# HYBRID QUANTISED OBSERVER FOR OSCILLATORY MULTI-INPUT-MULTI-OUTPUT NONLINEAR SYSTEMS

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**Abstract**

The problem of state estimation with limited information capacity of the coupling channel for oscillatory multi-input-multi-output nonlinear systems is analyzed for systems in Lurie form (linear part plus nonlinearity depending only on measurable outputs) with first-order coder-decoder. It is shown that for oscillatory systems the upper bound of the limit estimation error is proportional to the maximum rate of the coupling signal and inversely proportional to the information transmission rate (channel capacity). The results are applied to state estimation of a 3rd order nonlinear system: self-excited mechanical oscillator.

**Key words**

Nonlinear dynamics, State estimation, Communication constraints, Hybrid observer

## 1 Introduction

During the last decade substantial interest has been shown in *networked control systems* (NCS). The idea is to use serial communication networks to exchange system information and control signals between various physical components of the systems that may be physically distributed. NCS are real-time systems where sensor and actuator data are transmitted through shared or switched communication networks, see e.g. (Ishii and Francis, 2002; Goodwin *et al.*, 2004; Abdallah and Tanner, 2007; Matveev and Savkin, 2008). Transmitting sensor measurement and control commands over wireless links allows rapid deployment, flexible installation, fully mobile operation and prevents the cable wear and tear problem in an industrial environment. The possibility of NCS motivates development of a new chapter of control theory in which control and communication issues are integrated, and all the limitations of the communication channels are taken into account. The introduction of a communication network into a NCS can degrade overall control system performance through quantisation errors, transmission time delays and dropped measurements. The network itself is a dynamical system that exhibits characteristics which traditionally have not been taken into account in control system design. These special characteristics include quantization and time-delays and are a consequence of the fact that practical channels have limited bandwidth. A successful NCS design should take network characteristics into account. Due to the digital nature of the communication channel, every transmitted signal is quantized to a finite set (Ishii and Francis, 2002). Hence, we argue that the finite set nature of the data should be explicitly taken into account in the design of NCS.

Recently the limitations of estimation and control under constraints imposed by a finite capacity information channel have been investigated in detail in the control theoretic literature, see (Wong and Brockett, 1997; Nair and Evans, 2003; Nair *et al.*, 2004; Bazzi and Mitter, 2005; Nair *et al.*, 2007) and references therein. For unstable linear systems it was shown that stabilisation of the system at the equilibrium under information constraints is possible if and only if the capacity of the information channel exceeds the entropy production of the system at the equilibrium (so called *Data-Rate Theorem* (Nair and Evans, 2003; Nair *et al.*, 2004).

For discrete-time nonlinear systems, the concept of feedback topological entropy was introduced and the minimum data-rate for local stabilisation was given in (Nair *et al.*, 2004). In (Savkin, 2006) the concept of topological entropy was extended to uncertain systems. Using this concept, in (Savkin, 2006) necessary and sufficient conditions for the robust observability of a class of uncertain nonlinear systems, and the solvability of the optimal control problem via limited capacity communication channels were obtained. Continuous-time nonlinear systems were considered in (Liberzon and Hespanha, 2005; De Persis, 2005; De Persis and Isidori, 2004; De Persis, 2006; Cheng and Savkin, 2007), where several sufficient conditions for different estimation and stabilisation problems were obtained. In (De Persis, 2006), uniformly observable systems were considered and an "embedded-observer" decoder and a controller were designed, which semi-globally stabilizes this class of systems under data-rate constraints. In (Cheng and Savkin, 2007), an output feedback stabilisation problem of a class of nonlinear systems with nonlinearities satisfying the non-decreasing property was considered, encoding/decoding scheme was introduced and the sufficient conditions for the stabilisation problem were obtained.

In most of the above mentioned papers the coding-decoding procedure is rather complicated: the size of the required memory exceeds or equals to the dimension of the system state space. Such a draw-back was overcome in (Fradkov *et al.*, 2006), where a first order coder scheme was proposed for single-input-single-output systems. Complexity of the scheme of (Fradkov *et al.*, 2006) does not grow with the dimension of the system state. In addition, in (Fradkov *et al.*, 2006) the synchronization rather than stabilization problem was studied (synchronization is irredusable to stabilization).

In this paper we extend the results of (Fradkov *et al.*, 2006) to systems with many inputs and many outputs (MIMO systems). We establish limit possibilities of state estimation for a class of nonlinear systems under information constraints. Such systems are well studied without information constraints (Morgül and Solak, 1996; Nijmeijer and Mareels, 1997; Nijmeijer, 2001). Here we present a theoretical analysis for $n$-dimensional systems represented in the so called Lurie form (linear part plus nonlinearity, depending only on measurable outputs). It is shown that the upper bound of the limit estimation error (LSE) is proportional to the upper bound of the transmission error. As a consequence, the upper and lower bounds of LSE are proportional to the maximum rate of the coupling signal and inversely proportional to the information transmission rate (channel capacity). Optimality of the binary coding for coders with one-step memory is established.

## 2 Description of state estimation over the limited-band communication channel

A block-diagram for implementing state estimation via a discrete communication channel is shown in Fig. 1. To simplify exposition we will consider a system model in the so-called Lurie form: right-hand side is split into a linear part and a nonlinearity vector depends only on the measured output. Then the system is modeled as follows:

$$\dot{x}(t) = Ax(t) + \varphi\big(y(t)\big),\ y(t) = Cx(t), \qquad (1)$$

where $x$ is an $n$-dimensional (column) vector of state variables; $y$ is an $l$-dimensional output vector; $A$ is an $(n\times n)$-matrix; $C$ is $(l\times n)$-matrix, $\varphi(y)$ is a continuous nonlinear vector-function, $\varphi : \mathbb{R}^l \to \mathbb{R}^n$. We assume that the system is dissipative: all the trajectories of the system (1) belong to a bounded set $\Omega$ (e.g. attractor of a chaotic system). Such an assumption is typical for oscillating and chaotic systems.

The observer has the following form:

$$\dot{\hat{x}} = A\hat{x} + \varphi(y) + L(y - \hat{y}),\ \hat{y} = C\hat{x}, \qquad (2)$$

where $L$ is the vector of the observer parameters (gain). Apparently, the dynamics of the state error vector $e(t) = x(t) - \hat{x}(t)$ are described by a linear equation

$$\dot{e} = A_L e,\ y = Cx, \qquad (3)$$

where $A_L = A - LC$.

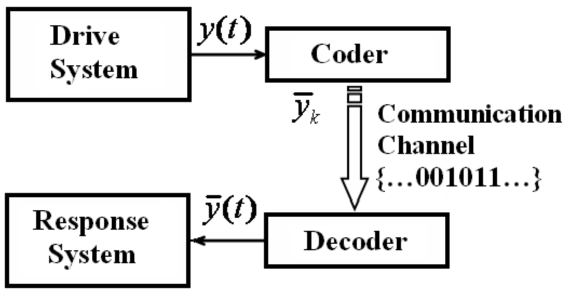


Figure 1. Block-diagram for state estimation over a discrete communication channel.

As is known from control theory, if the pair $(A, C)$ is observable, i.e. if $\mathrm{rank}[C^{\mathrm{T}}, A^{\mathrm{T}}C^{\mathrm{T}}, \ldots, (A^{\mathrm{T}})^{n-1}C^{\mathrm{T}}] = n$, then there exists $L$ providing the matrix $A_L$ with any given eigenvalues. In particular all eigenvalues of $A_L$ can have negative real parts, i.e. the system (3) can be made asymptotically stable and $e(t) \to 0$ as $t \to \infty$. Therefore, in the absence of measurement and transmission errors the estimation error decays to zero.

Now let us take into account transmission errors. We assume that the observation signal $y_i(t) \in \mathbb{R}^l$ is coded with symbols from a finite alphabet at discrete sampling time instants $t_{k,i} = k_i T_{s,i}$, where $i = 1, \ldots, l$, $k_i \in \mathbb{Z}$, $T_{s,i}$ are the sampling periods. Let the coded symbols $\bar{y}_i[k] = \bar{y}(t_{k,i})$ be transmitted over a digital communication channel with a finite capacity. To simplify the analysis, we assume that the observations are not corrupted by observation noise; transmissions delay and transmission channel distortions may be neglected. Therefore, the discrete communication channel with the sampling periods $T_{s,i}$ is considered, but it is assumed that the coded symbols are available at the receiver side at the same sampling instant $t_{k,i} = k_i T_{s,i}$, as they are generated by the coder.

Assume that *zero-order extrapolation* is used to convert the digital sequences $\bar{y}_i[k]$ to the continuous-time input of the response system $\bar{y}_i(t)$, namely, that $\bar{y}_i(t) = \bar{y}_i[k_i]$ as $k_i T_{s,i} \le t < (k_i+1)T_{s,i}$. Then the *transmission error vector* is defined as follows:

$$\delta_y(t) = y(t) - \bar{y}(t) \in \mathbb{R}^l. \qquad (4)$$

In the presence of transmission errors, equation (3) takes the form

$$\dot{e} = A_L e + \varphi(y) - \varphi\big(y + \delta_y(t)\big) - L\delta_y(t) \qquad (5)$$

Our goal is to evaluate limitations imposed on the estimation precision by limited transmission rate. To this end introduce an upper bound on the limit estimation error $Q = \sup \overline{\lim_{t\to\infty}} \|e(t)\|$, where $e(t)$ is from (5), $\|\cdot\|$ denotes the Euclidian norm of a vector, and the supremum is taken over all admissible transmission errors. In the next two sections we describe encoding and decoding procedures and evaluate the set of admissible transmission errors $\delta_y(t)$ for the optimal choice of coder parameters. It will be shown that $\|\delta_y(t)\|$ is bounded and does not tend to zero.

The properties of state estimation over a limited-band communication channel for single-output Lurie systems with one-step memory time-varying coder are studied in (Fradkov *et al.*, 2006). It is shown that the upper bound of the limit state estimation error is proportional to the upper bound of the transmission error. Under the assumption that a sampling time may be properly chosen, optimality of the binary coding in the sense of demanded transmission rate is established, and the relation between estimation accuracy and an optimal sampling time is found. On the basis of these results, the present paper deals with a binary coding procedure. We extend the results of (Fradkov *et al.*, 2006) for multi-output systems.

## 3 Coding procedure

We assume that the observation vector $y(t) \in \mathbb{R}^l$ is coded with symbols from a finite alphabet at discrete sampling time instants $t_{k,i} = k_i T_{s,i}$, $k_i \in \mathbb{Z}$, $i = 1, \ldots, l$, $T_{s,i}$ are sampling periods. We apply the coding/decoding procedure for each component $y_i(t)$ ($i = 1, \ldots, l$) of the system output $y(t) \in \mathbb{R}^l$ independently.

Let the coded symbols $\bar{y}_i[k_i] = \bar{y}_i(t_{k_i})$ ($i = 1, \ldots, l$, $k_i \in \mathbb{Z}$) be transmitted over a digital communication channel with finite capacity. To simplify the analysis, we assume that the observations are not corrupted by observation noise; transmissions delay and transmission channel distortions may be neglected. Therefore, the discrete communication channel with the sampling period $T_s$ is considered, but it is assumed that the coded symbols are available at the receiver side at the same sampling instant $t_k = kT_s$, as they are generated by the coder.

At first, introduce the memoryless (static) binary coder to be a discretized map $q_\varkappa : \mathbb{R} \to \mathbb{R}$ as

$$q_\varkappa(y) = \varkappa\,\mathrm{sign}(y), \qquad (6)$$

where $\mathrm{sign}(\cdot)$ is the *signum* function: $\mathrm{sign}(y) = 1$, if $y \ge 0$, $\mathrm{sign}(y) = -1$, if $y < 0$; parameter $\varkappa$ may be referred to as a *coder range* or as a *saturation value*. Evidently, $|y - q_\varkappa(y)| \le \varkappa$ for all $y$ such that $y : |y| \le 2\varkappa$. Notice that for binary coder each codeword symbol contains $R = 1$ bit of information. The discretized output of the considered coder is found as $\bar{y} = q_\varkappa(y)$ and we assume that the coder and decoder make decisions based on the same information.

In the present paper we use $l$ independent coders for components of an $l$-dimensional vector, transmitted over the channel. Each coder number $i$, $i = 1, \ldots, l$, has its particular sampling period $T_{s,i}$, transmission rate $R_i = 1/T_{s,i}$ and range $\varkappa_i$. The overall averaged rate $R$ is calculated as a sum $R = \sum_{i=1}^{l} R_i$.

The static coder (6) is a part of the time-varying coders with memory, see e.g. (Nair and Evans, 2003; Brockett and Liberzon, 2000; Tatikonda and Mitter, 2004; Fradkov *et al.*, 2006). Two underlying ideas are used for this kind of coder:

– reducing the coder range $\varkappa$ to cover the some area around the predicted value for the $(k+1)$th observation $y[k+1]$, $y[k+1] \in \mathcal{Y}[k+1]$. This means that the quantizer range $\varkappa$ is updated during the time and a time-varying quantizer (with different values of $\varkappa$ for each instant, $\varkappa = \varkappa[k]$) is used. Using such a "zooming" strategy it is possible to increase coder accuracy in the steady-state mode, and, at the same time, to prevent coder saturation at the beginning of the process;
– introducing memory into the coder, which makes it possible to predict the $(k+1)$th observation $y[k+1]$ with some accuracy and, therefore, to transmit over the channel only encrypted innovation signal.

Let us describe the first-order (one-step memory) coder . Introduce the sequence of central vectors (sequence of "*centroids*") $c[k] \in \mathbb{R}^l$, $k \in \mathbb{Z}$ with initial condition $c[0] = 0$ (Tatikonda and Mitter, 2004). At step $k$ the coder compares the current measured output $y[k]$ with the number $c[k]$, forming the deviation vector $\partial y[k] = y[k] - c[k] \in \mathbb{R}^l$. Then this vector is discretized with a given $\varkappa = \varkappa[k]$ according to (6). The output signal

$$\bar{\partial} y[k] = q_\varkappa(\partial y[k]) \qquad (7)$$

is represented as an $R$-bit information symbol from the coding alphabet and transmitted over the communication channel to the decoder. Then the central number $c[k+1]$ and the range parameter $\varkappa[k]$ are renewed based on the available information about the drive system dynamics. Assuming that the system output $y(t)$ changes at a slow rate, i.e. that $y[k+1] \approx y[k]$. we use the following update algorithms:

$$c[k+1] = c[k] + \bar{\partial} y[k], \quad c[0] = 0, \tag{8}$$

$$\varkappa[k] = (\varkappa_0 - \varkappa_\infty)\rho^k + \varkappa_\infty, \;\; k = 0, 1, \dots, \tag{9}$$

where $0 < \rho \leq 1$ is the decay parameter, $\varkappa_\infty$ stands for the limit value of $\varkappa$. The initial value $\varkappa_0$ should be large enough to capture all the region of possible initial values of $y_0$.

The equations (6), (7), (9) describe the coder algorithm. A similar algorithm is used by the decoder. Namely: the sequence of $\varkappa[k]$ is reproduced at the receiver node utilizing (9); the values of $\bar{\partial} y[k]$ are restored with given $\varkappa[k]$ from the received codeword; the central numbers $c[k]$ are found in the decoder in accordance with (8). Then $\bar{y}[k]$ is found as a sum $c[k] + \bar{\partial} y[k]$.

## 4 Evaluation of state estimation error

Now let us evaluate the estimation error

$$Q = \sup \overline{\lim_{t\to\infty}} \|e(t)\|, \tag{10}$$

where $\sup$ is taken over the set of transmission errors $\delta_{y,i}(t)$ not exceeding the corresponding level $\Delta_i$ where $i = 1, \dots, l$. Owing to nonlinearity of the equation (5) evaluation of the estimation error $Q$ is nontrivial and it may even be infinite for rapidly growing nonlinearities $\varphi(y)$. To obtain a reasonable upper bound for $Q$ we assume that the nonlinearity is Lipschitz continuous along all the trajectories of the system (1). More precisely, we assume existence of some positive real number $L_\varphi > 0$ such that $\|\varphi(y) - \varphi(y+\delta)\| \leq L_\varphi |\delta|$ for all $y = Cx$, $x \in \Omega$, where $\Omega$ is a set containing all the trajectories of the system (1), starting from the set of initial conditions $\Omega_0$, $|\delta| \leq \Delta$.

The error equation (5) can be represented as

$$\dot{e} = A_L e + \xi(t), \tag{11}$$

where $\|\xi(t)\| \leq \big(L_\varphi + \|L\|\big)\sqrt{n}\max_i \Delta_i$, i.e. the problem is reduced to a standard problem of linear system theory. Choose $L$ such that $A_L$ is a Hurwitz (stable) matrix and choose a positive-definite matrix $P = P^{\mathrm{T}} > 0$ satisfying the modified Lyapunov inequality

$$PA_L + A_L^{\mathrm{T}} P \leq -\mu P, \tag{12}$$

for some $\mu > 0$. Note that the solutions of (12) exist if and only if $\mu > \mu_*$, where $\mu_* = -\max \operatorname{Re} \lambda_i(A)$ is stability degree of matrix $A$. After simple algebra we obtain the differential inequality for the function $V(t) = e(t)^{\mathrm{T}} P e(t)$: $\dot{V} \leq -\mu V + e^{\mathrm{T}} P \xi(t) \leq -\mu V + \sqrt{V} \cdot \sqrt{\xi^{\mathrm{T}} P \xi}$. Since $\dot{V} < 0$ within the set $\sqrt{V} > \mu^{-1} \sup_t \sqrt{\xi(t)^{\mathrm{T}} P \xi(t)}$, the value of $\overline{\lim_{t\to\infty}} \sup V(t)$ cannot exceed $n \max_i \Delta_i^2 \big(L_\varphi + \|L\|\big)^2 \lambda_{\max}(P)/\mu^2$. In view of positivity of $P$, $\lambda_{\min}(P)\|e(t)\|^2 \leq V(t)$, where $\lambda_{\min}(P)$, $\lambda_{\max}(P)$ are minimum and maximum eigenvalues of $P$, respectively. Hence

$$\overline{\lim_{t\to\infty}} \|e(t)\| \leq C_e^+ \sqrt{l} \max_i \Delta_i, \tag{13}$$

where

$$C_e^+ = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \frac{L_\varphi + \|L\|}{\mu}. \tag{14}$$

The inequality (13) shows that the total estimation error is proportional to the upper bound of the norm $\sqrt{n}\max_i \Delta_i$ of the transmission error. As was shown in (Fradkov *et al.*, 2006), a binary coder is optimal in the sense of bit-per-second rate, and the optimal sampling times $T_{s,i}$ for each channel are

$$T_{s,i} = \varepsilon \Delta_i / L_{y,i} \cdot, \tag{15}$$

In (15), $L_{y,i}$ is the exact bound for the rate of $y_i(t)$, $L_{y,i} = \sup_{x\in\Omega} |C\dot{x}|$, where $\dot{x}$ is from (1), ($i = 1, \dots, l$); $\varepsilon$ is the constant number, $\varepsilon \approx 0.5923$. Consequently, the channel bit-per-second rate $R = \sum_i^l T_{s,i}^{-1}$ is as follows:

$$R = \sum_i^l r L_{y,i}/\Delta_i, \tag{16}$$

where $r = \varepsilon^{-1} \approx 1.688$, and this bound is tight for the considered class of coders. Taking into account the relation (16) for optimal transmission rate, the limit state estimation error can be estimated as follows:

$$\overline{\lim_{t\to\infty}} \|e(t)\| \leq C_e^+ r \cdot l \max(L_{y,i}/R_i), \tag{17}$$

i.e. it can be made arbitrarily small for sufficiently large transmission rate $R = \sum_i^l R_i$.

*Remark*. Relations (15), (16) are related to calculation of the partial bit-rates $R_i$ for transmission of $i$th component $y_i$ of the measured vector $y(t) \in \mathbb{R}^l$ for the given partial transmission error $\Delta_i$ and the bound $L_{y,i}$

of the rate of $y_i(t)$. The problem of reallocation of the given channel bit-rate $R$ between the components of the transmitted vector $y$, minimizing the estimate (17) of the limit estimation error may be posed. Simple calculations lead to the following optimal bit-rates $R_i^*$:

$$R_i^* = R \cdot L_{y,i} / \sum_i^l L_{y,i}, \quad i = 1, \ldots, l. \qquad (18)$$

## 5 Example. State estimation of nonlinear oscillator

Let us apply the above results to state estimation of nonlinear self-excited oscillator via a channel with limited capacity.

*System Equations.* Consider the following self-excited nonlinear oscillator:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\omega_1^2 \sin y_1 - \varrho x_2 + k_p \arctan(k_c y_2), \\ \dot{x}_3 = \omega_2 \cdot (x_1 - x_3), \end{cases} \qquad (19)$$

$$y_1 = x_1, \qquad y_2 = x_1 - x_3$$

where $y(t) \in \mathbb{R}^2$ is the sensor output vector (to be transmitted over the communication channel), $\omega_1$, $\omega_2$, $\varrho$, $k_p$, $k_c$ are system parameters, $x = [x_1, x_2, x_3]^{\mathrm{T}} \in \mathbb{R}^3$ is the state vector. The problem is to produce estimate $\hat{x}(t) \in \mathbb{R}^3$ of the system state vector $x(t)$ based on the signals $y_1(t)$, $y_2(t)$, transmitted over the communication channel.

System (19) has the form (1), where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\varrho & 0 \\ \omega_2 & 0 & -\omega_2 \end{bmatrix}, \quad C = \begin{bmatrix} 1, & 0, & 0 \\ 1, & 0, & -1 \end{bmatrix},$$

$$\varphi(y) = \begin{bmatrix} 0 \\ \omega_1^2 \sin y_1 + k_p \arctan(k_c y_2) \\ 0 \end{bmatrix}. \qquad (20)$$

For the considered case, the observer (2) has $(3 \times 2)$ gain matrix $L$. The matrices $A$, $C$ and function $\varphi(y)$ are given in (20). Matrix $L$ should be chosen so that the observer (2) stability conditions are satisfied, i.e. the characteristic polynomial $D_L(s) = \det(s\mathbf{I} - A_L)$ is Hurwitz. In our study the matrix $L$ was found through the solution $P$ of the Riccati equation as follows

$$PA^{\mathrm{T}} + A^{\mathrm{T}}P + PGP - C^{\mathrm{T}}C = 0, \quad L = P^{-1}C^{\mathrm{T}}, \quad (21)$$

where $G = G^{\mathrm{T}} > 0$ is $(3 \times 3)$-matrix, such as a pair $(A, \sqrt{Q})$ is controllable. Such a choice of the gain $L$ yields passivity of the observer (2) error dynamics (Shim *et al.*, 2003).

*System parameters.* For simulation the following parameter values of the oscillating system (19) were

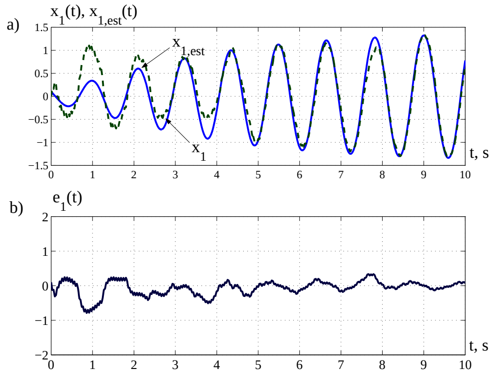

Figure 2. (Color online) Time histories: *a*) $x_1(t)$ (solid line), $\hat{x}_1(t)$ (dashed line); *b*) estimation error $e_1(t) = x_1(t) - \hat{x}_1(t)$ for $R = 50$ bit/s, ($T_{s,1} = 0.03$ s, $T_{s,2} = 0.06$ s.

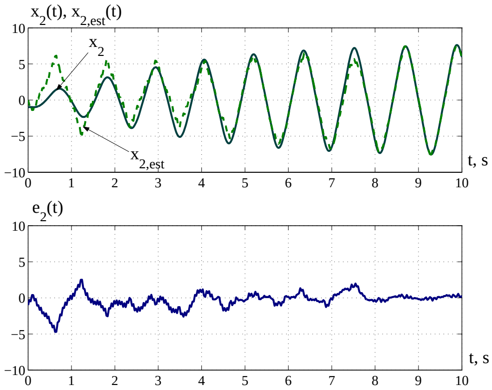

Figure 3. (Color online) Time histories: *a*) $x_2(t)$ (solid line), $\hat{x}_2(t)$ (dashed line); *b*) estimation error $e_2(t) = x_2(t) - \hat{x}_2(t)$ for $R = 50$ bit/s, ($T_{s,1} = 0.03$ s, $T_{s,2} = 0.06$ s.

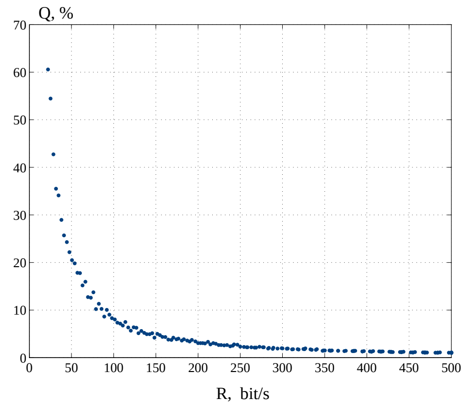

Figure 4. Estimation error $Q$ vs transmission rate $\bar{R}$.

taken: $\omega_0^2 = 40$ s$^{-2}$, $\varrho = 1$ s$^{-1}$, $k = 0.1$, $k_p = 5$, $k_c = 10$, $\tau = 0.1$ s, $L_{y,1} = 10$ s$^{-1}$, $L_{y,2} = 5$ s$^{-1}$, the decay parameters $\rho_i = \exp(-0.2T_{s,i})$, $i = 1, 2$. The matrix $G$ in (21) was taken $G = 0.1\mathbf{I}_3$, where $\mathbf{I}_3$ is an identity $(3 \times 3)$-matrix. The values of $\Delta_1 = \Delta_2$ have been taken from the interval $\Delta \in \{1, 40\}$. The sampling times $T_{s,i}$ for each $\Delta_i$ weretaken from (15). The initial values $\varkappa_{0,i}$ in (9) were $\varkappa_{0,1} = 2$, $\varkappa_{0,2} = 1$.

Some simulation results for the coder (6), (7), (9) are shown in Figs. 2–4. The time histories of $x_1(t)$, $\hat{x}_1(t)$, estimation error $e_1(t) = x_1(t) - \hat{x}_1(t)$ for $R = 50$ bit/s, ($T_{s,1} = 0.03$ s, $T_{s,2} = 0.06$ s are shown in Fig. 2. Respectively, the time histories for the second state variable are depicted in Fig. 3. Dependence of the estimation error $Q$ on the transmission rate $\bar{R}$ is shown in Fig. 4, demonstrating that the estimation error becomes small for sufficiently large transmission rates.

## 6 Conclusions

We have studied dependence of the error of state estimation for nonlinear Lurie systems over a limited-band communication channel both analytically and numerically. It is shown that in common with SISO systems, upper and lower bounds for limit estimation error depend linearly on the transmission error which, in turn, is proportional to the driving signal rate and inversely proportional to the transmission rate. Though these results are obtained for a special type of coder, it reflects peculiarity of the estimation problem as a nonequilibrium dynamical problem. On the contrary, the stabilisation problem considered previously in the literature on control under information constraints belongs to a class of equilibrium problems.


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