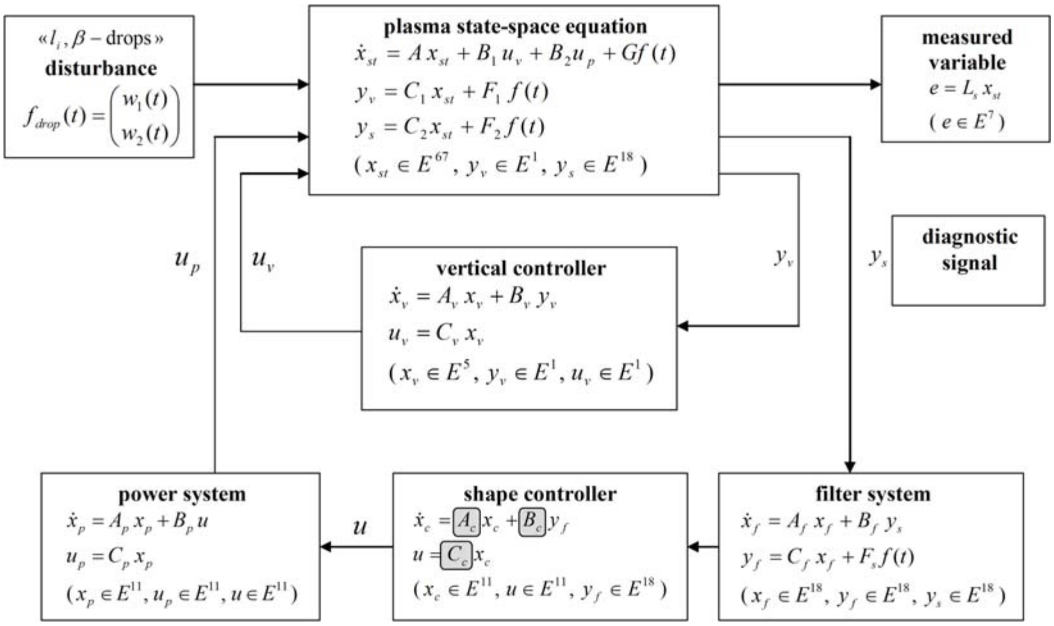

Figure 1. Structural model of ITER plasma control system.

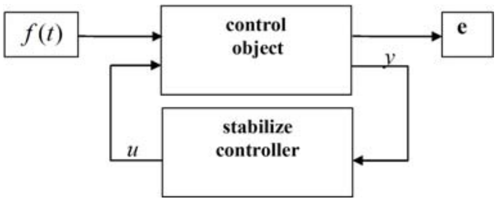

Figure 2. The control object closure by the obtained regulator.

constant matrices of the controller, which must be obtained. The control object closure by the obtained regulator is done in accordance with the scheme in Fig.2.

By the "regulator synthesis" we mean such a choice of component of controller matrices that gives us a stable closed object and sufficient quality of stabilization. The stabilization quality performance base on the numerical characteristics such as the integral of squared gaps

$$I_{gaps} = \int_0^T \sum_{i=1\ldots6} g_i^2(t)\ dt \tag{3}$$

and the settling time

$$t_{\text{settling}} = \min_{\widetilde{t}} \{\widetilde{t} : \max_{i=1\ldots6} |g_i(t)| \le 0.01, \quad \forall t \ge \widetilde{t}\}, \tag{4}$$

Also, we need take into account the nonlinear amplitude constraints on control voltage signals. For example, the ITER tokamak has 11 control coils with constraints on voltage amplitudes. These constraints have various numerical values and can be represented in the following form

$$u_i = \begin{cases} c_i, & u_i > c_i \\ u_i, & |u_i| \le c_i \\ -c_i, & u_i < -c_i \end{cases}, \ i = 1, ..., 11\ , \tag{5}$$

where i is index of coil, c_i is amplitude maximum for i-th coil. It complicates the design of the controller and allows no simple raise in the magnitude of control voltage signals.

Note that the controller design is analyzed by means of the linear model, however this controller is tested on the nonlinear model, and has to possess appropriate characteristics.

2 Parametric Optimization Method

In order to design the control system for plasma current, shape and position the structural parametric optimization of transient processes is suggested. In the framework of this approach, the optimization of transient processes of the full-sized control object that is closed by a regulator of a decreased dimension is conducted. It is suggested to use an integral performance criterion as a functional that allows optimizing the transient process, perturbed at the initial point set and the set of external disturbances.

As a rough approximation for optimization the shape controller can be obtained, for example, using the reduction procedure and the LQG-optimal synthesis, see [4-10] for more details. So, let us investigate the equations of the control object presented in Fig.1 with constantly applied perturbation (1), which is closed by the regulator of decreased dimension (2) and the another subsystem of the structural diagram. We combine the structural diagram of control system into a system of linear differential equations with disturbances. To do that, let us introduce the following vectors and matrices: extended state-space vector $x = \{x_{st},\, x_v,\, x_p,\, x_f,\, x_c\}$ that includes plasma states, vertical controller states, power system states, filter system states and shape controller states; matrices P and N with constant components such what P is a matrix of the linear part of the system mentioned above, and N is the coefficient of the non-linear part; these matrices can be easily identified in Fig.1; the matrices L and K for linear combinations with extended state-space vector. Further, the elements of matrices $A_c,\, B_c,\, C_c$ of the dynamic shape controller will be taken as parameters that are to be optimized and combined into a vector of parameters

$$p = \{p_k\} \longleftrightarrow \{A_c,\, B_c,\, C_c\}\,. \qquad (6)$$

So, by using the newly introduced variables, we represent the structural diagram of the control system shown in Fig.1 in the following form:

$$\begin{array}{l} \dot{x} = P(p)\,x + N(p)\,f(t), \\ x(0) = x_0 \\ f(t) = f(d_\beta, t_\beta, d_l, t_l,\ t), \\ e = L\,x(t,\, x_0,\, p), \\ u = \ K(p)\,x(t,\, x_0,\, p), \end{array} \qquad (7)$$

where $x \in E^{112}$ is the extended state space vector, $e \in E^7$ is the measurement variables vector, $u \in E^{11}$ is the control voltages vector, $P,\, N,\, L,\, K$ are the above introduced constant component matrices, $f(t)$ is the $l_i, \beta\, -\, drops$ disturbance, $p = \{p_k\}$ is a vector of parameters. Note, that the matrices $A_c,\, B_c,\, C_c$ of a designed regulator (2) will be taken as parameters that are to be optimized. We combine the elements of these matrices into a vector of parameters $p = \{p_k\}$, where each parameter has it own index. By labeling it $P(p)$ and $N(p)$ we emphasize that it depends on the parameters that are being optimized. Based on this differential system (7) we have measurement variables vector e and control voltages vector u.

It is suggested to use the following integral performance criterion that allows optimizing the transient processes perturbed by the initial point and external disturbance

$$I(p) = \int_0^T \{e^*(t)\,Q\,e(t)\, +\, u^*(t)\,R\,u(t)\,\}\ dt + \\ +\ e^*(T)\,Q_1 e(T) \to \min, \qquad (8)$$

where Q, R, Q_1 are symmetrical weight matrices. The minimization algorithm of this functional by parameters $p = \{p_k\}$ is suggested bellow. Let entered additional differential equation

$$\begin{array}{l} \frac{d\psi}{dt} = -P^*\psi + 2\,(L^*Q\,L + K^*R\,K)^* x(t), \\ \psi(T) = -2\,L^*Q_1\,L\,x(T). \end{array} \qquad (9)$$

Then, using vector $\psi(t)$ we obtain a representation for the gradient of the functional

$$\frac{\partial I(p)}{\partial p_k} = -\left(\int_0^T \psi^*(t)\ \left(\frac{\partial P}{\partial p_k} x(t)\, +\, \frac{\partial N}{\partial p_k} f(t)\right) - \right. \\ \left. -2x(t)^* \frac{\partial K^*}{\partial p_k} R\,K(p)\,x(t)\ dt \right) \qquad (10)$$

Based on the analytical expressions (6)–(10) a gradient optimization method for the functional (8) with respect to the parameters $p = \{p_k\}$ is implemented for C++ and MatLab environments.

3 Optimization Results

The optimization results are considered based on transient processes of checked clearances – *gaps*, which are the members of the measurement variables vector $e \in E^7$ for the control object (7). The modeling of transient processes of the control object closed with the optimized and initial controller is shown. This initial controller is obtained using the approaches described in [4,10] and already possesses proper characteristics. The graphical results of optimization are presented in Fig.3 and Fig.4. The Fig.3 shows control voltage signals $u_1, ...u_{11}$ for control object closed with initial and optimized controllers and Fig.4 shows transient processes of gaps $g_1, ..., g_6$ for control object closed with initial and optimized controllers of the nonlinear model with voltage limitations.The Fig.3 shows that by using equation for u_i we can choose maximums of control voltages according to their limitations, then raise them and obtain better performance for measurement variables e_i. This is illustrated in Fig.4. Also, we consider optimization numerical characteristics the integral of squared gaps and the settling time mentioned above which are presented in Table 1.

4 Conclusion

This work is dedicated to questions concerning synthesis and optimization of tokamak plasma control system. The structural model of ITER plasma control system is discussed and the structural parametric optimization method is suggested. We combine the structural diagram of control system into differential equations system with disturbances. The estimate for the transient process ensemble dynamics is proposed. Based

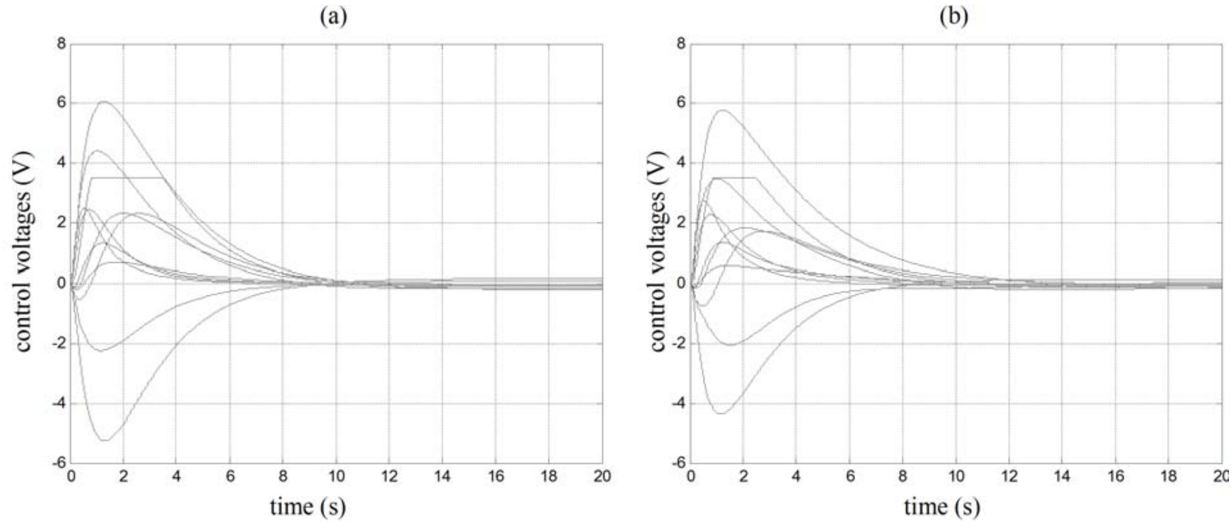

Figure 3. Control voltage signals $u_1, \ldots, u_{11}$ corresponding to a) initial controller and b) optimized controller.

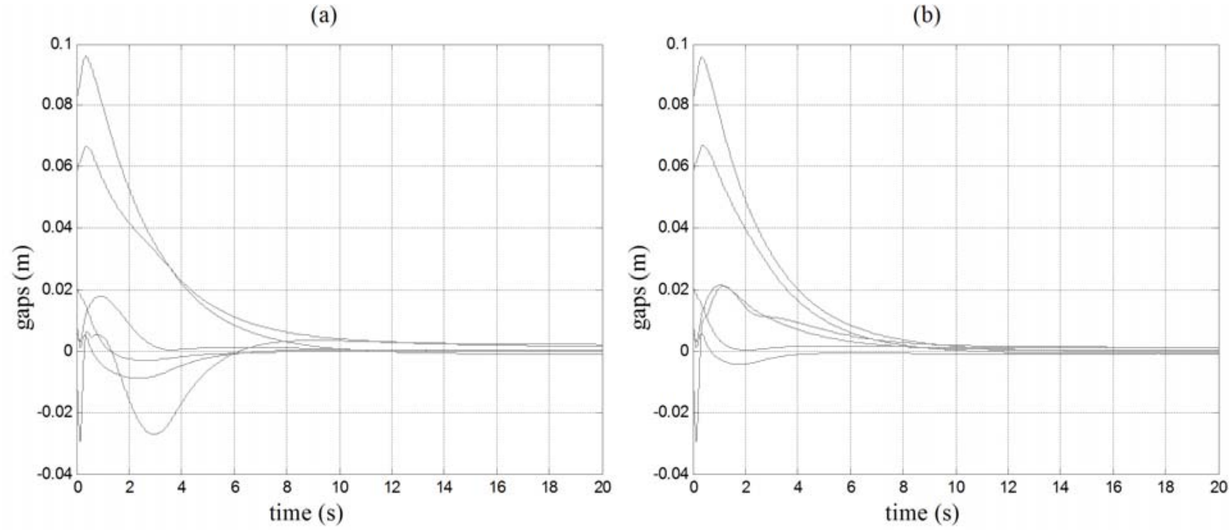

Figure 4. Transient processes of gaps $g_1, \ldots, g_6$ corresponding to a) initial controller and b) optimized controller.

Table 1. Settling time and integral of squared gaps for control object closed with initial and optimized controllers.

Control object closed with initial controller	Control object closed with optimized controller
$I_{gaps} = 0.027$	$I_{gaps} = 0.024$
$t_{settling} = 6.373$ sec.	$t_{settling} = 5.634$ sec.

on the analytical expressions a gradient method of optimization is implemented for C++ and Matlab environments. Results of the computations are obtained and discussed. Numerical characteristics such as the integral of squared gaps and the settling time are presented for the both initial and the optimized controllers. For the optimized controller the squared gaps and settling time are 11% and 12% lower, correspondingly.

References

1. D. A. Ovsyannikov, E. I. Veremey, A. P. Zhabko et al. *Mathematical methods of plasma vertical stabilization in modern tokamaks* // Nuclear Fusion. 2006. Vol 46. P. 652-657.

2. G. J. McArdle, V. A. Belyakov, D. A. Ovsyannikov, E. I. Veremey. *The MAST plasma control system* // Proc. 20^{th} Intern. Symp. on Fusion Tecnology SOFT'98. Marseille, France. 1998. P. 541–544.

3. D. A. Ovsyannikov, A. P. Zhabko, E. I. Veremey et al. *Plasma current and shape stabilization with control power reducing* // Proc. of the 5th Intern. workshop: "Beam Dynamics & Optimization". St.Petersburg, 1998. P. 103–111.

4. D. A. Ovsyannikov, E. I. Veremey, A. P. Zhabko et al. *Mathematical methods of tokamak plasma*

shape control // Proc. of the 3th Intern. workshop: ”Beam Dynamics & Optimization”. St.Petersburg, 1996. P. 218–229.
5. B. A. Misenov, D. A. Ovsyannikov, A. D. Ovsyannikov et al. *Analysis and Synthesis of Plasma Stabilization Systems in Tokamaks* // Control Application of Optimization: Preprints of the Eleventh IFAC Intern. Workshop. St.Petersburg, 2000. P. 249–254.
6. B. A. Misenov, A. D. Ovsyannikov, D. A. Ovsyannikov et al. *Non-linear model of tokamak plasma shape stabilization* // Intern. conf. on Informatics and Control (ICI&C’97). St.Petersburg, 1997. P. 382–388.
7. B. Misenov. *Computational aspects of plasma shape control synthesis problem* // Proc. of 2nd Intern. Workshop ”Beam Dynamics and Optimization”. St.Petersburg, 1995. P. 138–145.
8. N. F. Kirichenko. *Introduction to the motion stabilization theory*. Kiev, 1978, in Russian.
9. D. A. Ovsaynnikov. *Modeling and optimization of charged particle beam dynamics*. St.Petersburg, 1990, in Russian.
10. E. I. Veremey, N. A. Zhabko. *Plasma current and shape controllers design for ITER-FEAT tokamak* // Book of abstracts of Workshop on Computational Physics Dedicated to the Memory of Stanislav Merkuriev. St.Petersburg, 2003. P. 52.