DESIGN AND STABILITY OF ADAPTIVE SWITCHED SYSTEM

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Abstract
Adaptive control and stability problems for a certain class of SISO switched systems are considered. The dynamic system has piecewise constant parameter disturbances which change in arbitrary and unknown switching times. An interval between neighbouring switches is more than transition time. The system has the feedback on derivatives of the output variable. The stability problem is studied with the help of Lyapunov's function. A numerical example is given.

Key words
Discontinuous systems, nonlinear control.

1 Introduction

By a switched system, we mean a dynamical system consisting of a family of continuous-time subsystems and a rule that orchestrates the switching between them [Liberzon and Morse, 1999]. In common case the orchestrate rule can be unknown. Many systems exhibit switching between several subsystems that is dependent on various environmental factors. Some examples of such systems are discussed in [Wicks et al., 1998 and Mancilla-Aguilar, 2002]. Switched systems have numerous applications in the control of mechanical systems, automotive industry, aircraft control and many other fields. A survey of basic problems on stability and design of switched systems has been proposed in [Liberzon and Morse, 1999; Davrazos and Koussoulas]. There have been many researches about robust stabilization [Zhai et al., 2003; Mancilla-Aguilar, 2002; Narendra et al., 1994; Branicky, 1994]. The main approach to stability study presented in the literature for switched systems is Lyapunov's theory [Zhai et al., 2003; Narendra et al., 1994; Branicky, 1994]. Usually, if we choose Lyapunov's function approach, then we use either the technique of the common Lyapunov's function or the technique of multiple Lyapunov's functions [Davrazos and Koussoulas]. In this work an adaptive control and stability problems for SISO systems are considered. The dynamic system has piecewise constant parameter disturbances and arbitrary switching sequence. We suppose the time interval between neighbouring switches is more than transition time. For switched system control we design an adaptive controller by the method of derivative of higher order [Vostrikov, 1995, Vostrikov and Shpilevaya, 2004]. The system stability is studied by Lyapunov's method. A numerical example is presented to illustrate the system properties.

2 Problem Statement

Consider a class of SISO switched systems under arbitrary switching sequence described by the following equation

$$y^{(n)} + \sum_{i=0}^{n-1} a_i^j y^{(i)} = \sum_{l=0}^{m} b_l^j u^{(l)}, \qquad (1)$$

where y,u - output and input variables accordingly; $b_0^j \neq 0$, $\operatorname{sgn} b_l^j b_l^{j+1} = 1$ for $j = 1,2,3,\ldots,\overline{j}$, $b^j(p) = \sum_{l=0}^{m} b_l^j p^l$ - Hurwitz's polynomial of degree m, p - the operator of differentiation; $n > m$. For uncontrolled parameters a_i^j, b_l^j we enter a disturbance vector $\rho^j = \left(a_0^j,\ldots,a_{n-1}^j, b_0^j,\ldots,b_m^j\right)^T$, which satisfies the following conditions

$$\left|\rho_s^j(t)\right| < \varepsilon_s^j, \left|\dot{\rho}_s^j(t)\right| < \delta_s^j, \varepsilon_s^j, \delta_s^j = const < \infty,$$
$$\rho_s^j(t) = \rho_{os} + \overline{\rho}_s^j, \overline{\rho}_s^j = d\,\tilde{\rho}_s^j, d = const > 0, \qquad (2)$$

where $\tilde{\rho}_s^j = \tilde{\rho}_s^j(t)$ - a smooth function for $t_j < t < t_{j+1}$, $t_{j+1} = t_j + \tau_j$, $t_s \le t_j < t_f$ $\left(t_f - t_s\right) >> \tau_j > t_n$. Here τ_j, t_n - active time of *j*th subsystem and transition time conformably, t_s, t_f - initial and final time moments, t_j - a moment of jth switching. We suppose that ρ_{os} is known, $s = 0,1,2,3,\ldots$. The control purpose is the output regulation:

$$\lim_{t\to\infty} |r - y(t)| < e_s \quad ,$$

on assumption zero initial conditions

$$y^{(i)}(t_s) = 0,\, i = \overline{0, n-1},$$

where r is a constant reference signal, e_s - an allowable static error. The objective of this paper is the design and the stability study of a control system with the signal adjustment.

3 Main results

The adaptive system will be synthesized with using referents equation method and method of the derivative of higher order. We will study the system stability by the second method of Lyapunov.

3.1 Adaptive switched system design

According to (2), the system (1) is transformed to

$$y^{(n)} = -\sum_{i=0}^{n-1} a_{0i} y^{(i)} + \sum_{l=0}^{m} b_{0l}\, u^{(l)} + m^j , \qquad (3)$$

where $m^j(t)$ - a new uncontrolled disturbance vector, $m^j = \left[\bar{\rho}^j\right]^T \theta$, with the bounded norm and bounded first derivative for $t_j < t < t_{j+1}$,

$$\left|m^j\right| < \delta_{1m}^j , \left|\dot{m}^j(t)\right| < \delta_{2m}^j . \qquad (4)$$

Here $\delta_{rm}^j = const < \infty$ and θ is a measurement vector,

$$\theta^T = \left(-y, \ldots, -y^{(n-1)}, u, \ldots, u^{(m)}\right).$$

Let a reference model be described by the equation

$$\begin{aligned} y^{(n)} &= -a_{n-1}^* y^{(n-1)} - a_{n-2}^* y^{(n-2)} - \ldots - a_0^* y + \\ &+ a_0^* r = F(y^{(n-1)}, y^{(n-2)}, \ldots, y, r). \end{aligned} \qquad (5)$$

The coefficients (a_i^*) of the equation (5) are received according to the given quality performance of the transient. Equate right parts (3), (5), then replace unknown disturbance $m^j(t)$ on an adjustable coefficient (k_m) and defind a control law,

$$b_0(p)u = \left(-\alpha^*(p) + \alpha_0(p)\right) y + k_m + a_0^* r , \qquad (6)$$

where $\alpha^*(p) = \sum_{i=0}^{n-1} a_i^* p^i$, $\alpha_0(p) = \sum_{i=0}^{n-1} a_{0i}\, p^i$, $b_0(p) = \sum_{l=0}^{m} b_{0l}\, p^l$ - Hurwitz's polynomial, $p^h = \dfrac{d^h}{dt^h}$. Since $m^j(t)$ depends on θ, it is necessary to synthesize an adjustment algorithm which should be fast enough. Therefore, we choose the method of derivative of higher order [Vostrikov and Shpilevaya, 2004] and have the following algorithm

$$\dot{k}_m(t) = \gamma_m\, l_m (y^{(n)} - F) ,$$

where γ_m - is the adaptor gain and l_m - is the auxiliary function. Note the algorithm (7) can be

$$\dot{k}_m(t) = \gamma_m\, l_m \operatorname{sgn}(y^{(n)} - F) . \qquad (7)$$

3.2 Stability of adaptive switched system

Let us study the system given in (3), (6), and (7). There is the arbitrary switching sequence provided for the condition $\left(t_f - t_s\right) >> \tau_j > t_n$ is true. We enter new variables for the stability study of the system (3), (6) and (7). The deviation between the adjusted parameter (k_m) and the uncontrolled disturbance (m^j) is estimated according to the expression $e_m^j = k_m - m^j$ and the deviation of the system trajectory from the reference trajectory - $\varepsilon^j = y^{(n)}\left(\tau_j\right) - F$. Substituting control law (6) into (3), we have

$$y^{(n)} = -\sum_{i=0}^{n-1} a_{0i} y^{(i)} + F + \sum_{i=0}^{n-1} a_{0i}\, y^{(i)} - k_m + m^j ,$$

$$y^{(n)} = F - k_m + m^j \quad . \qquad (8)$$

From (8) we can see that $\varepsilon^j = -e_m^j$. Prove the convergence $\varepsilon^j \to 0$ or $e_m^j \to 0$ with the help of the function $V^j(e_m^j) = |\, e_m^j \,|$. The researched function derivative is equal to $\dot{V}^j = (\dot{k}_m - \dot{m}^j)\operatorname{sgn} e_m^j$. In the function obtained, substitute the expression (7) instead of $\dot{k}_m$:

$$\dot{V}^j = -\gamma_m l_m \left|e_m^j\right| - \dot{m}^j \operatorname{sgn} e_m^j$$

If we choose subsidiary function as

$$l_m = 1 \text{ and } \gamma_m > \delta_{2m}^j , \qquad (9)$$

the negative definiteness condition of the function $\dot{V}$ is carried out. Let us note the function l_m can be $l_m = l_0 + y^{-h}$ or $l_m = l_0 + e_m{}^h$, $l_0 = const \geq 0$, $h = 0, 2, 4, \ldots$. It depends on negative definiteness condition of the function $\dot{V}$ and influences on the system properties. We choose $l_m = 1$. Therefore, each subsystem is asymptotic stable on the interval τ_j . And according to the results given in [Mancilla-Aguilar, 2000], we can assert that the adaptive switched system (3), (6), and (7) is asymptotic stable for $t \in \left[t_s, t_f\right]$. But the system (3) is similar to the system (1). If the conditions (2) are correct, then the adaptive switched system (1), (6), and (7) is asymptotic stable too. In this way we have showed the truth of the following proposition.

Proposition 1: The j-th subsystem is asymptotically stable for arbitrary switching sequence on the interval τ_j , $\left(t_f - t_s\right) >> \tau_j > t_n^j$, if conditions $\gamma_m > \delta_{m2}^j$, $l_m > 0$ and

$$\left|\rho_s^j(t)\right| < \varepsilon_s^j, \quad \left|\dot{\rho}_s^j(t)\right| < \delta_s^j,$$

where $\left|\dot{m}^j(t)\right| < \delta_{m2}^j$, $t_j < t < t_{j+1}$, are satisfied.

Using **Proposition 1** and the theorem given in [Mancilla-Aguilar, 2000, 2002] we can formulate the second proposition for arbitrary switching sequence such, that $\left(t_f - t_s\right) >> \tau_j > t_n^j$.

Proposition 2: The closed loop system (1), (6) and (7) consisting of asymptotically stable subsystems is locally asymptotically stable on $t \in \left[t_s, t_f\right]$ for arbitrary switching sequence, if $\gamma_m > \delta_{\max}$, where $\delta_{\max} = \max_{1 \le j \le \bar{j}} \left|\delta_{m2}^j\right| < \infty$.

4 Example

In this section we consider the longitudinal motion control of an aircraft on two regimes [Lebedev and Chernobrovkin, 1973]. The dynamical description is

$y^{(3)} + a_2^j\, y^{(2)} + a_1^j\, y^{(1)} = -b_1^j\, u^{(1)} - b_0^j\, u,\ j = 1,2$,

where y is the angle of a pitch, u is the angle of elevator diversion. The first subsystem has parameters $a_1^1 = 3.263,\ a_2^1 = 1.287,\ b_1^1 = 2.32,\ b_0^1 = 1.339$, and the second subsystem parameters are $a_1^2 = 6.526$, $a_2^2 = 2.574,\ b_1^2 = 4.64,\ b_0^2 = 2.678$, $\tau_1 = \tau_2 = 10$. We take following controller parameters

$a_{01} = 3.3,\ a_{02} = 1.3,\ b_{01} = 2.3,\ b_{00} = 1.3,\ \gamma_m = 30$

and reference model parameters (5): $a_0^* = b_0 = 37.5,\ a_1^* = 37,\ a_2^* = 9.5$. Estimate the required output variable derivatives with the help of the low-inertia dynamic system such as

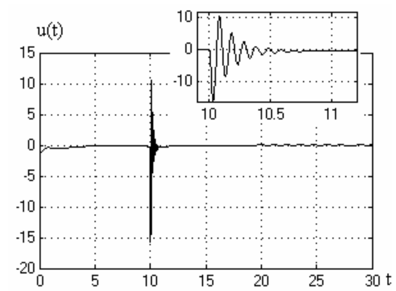
$$\mu^3 \bar{y}^{(3)} + d_2 \mu^2 \bar{y}^{(2)} + d_1 \mu \bar{y}^{(1)} + \bar{y} = y,$$

where μ, d_j and $\bar{y}^{(i)}$ are the fast time constant, the *j*-th damping coefficient, the estimation of *i*-th order derivative of the output variable accordingly. Using recommendations given in [Vostrikov and Shpilevaya, 2004] we have $\mu = 0.007$, $d_1 = 3.143$, $d_2 = 3.265$. System trajectories look as shown in Figure 1. As we can see the adaptive system has saved the stability under piecewise parameter disturbances. In this system we can influence the transition time by reference model parameters and the adaptor gain.

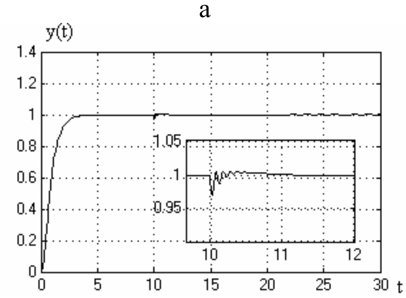
5 Conclusion

We considered one of the approaches to the SISO adaptive switched system design. Transformation of the equation of switched plant taking into account disturbance model is executed. It has allowed obtaining of stationary and non-stationary parts of the model. The switched system is stabilized with the help of the adaptive regulator, in which the signal adjustment is used. The regulator with additive adjustment is designed on the basis of velocity vector method. There is into account the disturbance rate, as result we have the “fast” adaptive algorithm. There are lower and upper bounds of the adaptor gain. After the upper bound the gain increase does not lead to quality improvement of output processes. Regulator parameters have to be met the requirements, which we found on the basis of stability study. We defined the stability conditions using the common Lyapunov’s function for arbitrary switching sequence when there is slow switching.

Using considered adaptive regulator we can get the required quality of the closed loop switched system on all work time interval. This approach gives more simple and cheap realization and tuning of the system.



a



b

Figure 1. Control (a) and output (b) processes.

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References

Liberzon, D. and Morse, S. (1999). Basic Problems in Stability and Design of Switched Systems, *IEEE Control System Magazine*, vol. 19, pp. 59-70.

Wicks, M.A., Peleties, P., and DeCarlo, R.A. (1998). Switched controller synthesis for the quadratic stabilization of a pair of unstable linear systems, *European J. Control*, vol. 4, pp. 140-147.

Mancilla-Aguilar, J.L. (2002). *Analysis and design of* *switching and fuzzy systems.* PhD thesis, Bilkent university.

Mancilla-Aguilar, J.L. (2000). A condition for the stability of switched nonlinear systems. *IEEE Trans. on Automatic Control*, vol. 45, pp. 2077–2079.

Davrazos, G. and Koussoulas, N. T. *A review of stability results for switched and hybrid systems*. http://med.ee.nd.edu/MED9/Papers/Hybrid_systems/med01-169.pdf. 8 p.

Zhai, G., L. Hai and Antsaklis, P. J. (2003). Quadratic stabilisability of switched linear systems of linear systems with polytopic uncertainties, *Int. J. Control*, 76, (7), pp. 747-753.

Narendra, K.S. and Balakrishnan, J. (1994). A common Lyapunov function for stable LTI systems with commuting A-matrices, *IEEE Trans. Automat. Control*, vol.39, pp. 2469-2471.

Branicky, M.S. (1994). Stability of switched and hybrid systems, Proc. *IEEE Conf. on Decision & Control,* pp. 4228-4234.

Vostrikov, A.S. (1995). *Synthesis of nonlinear systems by means of localization method.* Novosibirsk Publishing Office, Novosibirsk, Russia.

Vostrikov, A.S. and Shpilevaya, O.Ya. (2004). Nonlinear Control Systems with Fast Adaptive Algorithm. In *Proceedings of the IASTED International Conf. Modelling, Identification, and Control,* Grindelwald, Switzerland, pp. 444-449.

Lebedev, A.A. and Chernobrovkin, L.S. (1973). *Flight dynamics of pilotless vehicle*. Machine building, Moscow, Russia. (in Russian).