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STABILIZATION OF A NONLINEAR PROTOTYPICAL WING SECTION WITH SELF-IMPROVEMENT OF CONTROL PERFORMANCE

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Abstract

In this paper a novel intelligent control is proposed for the purpose of active instability suppression of a nonlinear 2-D wing-flap aeroelastic system in the incompressible flow field. Lyapunov's direct method is utilized to establish a set of feasible controllers that can stabilize the system. A learning module is integrated to find the controller with smaller control error and less input energy within the feasible sets, realizing the self-improvement of control performance. The proposed control approach requires neither measurements of all states nor exact knowledge or estimation of nonlinearities. The simulation results are given to show the performance of the proposed control in suppressing the system's limit cycle oscillations in comparison with the feedback linearization method.

Key words

Nonlinear system, self-improvement of performance , aeroelastic control

1 Introduction

The non-occurrence of aeroelastic instability is a mandatory requirement within the aeroelastic context. It is well-known that nonlinearities, no matter structural or aerodynamical, may exhibit a variety of responses that are typically associated with nonlinear regimes of response including Limit Cycle Oscillation(LCO), flutter, and even chaotic vibrations [Dowell, Edwards, Strganac, (2003)] in aeroelastic systems. Significant decays of the flutter speed may happen and cause unexpected or even fatal accidents due to the existence of nonlinearities or the other uncertainties. Therefore, it is necessary to take uncertainties and nonlinearities into account in discussing aeroelastic problems.

In the last two decades, the advances of control technologies have rendered the applications of active flutter suppression feasible. In studies of flutter suppression of nonlinear systems, an aeroelastic model has been developed based on the research of the Benchmark Active Control Technology (BACT) wind-tunnel model designed at the NASA Langley Research Center [Waszak, (1997)], [Scott, Hoadley, Wieseman, et al. (2000)], [Bennett, Scott, Wieseman, (2000)] and [Mukhopadhyay, (2000)]. For this kind of model a set of wind-tunnel tests have been performed to examine the effect of nonlinear structural stiffness. To suppress the instability caused by nonlinearities, control systems have been designed using feedback linearizing technique, model reference adaptive control approaches, back-stepping design methods, robust control design with PI-observers and so on [Zeng, Singh, 1998], [Singh, Wang, (2002)], [Xing, Singh, 2000] and [Zhang, Söffker, 2009]. These methods stand for general approaches dealing with the effect of structural nonlinearities in aeroelastic problems.

However, the methods mentioned above have their own limitations because of their fixed dynamical behaviors which cannot guarantee a required control performance especially in the case of unknown effects like modeling errors or unknown inputs acting on the system. On the other hand, classical linear optimal control approaches like LQR or similar methods cannot be applied to this system because of its overall nonlinear behavior. This contribution proposes a new control strategy that is endowed with learning abilities to cope with the demand of improving the control performance.

In this paper, the Lyapunov's direct method is firstly utilized to provide a set of feasible control input functions to stabilize the nonlinear system. Because the set of controllers is obtained only with the information of the states related to the concerned motion, it is not necessary to get an estimation of all the states nor the estimation of nonlinearities. On the other hand, the output controllability of the system is required, as well as a stable zero dynamics. Defining a suitable performance index for a successful control in advance, one arbitrary control input function within the set is applied to the system, and the control performance will be fed back to the learning module as an evaluation signal, thereby the interaction between the control input and the related system performance response is formulated. The learning module uses this to create a new control input function with better control performance. After several iterations, the most optimized control input function within the feasible set will be found and memorized. If the illustrated control algorithm works successfully, the proposed control strategy can not only stabilize the nonlinear system, but also can accommodate itself from the interaction history to improve the control performance.

This paper is organized as follows: in Section 2 and 3 the system to be controlled and the design of the proposed controller are detailed. The simulation results of the proposed method, as well as its comparison with the feedback linearization approach, is given in the last part of this paper.

2 The aeroelastic example

The BACT wing-flap model has been widely studied in the aeroelastic research. The configuration of the nonlinear 2-D prototypical aeroelastic wing is shown in Fig.1. The two degrees of freedom, the pitching movement and the plunging one, are respectively restrained by a pair of springs attached to the elastic axis(EA) of the airfoil. A single trailing-edge control surface is used to control the air flow, thereby providing more maneuverability to suppress instability. This model is accurate for airfoils at low velocity and has been confirmed by wind tunnel experiments.

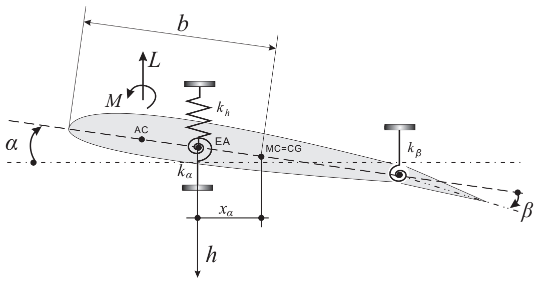

Figure 1. 2-D Wing-flap aeroelastic model.

The equations of motion governing the aerolastic system are given as

$$\begin{bmatrix} m_T & m_W x_\alpha b \\ m_W x_\alpha b & I_\alpha \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix}, \qquad (1)$$

where plunging and pitching displacement are denoted as h and α respectively. In Eq. (1) m_W denotes the mass of the wing, m_T represents the total mass of the wing and its support structure, b the semi-chord of the wing, I_α the moment of inertia, x_α the non-dimensional distance from the center of mass to the elastic axis, c_α and c_h the pitch and plunge damping coefficients respectively, k_α and k_h the pitch and plunge spring constants respectively, and M and L denote the quasi-steady aerodynamic lift and moment. In the case when the quasi-steady aerodynamics is considered, M and L should be written as

$$\left.\begin{aligned} L &= \rho U^2 b c_{l_\alpha}\left[\alpha + \tfrac{\dot{h}}{U} + (\tfrac{1}{2} - a) b \tfrac{\dot{\alpha}}{U}\right] \\ &\quad + \rho U^2 b c_{l_\beta} \beta \\ M &= \rho U^2 b^2 c_{m_\alpha}\left[\alpha + \tfrac{\dot{h}}{U} + (\tfrac{1}{2} - a) b \tfrac{\dot{\alpha}}{U}\right] \\ &\quad + \rho U^2 b^2 c_{m_\beta} \beta \end{aligned}\right\}, \qquad (2)$$

where c_{l_α} and c_{m_α} denote the lift and moment coefficients per angle of attack and c_{l_β} and c_{m_β} are lift and moment coefficients per angle of control surface deflection β.

The control objective is to drive the flap angle β properly so that the instability caused by structural nonlinearities can be suppressed in the vicinity of the nominal system flutter speed with smaller control errors and less input energy. It is supposed that the displacement and the velocity of the pitching motion, α and $\dot{\alpha}$, can be measured. The structural nonlinearity is supposed to exist in the pitching spring constant k_α and is assumed as to be a polynomial of α,

$$k_\alpha = \sum_{i=0}^{4} k_{\alpha_i} \alpha^i = k_{\alpha_0} + k_\alpha^*(\alpha) \;, \qquad (3)$$

where $k_\alpha^*(\alpha) = \sum_{i=1}^{4} k_{\alpha_i} \alpha^i$. The coefficients k_{α_i}, $i = 0, 1, ...4$ can be determined from experimental data [Singh, Wang, (2002)], which are assumed as unknown to the controller.

Due to the structural nonlinearity the open-loop response of the pitch motion is a limit cycle, as shown in Fig.2.

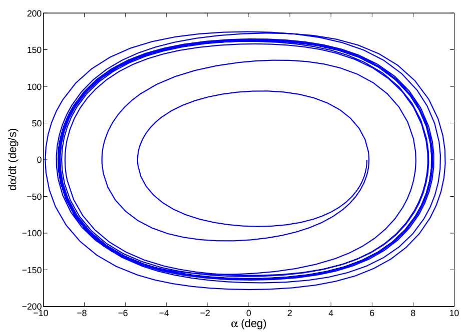

Figure 2. Limit cycle of system open-loop response.

3 Design of the control module

The design of the proposed intelligent control module is composed of two components: deriving the set of the feasible controllers that can stabilize the system, and finding the learning algorithm searching for the controller gains in order to accomplish self-improvement of control performance. These two components will be discussed in detail in the following two subsections.

3.1 Stabilization of the nonlinear system

It has been proved in [Behal, Marzocca, Dawson, et al. (2004)] that the wing-flap aeroelastic system is a minimum phase system. Choosing the state vector as

$$[\, z_i \,] = \begin{bmatrix} \alpha & \dot{\alpha} & h & -g_3\dot{h} + g_4\dot{\alpha} \end{bmatrix}^T, \tag{4}$$

where g_3 and g_4 are auxiliary coefficients, Eq.(1) can be transformed to the state space representation

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ f_n^1(z) \\ \frac{g_3}{g_4} z_2 + \frac{1}{g_4} z_4 \\ f_n^2(z) \end{bmatrix} + \begin{bmatrix} 0 \\ g_4 U^2 \\ 0 \\ 0 \end{bmatrix} \beta\,, \tag{5}$$

where $f_n^1(z)$ and $f_n^2(z)$ are nonlinear functions of the state vector z [Behal, Marzocca, Dawson, et al. (2004)].

It can be seen from Eq.(5) that the subsystem $[z_3,\ z_4]^T$ is not affected explicitly by the control input β. Moreover, it can be found that the zero dynamics of $[z_3,\ z_4]^T$ is asymptotically stable, which guarantees the local stability of the internal dynamics. This conclusion implies that if it can be found a suitable control input β with which the subsystem $[z_1,\ z_2]^T$ is asymptotically stable, the global system will also be asymptotically stable.

As a result, it is only necessary to consider the internal stability of the subsystem $[z_1,\ z_2]^T$. Therefore, the Lyapunov function is chosen as

$$V(z_1, z_2) = {z_1}^2 + {z_2}^2. \tag{6}$$

It can be seen that $V(z_1, z_2)$ is positive definite. The derivative of $V(z_1, z_2)$ with respect to t is

$$\dot{V}(z_1, z_2) = \dot{z}_1 z_1 + z_2 \dot{z}_2 = z_2(z_1 + \dot{z}_2). \tag{7}$$

According to the Lyapunov stability theory, a stable subsystem of states $[z_1,\ z_2]^T$ requires $\dot{V} < 0$, which is identical to the following equations

$$\left.\begin{array}{ll} z_1 + \dot{z}_2 > 0, & \text{if } \ z_2 < 0 \\ z_1 + \dot{z}_2 < 0, & \text{if } \ z_2 > 0 \end{array}\right\}. \tag{8}$$

Since the states z_1 and z_2 are measurable, the phase trajectory of the pitching motion can be generated and the velocity of leading point in the phase trajectory can be estimated. By this means the estimation of $\dot{z}_2$ at each time instant can be obtained numerically. From Eq.(5) it can be seen that the control input β can directly influence $\dot{z}_2$, thus it is reasonable to neglect the detailed dynamics of the subsystem $[z_1,\ z_2]^T$ and use directly the following control input to stabilize the system when $z_2(z_1 + \dot{z}_2) > 0$, the input can be chosen as

$$u_{in} = -K(z_1 + \dot{z}_2)\,, \tag{9}$$

where $u_{in} = g_4 U^2 \beta$, and $K > 1$.

Therefore, with a certain input gain K, the control input (9) can stabilize the nonlinear aeroelastic system. Apparently the input u_{in} does not require detailed information of the nonlinear function $f_n^1(z)$ and $f_n^2(z)$ in the system state space equations.

3.2 Self-improvement of control performance

The control performance is measured in the sense of Integral Squared Error (ISE) and Input Energy (IE), which are defined non-dimensionally as

$$\text{ISE} = \int_{t_0}^{t_1} \bar{z}^{\,T}\, \bar{z}\, \mathrm{d}t, \tag{10}$$

and

$$\text{IE} = \frac{1}{d} \int_{t_0}^{t_1} u_{in}^T\, u_{in}\, \mathrm{d}t, \tag{11}$$

where $\bar{z} = \frac{z}{d}$ and $d = \|{z_0}^T z_0\|_2$. Here z_o represents the distance in state space from the initial point to the stable equilibrium, as shown in Fig. (3)

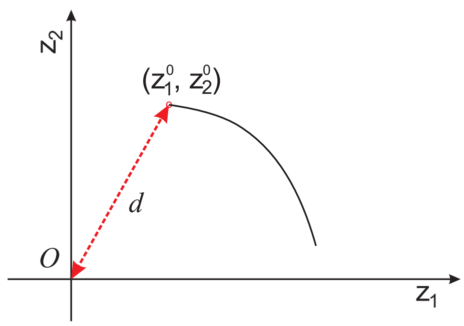

Figure 3. Definition of d in state space.

Similar as optimal control approaches with performance index of quadratic cost, the control performance measure is defined as the sum of ISE and IE with a predefined relation with each other, as

$$J = \text{ISE} + q\,\text{IE}, \tag{12}$$

where $q = \frac{\text{ISE}}{\text{IE}}$, denoting the pre-defined ratio between ISE and IE. The task of the performance self-improvement is to learn from the interaction between the control input and the system to find the best control input u_{in} , in order to minimize automatically the proposed performance measure.

The set S_K containing feasible control inputs is defined in Eq. (9). Different control inputs, represented by different values of K, can be located uniquely in the performance plane with coordinates comprising of ISE and IE.

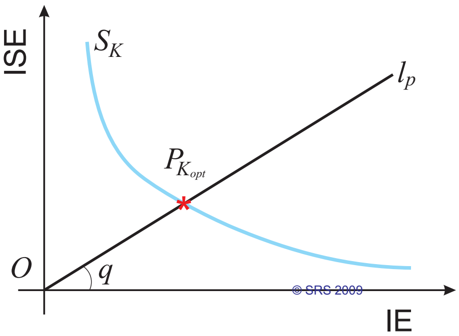

Figure 4. Searching for optimized control gain.

Depending on the demand for the relation between the input energy and the control errors, a constant value of q, denoted as q_{opt}, can be defined in advance as a constraint for the control performance. This relation can be shown as a straight line l_p with a slope of q_{opt} crossing the origin in the performance plane. The optimized control gain should be located at the point $P_{K_{opt}}$ which has the shortest distance to the origin among all the crossing points of the line l_p and the boundary of the set S_K as shown in Fig. 4.

The point $P_{K_{opt}}$ is obtained by the fixed-point iteration method which is well developed for solving nonlinear equations [Burden, Faires, 2001]. During the i-th iteration, for the specific control gain K_i, the closed loop system will generate a unique value of the ratio between ISE and IE, denoted as q_i. Therefore, the value of q_i can be seen as the value of an unknown function $q(K)$ when $K = K_i$, as

$$q_i = q(K_i). \tag{13}$$

Hence the learning process of searching for the optimized gain K_{opt} can be treated analog to the process solving the following equation

$$q_{opt} = q(K_{opt}). \tag{14}$$

Define the fix-point function $f(K)$ with the fix point $K = K_{opt}$ as

$$f(K) = K - (q(K) - q_{opt}). \tag{15}$$

By this way, the problem of approaching the point $P_{K_{opt}}$ in the performance plane is reformulated to solve the numerical nonlinear equation $f(K) = 0$ using the fix-point iteration technique. When the point $P_{K_{opt}}$ is found, the optimized gain K_{opt} will be stored as knowledge to realize a stable control with improved performance.

4 Simulation results

In this section, numerical results for the proposed control are presented. The values of the model parameters are taken from [Singh, Wang, (2002)]. The initial conditions for motions of the system are selected as $\alpha(0) = 5.75\,$deg, $h(0) = 0.01$m, $\dot{\alpha}(0) = 0\,$deg/s, and $\dot{h}(0) = 0$m/s.

The self-optimization process runs in the case of $q_{opt} = 0.2$. The gain for the first control input function is $K_1 = 1.35$. After 23 times of iteration the optimized gain $K_{opt} = 1.4012$ is found. The fixed-point iteration process is shown in Fig.(5)

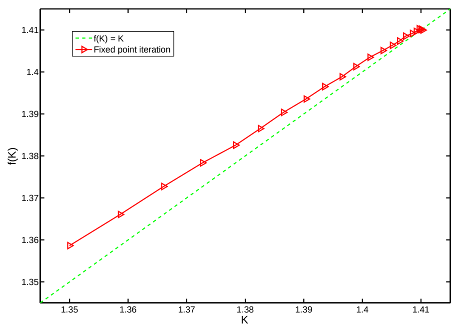

Figure 5. Fixed-point iteration searching for optimized gain.

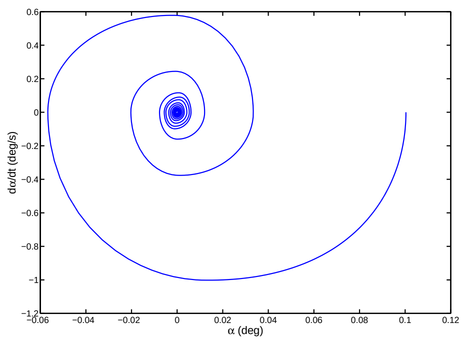

Figure 6. Phase trajectory of pitching motion.

The simulation of the proposed method stabilizing the nonlinear system is compared with feedback linearization method at wind speed $U = 20$m/s and with the

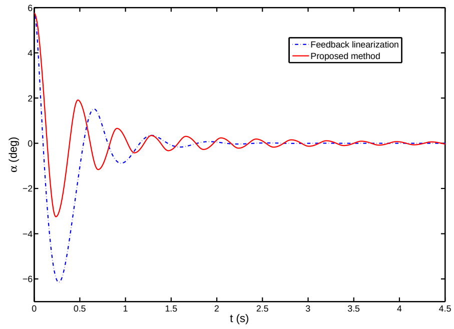

Figure 7.   Time history of pitching motion.

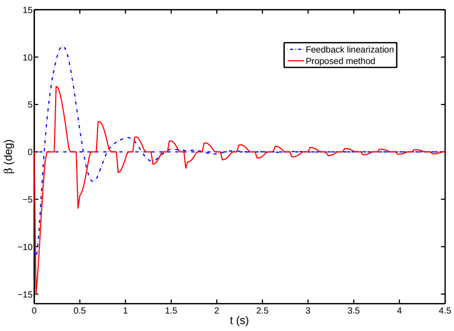

Figure 8.   Time history of flap motion.

non-dimensional distances from mid-chord to the elastic axis $a = 0.6847$. In Fig.(6) the stable phase trajectory of the pitching motion under the optimized control is shown . In Fig.(7) and (8) the time histories of the pitching motion α and flap motion β are shown for the case when the optimized control input function is applied, compared with the feedback linearization method.

From Fig.(7) and (8) it may not be clearly seen that the proposed method has great advantages compared with feedback linearization approach. However, the performance measure of these two methods varies prominently and starting from the performance point of view the superiority of the proposed method is straightforward for this example, as shown in Fig.(9). In Fig.(9) the position of the optimized control input and that of the feedback linearization in the performance plane are shown. This simulation is executed with the wind speed equals to 16m/s, 18m/s, and 20m/s separately. It can be seen that at all the different wind speeds the proposed method can not only reach the designed ratio between ISE and IE, but also stabilize the system using less input energy and smaller control error than the feedback linearization method.

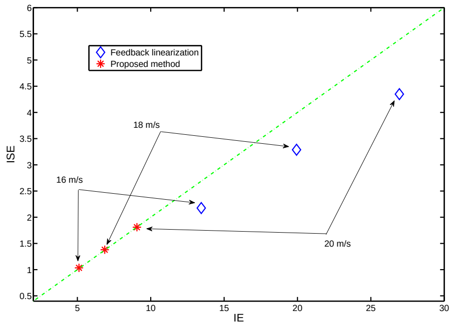

Figure 9.   Optimized control in performance plane.

5 Conclusion

In this paper, a new intelligent control algorithm is designed for the stabilization of a nonlinear wing-flap aeroelastic system for the first time. The proposed controller can not only suppress unstable motions, but also realize the self-improvement of control performance. Moreover, it requires no estimation of the structural nonlinearities, nor the fully measured states. It should be pointed out that the optimized control derived by the proposed method is not a global 'optimal' control. That is because the set of feasible control input functions is obtained by the Lyapunov direct method, which can only provide a sufficient condition for the system stability. Nevertheless, the simulation results show that compared with the feedback linearization method, the proposed controller which is endowed with more design degrees of freedom can stabilize the wing-flap system with smaller control error and less input energy.

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