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A MODIFIED GALERKIN APPROACH TO ADAPTIVE CONTROL DESIGN IN HEAT TRANSFER PROBLEMS WITH PARAMETER UNCERTAINTIES

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Abstract

Control problems for distributed systems, described by parabolic PDEs, are considered. The goal of the study is to develop an adaptive strategy including online parameter identification for efficient control of the systems. The developed strategy is based on the method of integrodifferential relations, a projective approach, and a suitable finite element technique. An adaptive control algorithm with predictive estimates of the desired output trajectories is proposed and its specific features are discussed.

Key words

Heat Trancfer, Adaptive Control, FEM.

1 Introduction

The design of adaptive control strategies for dynamic systems with distributed parameters has been actively studied in recent years. Processes such as heat transfer, diffusion, and convection are part of a large variety of applications in science and engineering. The theoretical foundation for optimal control problems with linear partial differential equations (PDEs) and convex functionals was established in [Lions, 1971]. In [Tao, 2003], some common and efficient adaptive control approaches, including model reference adaptive control, adaptive pole placement control, and adaptive backstepping control are presented and analyzed. The book [Krstic and Smyshlyaev, 2010] introduces a comprehensive methodology for adaptive control design of parabolic PDEs with unknown functional parameters, including reaction-convection-diffusion systems ubiquitous in chemical, thermal, biomedical, aerospace, and energy systems.

Different approaches to discretization of dynamical models with distributed parameters are developed to reduce the original initial-boundary value problem to an ODE system. It is worth noting the variational and projection methods used to solve control problems for such systems. The method of integrodifferential relations (MIDR) is proposed in [Kostin and Saurin, 2006] for the optimal control design of elastic beam motions. Variational principle on the basis of the MIDR are applied in [Aschemann et al, 2010] to parabolic PDE systems. A projective approach is developed in the frame of the MIDR for a heat transfer system in [Rauh et al, 2010]. In the paper this approach and FEM technique is applied to design an adaptive control strategy including online parameter identification and predictive estimates of the desired output trajectories.

2 Statement of the control problem

Consider a one-dimensional controlled process in a rod with the length L described by the following parabolic system of PDEs with boundary and initial conditions:

$$\begin{aligned}
&\xi = q + \lambda\theta' = 0, \\
&q' + \kappa_1\dot{\theta} + \kappa_2\theta = a(x)u(t) + b(x)v(t), \\
&q(0,t) = 0, \quad q(L,t) = 0, \\
&\theta(x,0) = \theta_0(x) \ .
\end{aligned} \tag{1}$$

Here $\theta(x,t)$ and $q(x,t)$ are the state variables; λ, κ_1, κ_2 are some material coefficients; u is the control input, v is the function of external disturbances; θ_0, a, and b are known functions of the spatial coordinate x. The dotted symbols denote the partial derivatives with respect to the time t , and the primed symbols stand for the partial derivatives with respect to the coordinate x .

We assume that $x = z_d$, $0 \le z_d \le L$, denotes the output position of the system. The goal of the control strategy is the computation of the control input $u(t)$ such that the output $\theta(z_d, t)$ coincides with a sufficiently smooth profile $y_d(t)$. To solve the initial-boundary value problem , we apply the MIDR, in which the local equality $\xi = 0$ and initial conditions are replaced by integral relations, whereas the first equation and boundary conditions are satisfied exactly in (1).

3 Discretization algorithm

Let us eliminate the function q taking into account the first equation and boundary conditions at $x = 0$ in (1) as follows

$$q(x,t) = \int_0^x [\ a(z)u(t) + b(z)v(t) - \kappa_1\dot{\theta} - \kappa_2\theta\ ]\, dz \tag{2}$$

and define a space mesh with the nodes: $x_0 = 0$, $x_M = L$, $0 \le x_{j-1} < x_j$, $I_j = (x_{j-1}, x_j)$, $j = 1, \dots, M$. To find an approximate solution of the problem (1), the function θ is approximated by piece-wise polynomial space splines

$$\begin{aligned} \theta \in S_w^{(N+2)} = \{\ \theta(t,x) :& \\ & \theta = \sum_{i=0}^{N} \vartheta_{ij}(t)\,(x/L)^j\,, \\ & x \in I_i,\ i = 1, \dots, M; \\ & \theta \in C^0, \quad x \in [0, L]\ \}\,, \end{aligned} \tag{3}$$

$$\vartheta(t) = \{\vartheta_{10}, \vartheta_1, \dots, \vartheta_M\}\,,$$
$$\vartheta_i = \{\vartheta_{i1}, \dots, \vartheta_{i,N}\}\,, \quad i = 1, \dots, M\ ,$$

where $\vartheta(t)$ is the vector-function defining the unknown state variable θ .

A projective approach is used to reduce the original PDE system to a system of ODEs with initial conditions in the form

$$\begin{aligned} & \int_0^L \xi(x, \vartheta, u, v)\chi(x)dx = 0, \\ & q(L, \vartheta, u, v) = 0, \\ & \int_0^L [\vartheta(x, \vartheta(0)) - \vartheta_0(x)]\,\chi(x)dx = 0 \\ & \forall \chi \in S_\chi^{(N)} = \{\ \chi(x) :\ \chi = \sum_{i=0}^{N-1} \chi_{ij} x^j / L^j, \\ & \qquad x \in I_i,\ i = 1, \dots, M\ \}\,. \end{aligned} \tag{4}$$

Here ξ is obtained by substituting relations (2) and (3) in (1). The following integral error is proposed to estimate the quality of this approximation

$$\begin{aligned} & \Delta = \Phi/\Psi, \quad \Phi = \int_0^T \int_0^L \varphi(t,x)dxdt, \\ & \Psi = \int_0^T \int_0^L \psi(t,x)dxdt, \\ & \varphi = \frac{\xi^2}{2}, \quad \psi = \frac{(\lambda\vartheta')^2}{2} \end{aligned} \tag{5}$$

4 Adaptive control strategy

The proposed adaptive control strategy takes into account a sequence of time steps $t \in [t_{k-1}, t_k]$, $t_k = kt_c$. At the initial time the vector $y = \{y_1, \dots, y_{N_y}\}$ of measurements $y_i = \theta(t, z_i^y)$, $z_i^y \in [0, L]$, $i = 1, \dots, N_y$, and a value v_1 for the function of external disturbances $v(t)$ are given. Using the current vector $y(t_{k-1})$, identified beforehand external function $v_k(t) = \mathrm{const}$, and the desired profile $y_d(t)$, the control $u_k(t) = \mathrm{const}$ is found in the *k*-th step by the following minimization

$$\begin{aligned} & u_k^* = \arg\min_{u_k} \left\{ \int_{t_k}^{t_k+t_p} \Delta y^2 dt \right\}, \\ & \Delta y = \theta(t, z_d, u_k, v_k) - y_d(t) \end{aligned} \tag{6}$$

and applied to the system at the beginning $t = t_k$ of the time step. At the end of this step, the vector $y(t_k)$ is measured and used together with the values $y(t_{k-1})$ and u_k to produce a new value v_{k+1} as follows

$$\begin{aligned} & v_{k+1}^* = \\ & \arg\min_{v_k} \left\{ \sum_{i=1}^{N_y} \left(\theta(t_k, z_i^y, u_k, v_k) - y_i(t_k)\right)^2 \right\}. \end{aligned} \tag{7}$$

After that, the current vector $y(t_{k-1})$ and the identified function v_{k+1} are used as the initial data for the next step. The control (t_c) and predictive (t_p) horizons are chosen to guarantee the stability of the control process.

5 Numerical results

Consider the heating system which has been built up at the Chair of Mechatronics of the University of Rostock [Aschemann et al, 2010; Rauh et al, 2010]. The following data for the problem and approximation are

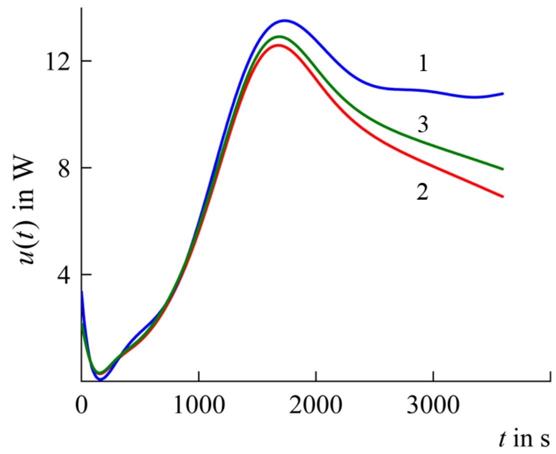

Figure 1. Control functions $u(t)$.

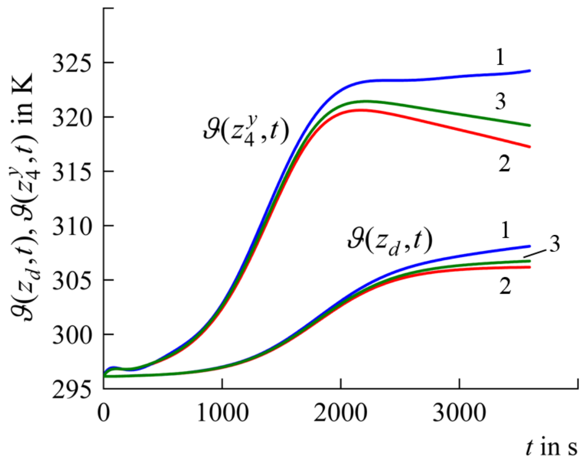

Figure 2. Temperature trajectories at the input and output positions.

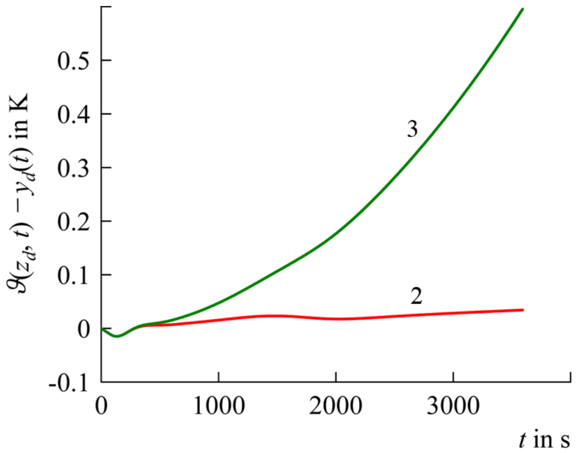

Figure 3. Temperature deviations from the desired profile for the adaptive control with (2) and without (3) identification

given:

$$\lambda = 110\ \mathrm{W/(m \cdot K)},$$

$$\kappa_1 = \ 3.276 \cdot 10^6 \mathrm{J/(m^3K)},$$

$$\kappa_2 = \ 4170\ \mathrm{W/(m^3K)},$$

$$L = \ 0.32\ \mathrm{m}, \quad x_n = nL/4, \quad n = 1, 2, 3,$$

$$z_i^y = (2i-1)L/8, \quad i = 1, \ldots, 4, \quad z_d = z_1^y$$

$$y_d\,(t) = \theta_0 + 5\,(1 + \tanh[\sigma(t - T/2)]\coth[\sigma T/2])\,,$$

$$\sigma = 0.0015, \quad T = 3600\,\mathrm{s}, \quad b(x) = \kappa_2,$$

$$v(t) = \theta_0 + 3t^2T^{-2}.$$

The control input is provided by a Peltier element so that if $x \in [3L/4, L]$ then $a(x) = 6510\,\mathrm{m}^{-3}$ else $a(x) = 0$. In Fig. 1, three control functions, optimal polynomial feedforward control ($v_k = \vartheta_0$, see [2]), adaptive with and without ($v_k = \vartheta_0$) identification, are presented by the curves 1, 2, and 3, respectively.

The temperatures at the output position ($z = z_d$) and at the middle of the control segment ($z = z_4^y$) are shown for these controls in Fig. 2 by curves with the same numbers. The relative error of the numerical solution is equal to $\Delta = 1.5 \cdot 10^{-4}$. Note that adaptive control strategy with identification gives the least deviation from the desired profile y_d.

In Fig. 3 the deviations of the temperature trajectories $\vartheta_1(z_d, t)$ from the desired profile $y_d(t)$ for the two adaptive controls are presented. The trajectory deviations for the feedforward control are rather large and not shown in this figure.

It is seen in Fig. 1–3 that the control obtained via feedforward strategy is the worst because no data about the rise of the ambient temperature is used in the optimization algorithm. By contrast, the adaptive control gives a better output trajectory even without any identification procedure, since the information about external disturbances is implicitly passed on the adaptive controller by means of temperature measurements $y(t)$. If the adaptive strategy involves the parameter identification the mathematical model is corrected in the adaptive controller during the process and can provide more accurate output trajectories.

The identified ambient temperature $\tilde{v}(t)$ and its error $\tilde{v}(t) - v(t)$ for the adaptive strategy with identification are given in Fig. 4 and 5, respectively. The deviations of the identified temperature from its actual values (Fig. 5) is much smaller then the maximal changing of the external temperature in the control process. The numerical simulations show that this error decreases if the time control horizon t_c becomes shorter. It also seen that the identification accuracy goes down if the rate of the temperature growth increases. This circumstance imposes certain constraints on the applicability of the adaptive algorithm proposed.

The local error distribution $\varphi(z, t)$ introduced by the

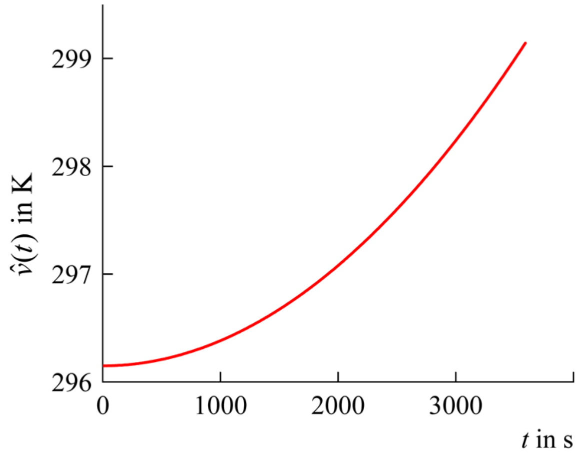

Figure 4. Identified ambient temperature $\hat{\vartheta}$.

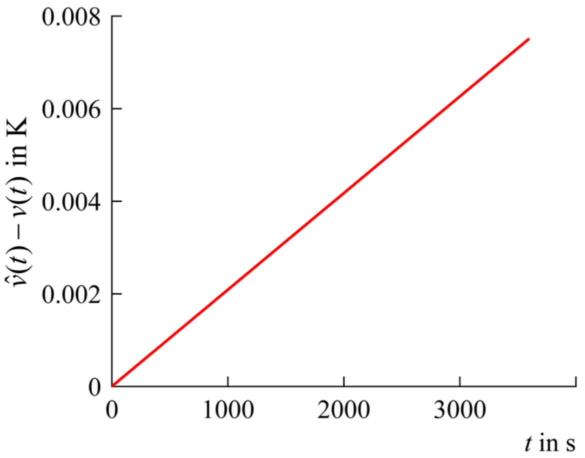

Figure 5. Deviations of the identified ambient temperature from its actual values.

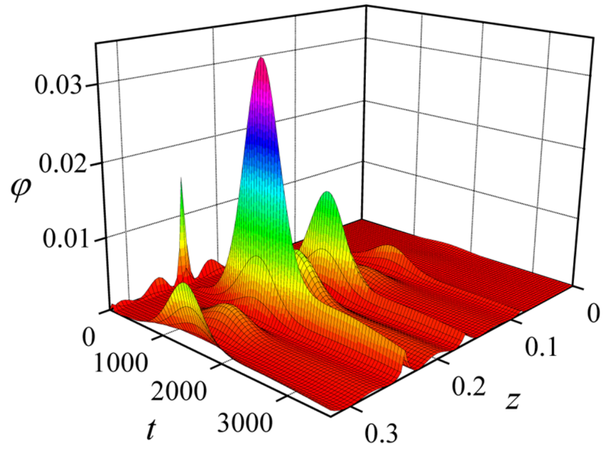

Figure 6. Local error distribution

Eq. (5) and obtained from numerical experiments for the adaptive control strategy with identification is depicted in Fig. 6 in the case when the order of the polynomial approximations on each finite element is $M = 2$. The corresponding relative integral error defined by Eq. (5) is small enough: $\Delta = 1.5 \cdot 10^{-4}$. If the order of approximations M is increased then the integral error is decreased notably. For example, for $M = 3$ and $M = 4$ the relative errors are equal to $\Delta = 1.6 \cdot 10^{-6}$ and $\Delta = 1.1 \cdot 10^{-7}$, respectively. Note that the function $\varphi(z, t)$ exposes imperfection of the applied finite-dimensional model and give one the possibility to develop new strategies of model refinement.

6 Conclusion

In this paper, the adaptive control algoritm with parameter identification for trajectory tracing in the distributed heating system is proposed and discussed. This control strategy is based on the method of integrodifferential relations, projective approach, and the finite element technique. The principle scheme of the adaptive control structure is worked out and its specific features are considered. A verification of control laws proposed is performed in numerical simulations taking into account the explicit local and integral error estimates resulted directly from the MIDR.

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References

Lions, J. L. (1971). *Optimal Control of Systems Governed by Partial Differential Equations*. Springer Verlag. New York.

Tao, G. (2003). *Adaptive Control Design and Analysis*. Wiley. New Jersey.

Krstic, M. Smyshlyaev, A. (2010). *Adaptive Control of Parabolic PDEs*. Princeton University Press.

Kostin, G. V. and Saurin, V. V. (2006) The optimization of the motion of an elastic rod by the method of integrodifferential relations. *J. Comp. Sys. Sci. Int.*, **45**(2), pp. 217–225.

Aschemann, H., Kostin, G. V., Rauh, A. and Saurin, V. V. (2010) Variational approaches to solving initial-boundary-value problems in the dynamics of linear elastic systems. *J. Comp. Sys. Sci. Int.*, **49**(3), pp. 380–391.

Rauh, A., Kostin, G. V., Aschemann, H., Saurin, V. V. Naumov, V. (2010) Verification and experimental validation of flatness-based control for distributed heating systems. *Int. Rev. Mech. Eng.*, **4**(2), pp. 188–200.