Autowaves on Locally Coupled Relaxation Oscillators

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Abstract—In this work, we introduced a programmable network with a cloning template in order to generate autowaves. The presented model of the network can be easily implementable. The waves diffract from sources can be located on the network using the inputs of the network. Furthermore new waves on the network have been observed. Propagation of autowaves on the inhomogeneous network, formed by the fixed-state map on the network is presented.

1. Introduction

Autowaves are well described by reaction-diffusion equations. Because of easy implementation of the reaction-diffusion on Cellular Neural Networks (CNNs)[2] the CNN based model has been widely used for generating autowaves. In [4], a review on the study of spatial-temporal behavior on an array of Chua's circuits, where Chua's circuit is used as the reaction term in the model, has been presented. Manganaro *et al.* [3] have used a second-order system for the reaction term and have observed autowaves, also spiral waves and Turing patterns on a two layer CNN with a Chua-Yang model. Furthermore the spatiotemporal phenomena in a two layers Analogic Cellular Engine (ACE) chip [1] which is designed based on CNN-Universal Machine (CNN-UM) architecture [5] were also reported recently. More recently spatiotemporal pattern phenomena on an ACE16k chip (which was designed as an array of locally coupled first order 128×128 cells) has been reported by Yalçın *et al.* in [6].

In this paper a new network model is studied which is inspired by the ACE16k chip. The simulations in [6] show that the states of each cells in ACE16k chip are oscillating when they are uncoupled. In [7] a new cell model has been presented taking a parasitic capacitance and resistance effects into account for ACE16k CNN chip. Because of the parasitic effect the cell model is turn to a second order system and the cell can oscillate for a given parameter set. In fact the new cell model is a relaxation oscillator. In [7] by taking a one-dimensional CNN consisting of the new cell model, travelling waves are obtained. In this work we present the experimentally observed autowaves on an 2D-array of locally coupled the new cells.

This paper is organized as follows. Section 2 briefly describes the new cell model and the network model. Section 3 presents autowaves on the network. Section 4 concludes the work.

2. Locally Coupled Relaxation Oscillators

The cells dynamics are described by

$$\begin{aligned} \dot{x}_{i,j} &= (a+1)x_{i,j} + by_{i,j} - g(x_{i,j}) \\ &+ \sum_{k=-1}^{1}\sum_{l=-1}^{1} a_{k,l}x_{i+k,j+l} + u_{i,j} \\ \dot{y}_{i,j} &= cx_{i,j} + dy_{i,j} \end{aligned} \qquad (1)$$

where $a, b, c, d \in R$ are the parameters of the cell $i = 1, 2, ..., N$ and $j = 1, 2, ..., M$. The state variables of the cells are $x_{i,j}$ and $y_{i,j}$. The input of the cell is $u_{i,j}$. $g(x)$ is the nonlinearity of the cell which is given in Figure 1. The cloning template (A) is defined by

$$A = \begin{bmatrix} a_{-1,-1} & a_{-1,0} & a_{-1,1} \\ a_{0,-1} & a_{0,0} & a_{0,1} \\ a_{1,-1} & a_{1,0} & a_{1,1} \end{bmatrix}.$$

Dynamical behaviour of the uncoupled cell was studied in [7]. Figure 2 shows the limit cycle of the uncoupled cell.

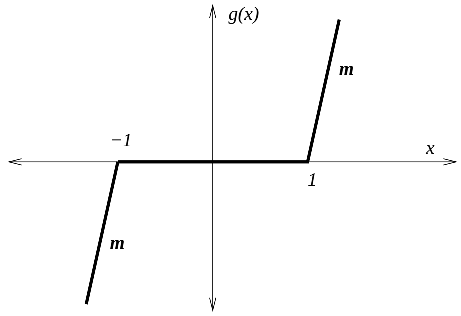

Figure 1: The nonlinear function for $g(x)$.

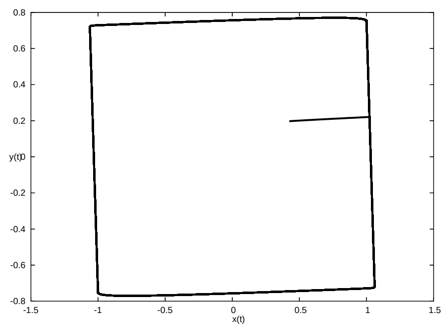

Figure 2: Limit cycle for the uncoupled cell.

3. Autowaves on the network

During our experiments, the following cloning template

$$A = \begin{bmatrix} 0 & \alpha & 0 \\ \alpha & 0 & \alpha \\ 0 & \alpha & 0 \end{bmatrix}, \tag{2}$$

were chosen. In order to visualize the results obtained on the network, the states $(x_{i,j}(t))$ of the network at time t map to an image with the function

$$f(x_{i,j}(t)) = \begin{cases} 1, & x_{i,j} \geq 1 \\ 0.5, & -1 < x_{i,j} < 1 \\ 0, & x_{i,j} \leq -1. \end{cases} \tag{3}$$

Figure 3 shows six consecutive snapshots $(f(x_{i,j}(t)))$ depicting the obtained autowave during the time evolution for the cloning template with fixed boundary condition and random initial condition. Four wave sources shown in Figure 3 which are located at the corners have been observed.

In a second experiment, it is shown that new autowave sources can be imposed at any place on the network. A source can be located with taking the input value equal to 1 (the rest equal to 0.5) In Figure 4

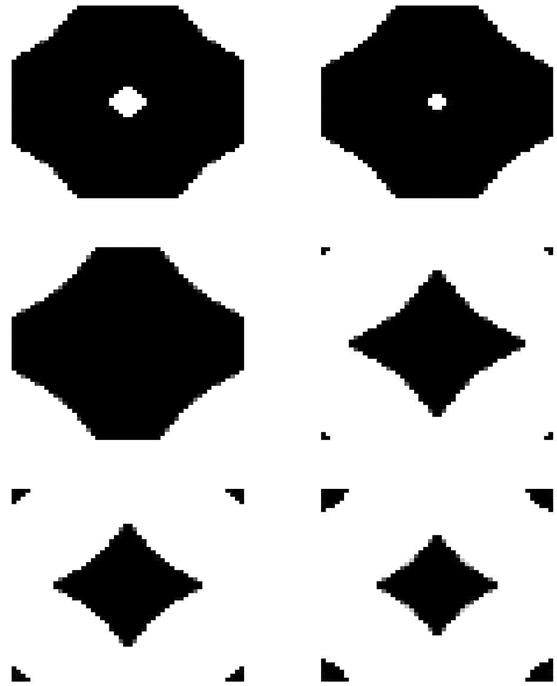

Figure 3: Time evolution for the cloning template. The waves are diffracting form the corners. The shape and amplitude of the waves remains constant during propagation. There is no interference between waves and the colliding waves annihilate. Furthermore, the waves do not reflect from the boundaries of the network.

two sources are located on the network. Two consecutive snapshots depict the resulting autowave in time.

The fixed-state map of the network is a binary image. It specifies which cells of the network are in a certain active or inactive state for all time. The state variables of these cells are frozen to fixed values and do not change in time. The propagation of autowaves from the top-left corner is shown in Figure 5. The waves continue propagating around the frozen cells by the fixed-state map.

Furthermore new waves on the network have been observed. During the presentation we will present the new waves which are observed on the network [8]. We will discuss possible engineering applications of these waves.

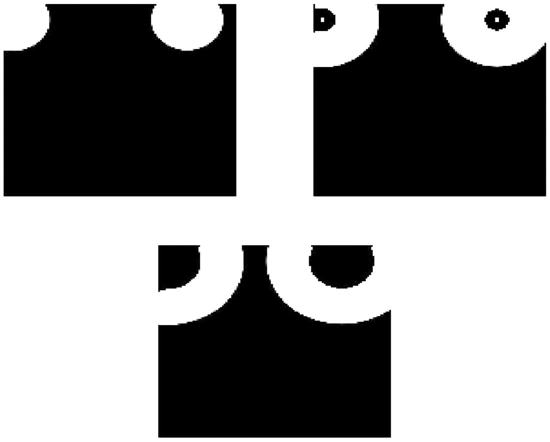

Figure 4: Time evolution for the cloning template with two new autowave sources which are defined by the input.

Figure 5: Propagation of autowaves from the source which are located at top-left corner on the network. The waves continue to propagete around the frozen cells on the array.

4. Conclusion

It has been shown that autowaves can generate on locally coupled relaxation oscillator. Furthermore the experimental observation on ACE16k chip and the observation on the presented network show similarity. The presented model of the network can be easily implementable with electrical circuits because of local coupling. Therefore it is the first simple network model to generate autowaves.

Acknowledgments

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