LINEAR CONTROL MODELS FOR GUTTA TOKAMAK

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Abstract—**Software package for constructing linear models of plasma for various tokamaks is under consideration. Results of comparison of different linear models built using this package for ITER and Gutta tokamaks are presented.**

I. Introduction

New experiments on plasma in thermonuclear devices such as tokamaks cannot be carried out without control systems for plasma parameters. Such systems may have different ideology and complexity, but all of them involve feedback control of plasma location, current and shape [1, 2, 3, 4].

Main directions in creating such feedback control system may be reduced to construction of simplified model that describes dynamics of plasma parameters, and selection of control laws (regulators) which allow to stabilize these parameters.

The need of achieving almost critical values of plasma parameters in experiments issues the challenge of creating effective control systems for plasma parameters. The quality of control system is most critical for large devices with high-energy plasma, where problems in discharge may lead to substantial decrease of service life and damage the unit. The problem of optimization of programmed control for voltage on poloidal currents and the problem of optimization of feedback control can be solved only by virtue of the mathematical model of plasma filament capable of describing the reaction of the filament on external disturbances and control actions with sufficiently high precision.

Rapid progress in computer technology extends ability in analysis and increases complexity of the plasma control systems in tokamaks.

II. Description of the Software Package

The construction of linear model can be divided into 4 stages.

1. construction of geometrical model of tokamak;
2. calculation of matrices of inductivities and resistances of the circuits;
3. reconfiguration of database for plasma steady states;
4. construction of linear model matrices based on the equilibriums' database;

Before starting the calculation of linear model, it is necessary to represent the geometry of the most important, from the point of view of stabilization of plasma, conducting structure of tokamak in the form of PET code that will be used later on reconfiguration of database for plasma equilibrium.

Since the whole construction is axially symmetrical, we will analyze the section by the (r, z) plane that passes through the device axis—the poloidal section of a tokamak.

The passive structure of the tokamak has to be divided into several circuits, whose induced currents together with the current in control windings are the states in the linear model. The number of circuits that the passive structure is being divided into is chosen so that the transient processes that are being described by the linear model would not be so dependent of digitalization.

The section of each circuit that is included in the linear model can be geometrically presented by one (active circuit) or several (walls of vacuum chamber) rectangles. The division of circuits that refer to the walls of the vacuum chamber into several rectangles allows to approximate the geometry of chamber walls more precisely.

The programs for calculation of geometry make the first part of the software package that is discussed here.

A very important stage of construction of the linear model is to choose of the base equilibrium. In some cases, the base equilibrium can be known in advance as a result of finding solutions of other modeling problems (as, for example, for ITER tokamak). In other cases, for given plasma parameters, an inverse problem of equilibrium is being solved and the currents of control coils are being calculated (as it was done for Gutta tokamak). Notice that the inverse problem was solved with the use of DIALEQT-C code.

In order to calculate plasma equilibrium near the base equilibrium, direct problem of equilibrium has to be solved. Let us assume that coil currents and physical parameters of plasma (currents, "beta poloidal", etc.) are known. It is necessary to restore the magnetic surface for plasma equilibrium. According to these calculations a linear model of plasma behavior near the equilibrium point will be constructed.

Let's write the Grad-Shafranov equation and use it as the boundary conditions at infinity and at the symmetry axis. Then the problem is given by [5]:

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$$\Delta^* \psi = \begin{cases} -\dfrac{8\pi^2}{c} r j_\varphi \text{ inside } S_p \\ -\dfrac{8\pi^2}{c} \sum\limits_{i=1}^{L} r I_i \delta(r - r_i)\delta(z - z_i) \text{outside } S_p, \end{cases}$$

where $\psi \to 0$ when r or $z \to \infty$ and when $r \to 0$.

Here L is the number of external stationary conductors, (r_i, z_i) and I_i are the location and the current intensity of these conductors respectively, δ is the Dirac delta-function. In our case, the delta-function describes current distribution: at points $(r_i, z_i), i = \overline{1, L}$ the values of currents equal to I_i, there are currents in other points outside of the chamber.

The position of plasma border S_p is not given in such problem statement and is determined by the problem solution. Because of that it is always non-linear even in cases when the right part of the equation is linear.

The code PET was used in order to find the solution of the equilibrium problem.

The A, B, C, D representation of control object equation is the most common one to be used in the theory of regulator synthesis. In our case it is:

$$\begin{cases} \dot{x} = Ax + B\,\delta U + AE\,\delta\xi \\ \vec{g} = Cx + D\,\delta U + (CE + F)\,\delta\xi \end{cases}$$

Here x is the state vector (contains currents in active and passive structures), g is the observation vector (describes the deviation of different plasma parameters that are being observed form their equilibrium values), δU is the voltage vector, $\delta\xi$ is the disturbance vector. Notice that in our case, D is a null matrix.

The software of the discussed package is used to calculate the matrixes of the linear model.

III. Construction and Comparison of Linear Models of Various Orders for ITER

The first problem solved using the software package was building linear models for ITER.

The schematic model of ITER that was used during linear models construction is depicted in Fig. 1.

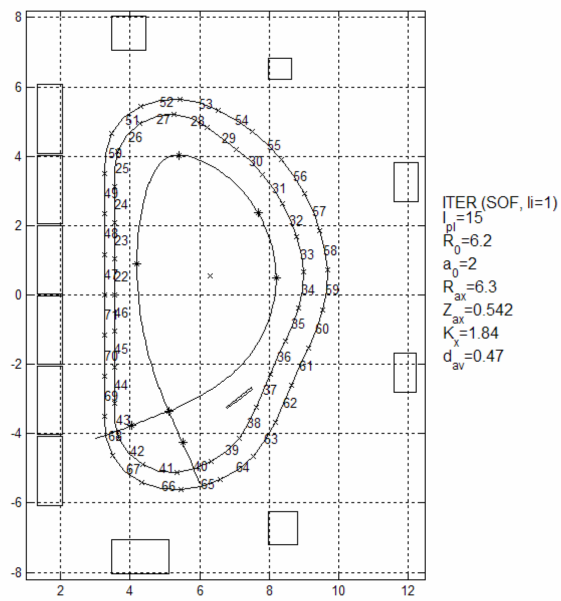

Fig. 1. Poloidal section of the ITER

Models with various degree of division of passive elements of the tokamak were computed (for divisions into 131 and 71 circuits) and then their comparison was performed.

For base equilibrium one of the standard ITER plasma equilibriums was selected (see fig. 1).

For comparison of two linear models with different degree of division of passive structures (into 71 and 131 elements) positive eigenvalues of A-matrices were computed for each model as well as the corresponding eigenvectors.

As a result the only positive eigenvalue for linear model corresponding to 131 circuit is 12.44, and for linear model corresponding to 71 circuit – 12.541.

Resume:

1. Difference between positive eigenvalues of A-matrices for different models is less than 1%.
2. Double-ply increase of passive contours does not result in essential increase of model parameters accuracy, so for practical computations it is enough to use the model with 71 contours.

IV. Construction and Comparison of Linear Models of Various Orders for Gutta

Geometry of the Gutta tokamak is shown at Fig. 2.

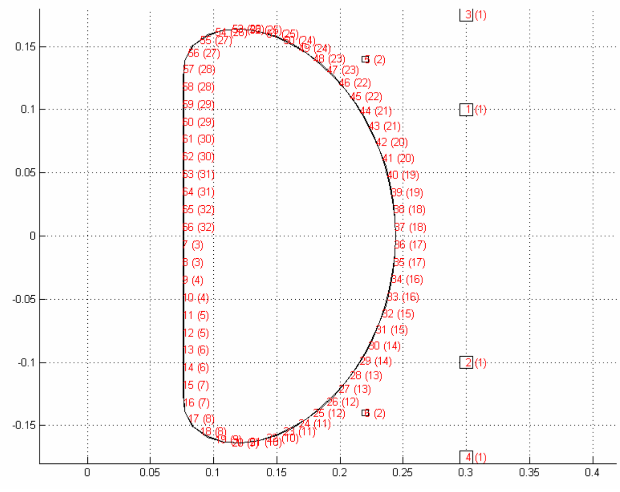

Fig. 2. Poloidal section of the Gutta

For linear model construction the following plasma parameters were chosen:

TABLE I
REFERENCE EQUILIBRIUM PARAMETERS

Parameter	Value
Minor radius	7.9 cm
Major radius	16.2 cm
Elongation	1.33
Triangularity	0.08
Poloidal beta	0.2
Internal inductance	0.87
Plasma current	25 kA

For these prescribed plasma parameters inverse equilibrium problem was solved using DIALEQT-C code so the currents in the control coils corresponding to reference equilibrium were computed.

Models of order 103, 93, 83, 73, 63, 53, 43, 33, 23, 21, 18, 16 and 13 that were built using considered software package were compared. State vectors for all these models represent deviations of currents in the control coils (Gutta contains 2 independent active coils), currents in passive structures and plasma current from the reference equilibrium. For all models the command vector $x \in E^2$ contains 2 elements that correspond to voltage on control coils, and the vector of observations $y \in \mathrm{E}^{14}$ that contains deviations of plasma boundary from 5 control points (5 gaps), radial and vertical coordinates of plasma center, major and minor radiuses, plasma triangularity and elongation, currents in the control coils, plasma current and electrical power on the control coils.

The central question is to see how models of different orders agree. Such MIMO-models can be compared using some characteristics of the whole system, which does not depend on number of inputs and outputs. For this purpose for MIMO-system with transmission matrix P(s) one can take singular characteristic – frequency portrait of maximum singular number of matrix $P(j\omega)$, defined as $\bar{\sigma}(\omega) = \max_i \lambda_i\left(P(j\omega)P^*(j\omega)\right)$, where λ_i's are the eigenvalues of the matrix. It is known that maximum singular number of the matrix $P(j\omega)$ can be treated as a generic amplification coefficient of the system. Singular characteristics computed for all considered linear models in the frequency operating band the $\omega \in \left[1, 10^6\right]$ rad/s are shown in Fig. 3. Assume that linear model of highest order (103) characterizes dynamics of the system most closely. However such high order entails problems during the investigation of the dynamical properties of the system. As a result it is necessary to choose linear model of lower order. Figure 3 shows that the reduction of order below 23 leads to essential deviation of the corresponding singular characteristic from the singular characteristics that correspond to higher orders.

As a result the model of order 33 was selected as the base model, which provides compromise between order of the model and closeness of the singular characteristic of the model to singular characteristics of the higher-order models. Fig. 4 shows absolute deviations of singular characteristics for different models from the singular characteristic of the base model in the working frequency range.

Results of the simulation in the MATLAB-Simulink framework also show consistency of considered models. In the simulation the perturbations of types l_i, β-drops, which simulate discontinuous variation of integral characteristics of plasma during discharge, were used. Fig. 5 shows how models of different order suppress such perturbations, plasma current is shown as output. One can see that in such situation all linear models behave in a similar way.

Moreover, Fig. 6-7 depict amplitude-frequency responses from two inputs of the system to the output that corresponds to the plasma current.

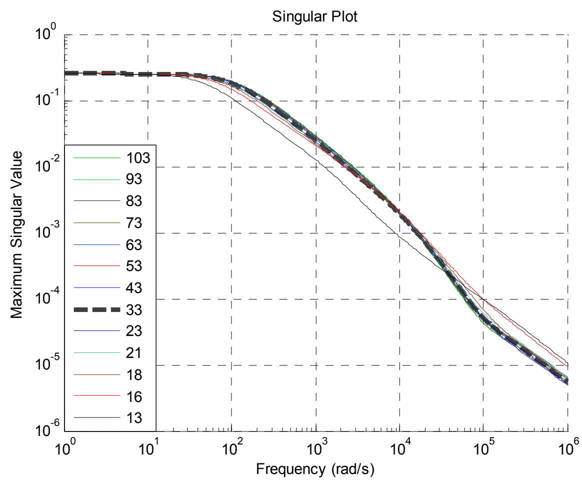

Fig. 3. Dependency ofmaximum singular value on frequency

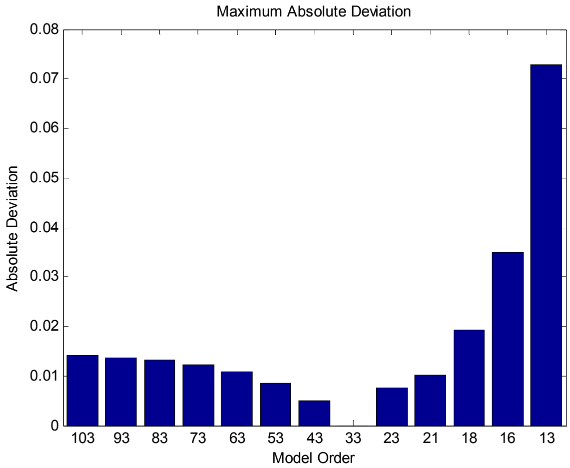

Fig. 4. Deviations of singular characteristics for different models from singular characteristic of the base model in the working frequency range

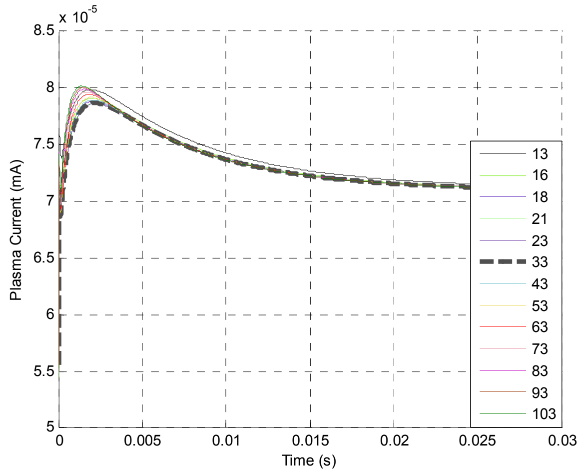

Fig. 5. Reaction of different models on the perturbations

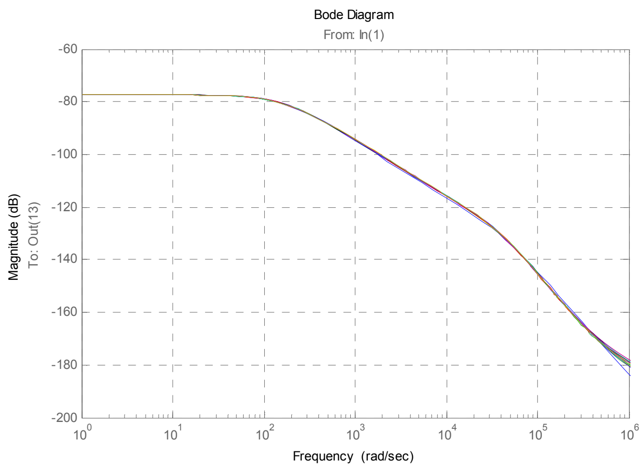

Fig. 6. Amplitude-frequency responses from input #1 of the system to the output that corresponds to the plasma current.

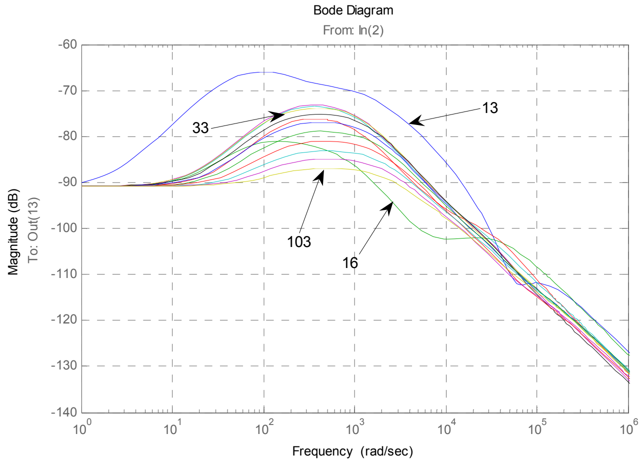

Fig. 7. Amplitude-frequency responses from input #2 of the system to the output that corresponds to the plasma current.

V. Conclusion

Presented software package is useful for construction of linear models for plasma current and shape control on various tokamaks. It iss significant that for Gutta tokamak linear models were constructed for the first time.

The complex also includes educational materials that help in preparing skilled specialists in the field of plasma control.

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