Autonomous tracking system for carrier rocket orbital stage motion: structure and algorithms

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Abstract: The autonomous tracking system (ATS) for launcher orbital stage motion, intended for determination of characteristics its motion (spatial orientation angles, motion parameters) and a current condition of the upper atmosphere density (ballistic coefficient) for increase of motion forecasting accuracy and for specification of the atmospheric entry moment is offered. For transmission maintenance of the high information transfer efficiency to the Earth and depreciation of flight tracking it is offered to use a low-altitude satellite communication system (LSCS) Globalstar and the Internet. The fixed-frequency two-system navigational receiver, the magnetometer, and the satellite modem is included in ATS structure. The complex of the algorithms using the picked structure of measuring devices is described.

Keywords: Tracking system, navigation algorithm, attitude algorithms, orbital stage, launcher.

1. INTRODUCTION

Now the tracking of motion of orbital stages of launchers (OSL) making uncontrolled flight after an output into a reference orbit and separations of payloads, implements with use of ground measuring centers. In view of boundedness of the locations of such centers and the high cost of their utilization, the organization of constant and low-cost monitoring behind OSL motion which is necessary for definition of the moment of the beginning of heavy braking and atmospheric entry is hindered. It does not allow to estimate operatively prospective impact area of fragments of a stage which are «space debris» after finishing of launching, and to lower risks and damage for the nature and economic activities.

In this report the structure and algorithmic maintenance of small-sized ATS, allowing to determine parameters of an orbital stage motion and to transmit information to the Earth, using the satellite modem for connection through LSCS Globalstar which application is certificated in territory of the Russian Federation, is described.

2. DESCRIPTION OF THE AUTONOMOUS MOTION TRACKING SYSTEM

The monitoring system of motion consists of the electronics unit, power supply unit, receiving antenna for signal from GLONASS/GPS systems and a transmitting antenna for connection with LSCS Globalstar.

In the electronics unit are combined the fixed-frequency two-system navigational receiver; the three-axis magnetometer; the satellite modem, the onboard computer.

For determination of OSL mass center motion parameters the navigational solutions incoming from the navigational receiver are used. The same solutions accumulated on the interval of one orbit, are used for operative specification of the ballistic coefficient that characterizes the current condition of the Earth upper atmosphere density.

One orbit of flight picked in connection with that potential accuracy of the navigational receiver allows to reveal braking effect from atmosphere.

For determination of OSL orientation the measurements incoming from the magnetometer are used. It is known, that measurements not less than two vectors is necessary for unambiguous definition of body orientation in the space. In this connection it is offered to use the information on visibility and invisibility of GPS and GLONASS satellites which together with data about their euhemerizes and OSL position allow to estimate orientation of phase center of the receiving antenna established on an orbital stage surface.

On fig.1 the three-level structure of ATS algorithmic maintenance is shown.

At the first level the OSL current orientation is determined, on second level the ballistic coefficient is updated under accumulated data about parameters of motion, at the third level the OSL motion forecasting and estimation of atmospheric entry time is realized.

3. ALGORITHMIC COMPLEX FOR ORBITAL STAGE LAUNCHER ATTITUDE DETERMINATION

At formulation and solution of a OSL attitude determination problem the right orthogonal frames with the center located in OSL mass center are used: state coordinate system (SCS) $OX_1Y_1Z_1$ (an axis OX_1 – OSL longitudinal axis); orbital coordinate system (OCS) $OX_2Y_2Z_2$ (an axis OZ_2 is directed on OSL radius - vector, the axis OY_2 is directed on vector of angular momentum of OSL orbital motion, the axis OX_2 supplements a system up to right).

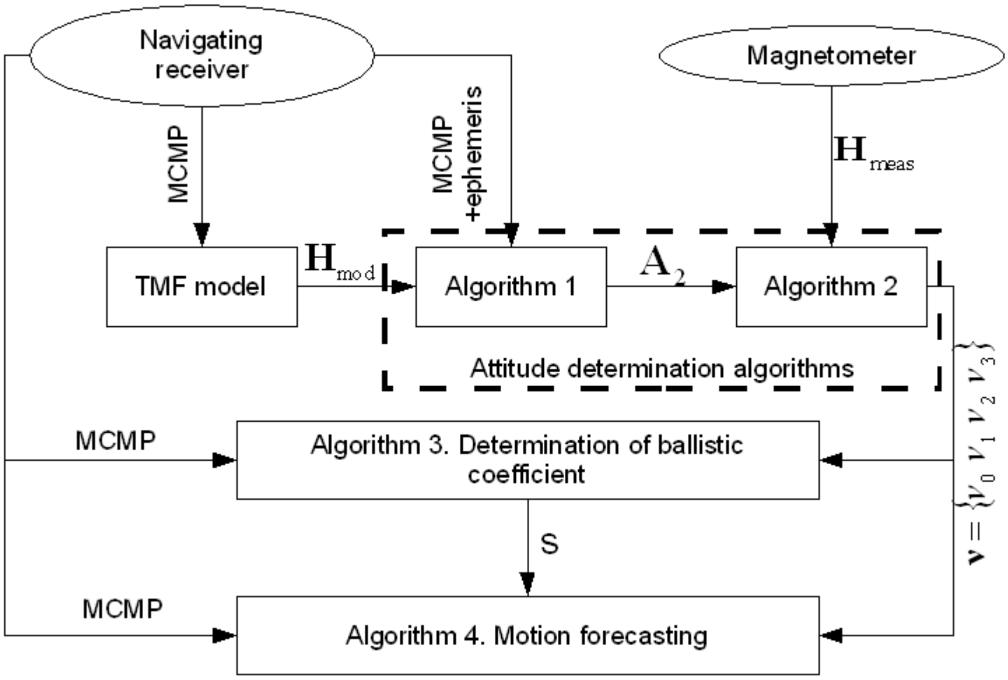

Fig.1. ATS algorithmic maintenance structure (MCMP- mass center motion parameters)

The SCS $OX_1Y_1Z_1$ position relatively of OCS $OX_2Y_2Z_2$ position is set with the help of the quaternion $v = (v_0, v_1, v_2, v_3)$, having the single norm: $v_0^2 + v_1^2 + v_2^2 + v_3^2 = 1$. The transition matrix from $OX_2Y_2Z_2$ to $OX_1Y_1Z_1$ is marked out for $\mathbf{M}_{X_1X_2}$. Elements of this matrix express through components of a quaternion v by means of formulas:

$$\begin{aligned} &m_{11} = v_0^2 + v_1^2 - v_2^2 - v_3^2;\ m_{12} = 2\cdot(v_1v_2 + v_0v_3);\\ &m_{13} = 2\cdot(v_1v_3 - v_0v_2);\ m_{21} = 2\cdot(v_1v_2 - v_0v_3);\\ &m_{22} = v_0^2 - v_1^2 + v_2^2 - v_3^2;\ m_{23} = 2\cdot(v_0v_1 + v_2v_3);\\ &m_{31} = 2\cdot(v_1v_3 + v_0v_2);\ m_{32} = 2\cdot(v_2v_3 - v_0v_2);\\ &m_{33} = v_0^2 - v_1^2 - v_2^2 + v_3^2. \end{aligned} \tag{1}$$

The problem of OSL attitude determination is considered as a problem of a determination of a quaternion v.

By development of attitude determination algorithms, the approach that has been set up on the coordination of measurements of various vectors in two coordinate systems which mutual to orientation is subject to definition (Wertz, 1978; Wahba, 1965) is widely applied. At a solution of a problem of attitude determination the terrestrial magnetic-field vector is taken as the first vector $\mathbf{U}^1$ and the vector of direction cosines of phase center of navigational antenna is taken as the second vector $\mathbf{U}^2$. Definition of vector $\mathbf{U}^2$ in essence is possible under the analysis of a spatial disposition of visible and invisible GPS and GLONASS satellites.

3.1 Algorithm of orientation determination of an orbital stage longitudinal axis

This algorithm allows to determine the vector $\mathbf{U}^2$ in OCS. On fig.1 it is marked out as Algorithm 1.

Input dates of algorithm: $x, y, z, \dot{x}, \dot{y}, \dot{z}$ – - parameters of OSL center mass motion; x_i, y_i, z_i – ephemeris of GPS and GLONASS satellites ($i = \overline{1, N}$, N – common number of navigational satellites); numbers of visible/invisible navigational satellites; $\mathbf{A}_1 = (x_1, y_1, z_1)^{\mathrm{T}}$ – vector of direction cosines of phase center of a navigational antenna in SCS.

The algorithm of definition of spatial orientation of OSL longitudinal axis is based on use of the information on positions of GPS and GLONASS satellites (Belokonov, 2007).

For further actions is considered, that navigational antenna is located on OSL longitudinal axis. The problem of definition of orientation of OSL longitudinal axis is reduced to searching an estimation of vector of direction cosines of phase center of navigational antenna $\mathbf{A}_2 = (x_2, y_2, z_2)^T$, located on OSL longitudinal axis, from a condition of a minimum of object function $\Phi(x_2, y_2, z_2)$, reflecting conditions of visibility / invisibility of navigational satellites, in view of a normality condition of direction cosines $x_2^2 + y_2^2 + z_2^2 = 1$,

$$\begin{cases} \cos\left(\mathbf{A}_2, \mathbf{grad}_{B_i}\right) \geq \cos(\alpha), & \left(i = \overline{1, N_{Vis}}\right); \\ \cos\left(\mathbf{A}_2, \mathbf{grad}_{HB_j}\right) < \cos(\alpha), & \left(j = \overline{1, N_{Inv}}\right), \end{cases} \tag{2}$$

where $\mathbf{grad}_i = \{x_{2i}, y_{2i}, z_{2i}\}$ – Unit vector of distance up to i-th navigational satellite, in projections to axes OCS; N_{Vis}, N_{Inv} – accordingly, an amount of visible and invisible navigational satellite; α – half-angle of shading cone of a navigational antenna.

The procedure of a problem solution of OSL longitudinal axis orientation definition consists of following steps:

1. Conversion of ephemeris of invisible navigational satellites on times of a solution of attitude determination.

2. Conversion of distances up to visible/invisible navigational satellites from an absolute coordinate system to OCS.

3. Elimination of reviewing invisible satellites, those that shaded Earth that corresponds to realization of a condition:

$$z_{2k} < 0 \quad u \quad |z_{2k}| > \cos\left(\arcsin\left(\frac{R_з}{R_з + h}\right)\right), k = \overline{1, N}. \qquad (3)$$

4. Searching an estimation of vector of direction cosines of phase center of navigational antenna from a condition of a minimum of the object function reflecting conditions of visibility / invisibility of navigational satellites (2) (fig. 2)

$$\Phi(x_2, y_2, z_2) = \sum_{i=1}^{N_{Vis}} (x_{2i}x_2 + y_{2i}y_2 + z_{2i}z_2 - 1)^2 + \\ + \sum_{j=1}^{N_{Inv}} (x_{2j}x_2 + y_{2j}y_2 + z_{2j}z_2 + 1)^2 . \qquad (4)$$

In view of a normality condition of direction cosines of phase center of navigational antenna $x_2^2 + y_2^2 + z_2^2 = 1$.

$$J(\mathbf{M}_{X_1X_2}) = \sum_{i=1}^{2} \alpha_i \left(\mathbf{U}_1^i - \mathbf{M}_{X_1X_2}\mathbf{U}_2^i\right)^{\mathrm{T}} \left(\mathbf{U}_1^i - \mathbf{M}_{X_1X_2}\mathbf{U}_2^i\right) \qquad (5)$$

where $\mathbf{M}_{X_1X_2}$ – a matrix circumscribing connection between OCS and SCS, parameterized by means of quaternion; $\mathbf{U}_1^1 = \mathbf{H}_1$ – terrestrial magnetic field vector in SCS measured by magnetometer; $\mathbf{U}_2^1 = \mathbf{H}_2$ – terrestrial magnetic field vector in OSC, calculated on IGRF-2005 model; $\mathbf{U}_1^2 = \mathbf{A}_1$ – known vector of direction cosines of phase center navigational antenna in SCS; $\mathbf{U}_2^2 = \mathbf{A}_2$ – vector of direction cosines of phase center navigational antenna in OCS; α_i – a weight coefficient ($\alpha_i \neq 0$),taking into account relative significance magnitometric and satellite radio navigational measurements.

Orientation angles of OSL are determined on measurements of terrestrial magnetic field vector $\mathbf{H}_1 = \left(h_{X_1}, h_{Y_1}, h_{Z_1}\right)^{\mathrm{T}}$; on the vector of direction cosines of phase center of navigational antenna calculated in OCS $\mathbf{A}_2 = (x_2, y_2, z_2)^{\mathrm{T}}$, which are results of Algorithm 1; on the terrestrial magnetic field vector

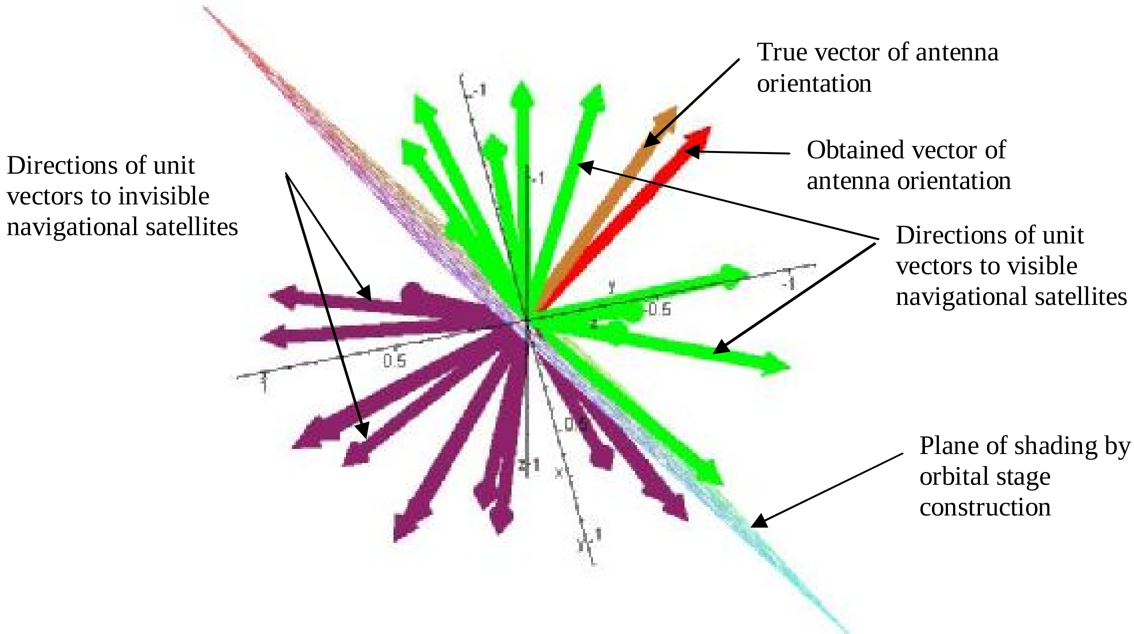

Fig. 2. Definition illustration of vector of direction cosines of navigational antenna phase center

in OCS, calculated with use of IGRF-2005 model $\mathbf{H}_2 = \left(h_{X_2}, h_{Y_2}, h_{Z_2}\right)^{\mathrm{T}}$; on known vector direction cosines of phase center of navigational antenna in SCS $\mathbf{A}_1 = (x_1, y_1, z_1)^{\mathrm{T}}$.

The procedure of minimization of object function (4) is reduced to a solution of a system of three linear equations.

3.2 Algorithm of determination of an orbital stage orientation

On fig.1 this algorithm is marked out as Algorithm 2. For searching a quaternion v the method described in (Wertz, 1978) is used. The essence of a method consists of minimization of the criterion representing the sum suspended with weights a_i of quadrates of differences to values of two vectors, given in two coordinate systems (Wahba, 1965):

The required quaternion is defined from a condition of a minimum of criterion (5) in view of the additional equation ensuring normality condition for elements of a quaternion: $v_0^2 + v_1^2 + v_2^2 + v_3^2 = 1$. It is shown in (Wertz, 1978), that minimization of criterion (5) under condition of normalizations of quaternion elements are reduced to a determination minimum eigenvalue of a four-dimensional matrix:

$$\mathbf{B} = \sum_{i=1}^{2} \frac{1}{\alpha_i} \begin{bmatrix} \mathbf{S} & \mathbf{Z} \\ \mathbf{Z}^{\mathrm{T}} & t \end{bmatrix}, \qquad (6)$$

where $\mathbf{S} = \mathbf{I}\left(\left(\mathbf{U}_1^i\right)^{\mathrm{T}} \mathbf{U}_2^i\right) - \mathbf{U}_2^i \left(\mathbf{U}_1^i\right)^{\mathrm{T}} - \mathbf{U}_1^i \left(\mathbf{U}_2^i\right)^{\mathrm{T}}$;

$\mathbf{Z} = -\left(\mathbf{U}_1^i \times \mathbf{U}_2^i\right)$, $t = -\left(\mathbf{U}_1^i\right)^{\mathrm{T}} \mathbf{U}_2^i$, $\mathbf{I}$ – identity matrix.

Thus the required quaternion represents an eigenvector corresponding to the least eigenvalue of a matrix (6).

The quaternion v_k, in an instant t_k assigning OSL orientation is determined to within a sign. Signs of quaternion elements v_k get out of a condition:

$$v_0^{(k)} > 0, \quad \sum_{i=0}^{3} v_i^{(k-1)} v_i^{(k)} > 0 \quad \left(k = \overline{1, S}\right).$$

After searching a quaternion projection v of an absolute angular velocity $\boldsymbol{\omega}$ в SCS $OX_1Y_1Z_1$ are defined with the help of numerical derivation for found quaternion and the kinematics equations

$$\begin{aligned} \omega_1 &= 2\left(v_0 \dot{v}_1 - v_1 \dot{v}_0 + v_3 \dot{v}_2 - v_2 \dot{v}_3\right), \\ \omega_2 &= 2\left(v_0 \dot{v}_2 - v_3 \dot{v}_0 + v_1 \dot{v}_3 - v_3 \dot{v}_1\right), \\ \omega_3 &= 2\left(v_0 \dot{v}_3 - v_3 \dot{v}_0 + v_2 \dot{v}_1 - v_1 \dot{v}_2\right). \end{aligned}$$

4. NAVIGATION FOR MOTION FORECASTING FOR LAUNCHER ORBITAL STAGE

It is considered, that atmospheric density ρ is a function of an flight altitude and an index of solar activity $F_{10,7}$. An estimation $\hat{S}$ of ballistic coefficient S, which is interpreted, as a matched ballistic coefficient between a variable of atmospheric density $\rho\left(H, F_{10,7}\right)$ and its nominal value $\rho_M(H)$, calculated with use of model (GOST 4401-81 Standard Atmosphere. Parameters) is defined. Thus, calculating an estimation of a ballistic coefficient $\hat{S}$ at set models of atmospheric density $\rho_M(H)$, actually we determine an estimation of products $S \cdot \rho$, as takes, that $S \cdot \rho = \hat{S} \cdot \rho_M(H)$.

Interval of smoothing of navigational measurements for an evaluation of estimation $\hat{S}$ gets out in view of an accuracy of definition of motion parameters, ensured by navigational receiver.

Thus, it is required to find a matched ballistic coefficient $\hat{S}$ and vector of a navigational estimation $\hat{\mathbf{q}}(t_N)$ at the moment of the termination of an smoothing interval t_N, which ensure a minimum to a functional

$$\begin{aligned} I\left(\hat{\mathbf{q}}(t_n), \hat{S}\right) = \sum_{j=1}^{N} \Big\{ & \left[L_p\left(t_j, \hat{\mathbf{q}}(t_N), \hat{S}\right) - \mathbf{q}(t_j)\right]^{\mathrm{T}} \times \mathbf{D}_j^{-1} \times \\ & \times \left[L_p\left(t_j, \hat{\mathbf{q}}(t_N), \hat{S}\right) - \mathbf{q}(t_j)\right] \Big\}, \end{aligned} \qquad (7)$$

where $L_p\left(t_j, \hat{\mathbf{q}}(t_N), \hat{S}\right)$ – an forecasting operator for OSL motion which enumerates the vector $\hat{\mathbf{q}}(t_N)$ on instants of receiving of navigational solutions; p=4, 8, 16 – the order of used harmonics in model of the Earth gravitational field;

$\mathbf{D}_j = \mathbf{K}_p(t_j)$ – matrix of weight coefficients which usually takes equal priory covariance matrix of errors of navigational solutions in the moment t_j. Various versions of realization of algorithm for a joint evaluation of estimations $\hat{\mathbf{q}}(t_N)$ и $\hat{S}$ are possible. According to one of them, widely used in practice of navigational maintenance the vector $\hat{\mathbf{q}}(t_N)$ and parameter $\hat{S}$ are determined jointly as the expanded seven-measurement vector $\left(\hat{\mathbf{q}}(t_N), \hat{S}\right)$. However, in this case the outcome will depend on a relation between a priori dispersions of components of vector $\hat{\mathbf{q}}(t_N)$ and an a priori dispersion of $\hat{S}$. It is offered to use two-stage algorithm, which is, deprived the above-mentioned shortage. Searching of a minimum of a functional (7) is divided on two matched processes of minimization with serial searching parameters $\hat{\mathbf{q}}(t_N)$ and $\hat{S}$: one-parameter minimization of a functional (7) on a ballistic factor $\hat{S}$, on which each step the navigational estimation $\hat{\mathbf{q}}_N = \hat{\mathbf{q}}(t_N)$, ensuring a minimum (7) at fixed value $\hat{S}$ is defined. To estimate a prize on an accuracy of this algorithm it is possible by means of relations reduced in (Eliasberg, 1976) which determine the errors of a method of least squares connected to omission of adopted assumptions.

6. CONCLUSIONS

The chosen structure of measuring tools and the developed software allow to realize monitoring of launcher orbital stage motion. Use of satellite modem LSCS Globalstar will allow to transmit operatively data to the Earth and to estimate prospective impact area of fragments of an orbital stage. It will allow to lower risks for economic activities and damage for the nature.

Now the possibility of arrangement of ATS at the third stage of the launcher «Soyuz» is investigated.

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REFERENCES

Wertz, J.R (1978). *Spacecraft Attitude Determination and Control*. Dordrecht, The Netherlands.

Wahba, G.A. (1965). *Least Squares Estimate of Spacecraft Attitude*. SIAM Review.Vol.7, №3. – p. 409.

Belokonov, I.V., Kramlikh, A.V. (2007). *Determination of possible orientation of a longitudinal axis of* *microgravitational space platform "Foton-M2" on satellite radio navigational measurements.* Motion control and navigation of aircraft: Proceedings of XIII All-Russian Meeting on Motion Control and Navigation of Aircraft. p. 83-89.

GOST 4401-81 Standard Atmosphere. Parameters.

Eliasberg, P.E. (1976). *Definition of motion by results of measurements*. M.: Nauka.