# TOWARDS APPLIED NONLINEAR ADAPTIVE CONTROL

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Abstract: A novel approach for the solution of nonlinear adaptive control problems is proposed. This approach does not rely on structural assumptions on the system to be controlled nor on linear parameterization. The approach is illustrated by means of several applications, including wing rock elimination, an adaptive visual servoing problem, the control of a power converter, and a flight control problem. Simulations and experimental results highlight the validity of the proposed methodology. 

Keywords: Adaptive control, nonlinear systems, visual servoing, power systems, flight control.

## 1. INTRODUCTION

This paper surveys a recently developed method for designing adaptively stabilising control laws for general nonlinear systems and demonstrates how such a method can be exploited to design control laws for physically motivated systems. The method relies upon the notions of system immersion and manifold invariance and, in principle, does not require the knowledge of a (control) Lyapunov function. The resulting adaptive control schemes counter the effect of the uncertain parameters adopting a robustness perspective. This is in contrast with some of the existing adaptive designs that (relying on certain matching conditions) treat these terms as disturbances to be rejected.

Finally, the proposed construction does not invoke certainty equivalence, nor requires a linear parameterisation and provides a systematic procedure to add cross terms between the parameter estimates and the plant states in the Lyapunov functions used to assess the properties of the adaptive system.

The paper is organized as follows. Section 2 introduces the basic tools for solving adaptive control problems and illustrates them on a simple example. Sections 3, 4, 5, 6 present four case studies. In the first one we solve the problem of aircraft wing rock elimination. The second one deals with a robot tracking problem using visual information. In the third one we design an adaptive output feedback voltage regulator for a DC–DC converter. The fourth one tackles the design of a complete flight control system for an autonomous aircraft. Finally, Section 7 wraps up the paper with some concluding remarks.

## 2. ADAPTIVE STABILIZATION VIA IMMERSION AND INVARIANCE

In this section the so-called Immersion and Invariance (I&I) methodology [1] is introduced. In particular, we present a generalization of the main theorem in (Astolfi and Ortega, 2003).

The basic idea in the I&I methodology is to achieve stabilisation by *immersing* the plant dynamics into a stable (lower-order) target system. Roughly speaking, a system $\Sigma_1$ is said to be immersed into a system $\Sigma_2$ if the input-output mapping of $\Sigma_2$ is a restriction of the input-output mapping of $\Sigma_1$, *i.e.,* any output response generated by $\Sigma_2$ is also an output response of $\Sigma_1$ for a restricted set of initial conditions, see, *e.g.,* (Byrnes *et al.*, 1997; Jouan, 2003).

[1] See (Astolfi and Ortega, 2003) and the forthcoming monograph (Astolfi *et al.*, 2007).

The main stabilization ideas in (Astolfi and Ortega, 2003) can be used to design adaptive stabilising controllers for classes of nonlinear systems with parametric uncertainties. To this end, consider the system

$$\dot{x} = f(x, u, \theta), \tag{1}$$

with state $x \in \mathbb{R}^n$, control $u \in \mathbb{R}^m$, unknown parameter $\theta \in \mathbb{R}^q$, and an equilibrium $x^*$ to be stabilised, and define the augmented system

$$\dot{x} = f(x, u, \theta), \qquad \dot{\hat{\theta}} = w, \tag{2}$$

where $\hat{\theta} \in \mathbb{R}^q$, and $w \in \mathbb{R}^q$ is a new control signal. The adaptive stabilisation problem can be posed, informally, as follows.

Find (if possible) a state feedback control law described by equations of the form

$$w = \omega(x, \hat{\theta}), \qquad u = v(x, \hat{\theta}) \tag{3}$$

such that all trajectories of the closed-loop system (2)-(3) are bounded and

$$\lim_{t\to\infty} x(t) = x^*. \tag{4}$$

Note that, since $f(\cdot)$ is only partially known, it is not required that $\hat{\theta}$ converges to any particular equilibrium, but merely that it remains bounded. The following theorem provides conditions under which the above problem is solvable using the I&I methodology.

*Theorem 1.* Consider the system (2) and a point $x^* \in \mathbb{R}^n$. Let $p \leq n$, $\xi \in \mathbb{R}^p$, $\zeta \in \mathbb{R}^{n-p}$ and $z \in \mathbb{R}^q$. Assume that there exist smooth mappings $\alpha(\xi, \theta) \to \mathbb{R}^p$, $\pi(\xi, \theta) \to \mathbb{R}^n$, $\phi(x, \theta) \to \mathbb{R}^{n-p}$, $c(\xi, \theta) \to \mathbb{R}^m$, $u(x, \zeta, z+\theta) \to \mathbb{R}^m$, $\omega(x, \zeta, z+\theta) \to \mathbb{R}^q$ and $\beta(x) \to \mathbb{R}^q$ such that the following hold.

*(H1)* The system

$$\dot{\xi} = \alpha(\xi, \theta) \tag{5}$$

has a globally asymptotically stable equilibrium at $\xi^* \in \mathbb{R}^p$ and $x^* = \pi(\xi^*, \theta)$.

*(H2)* For all $\xi \in \mathbb{R}^p$,

$$f(\pi(\xi, \theta), c(\xi, \theta), \theta) = \frac{\partial \pi}{\partial \xi}\alpha(\xi, \theta). \tag{6}$$

*(H3)* The set identity

$$\begin{aligned} \mathcal{M} &= \{x \in \mathbb{R}^n \mid \phi(x, \theta) = 0\} \\ &= \{x \in \mathbb{R}^n \mid x = \pi(\xi, \theta),\ \xi \in \mathbb{R}^p\} \end{aligned} \tag{7}$$

holds.

*(H4)* All trajectories of the system

$$\begin{aligned} \dot{\zeta} = {} & \frac{\partial \phi(x, \hat{\theta})}{\partial x} f(x, u(x, \zeta, z+\theta), \theta) + \\ & \frac{\partial \phi(x, \hat{\theta})}{\partial \hat{\theta}} \omega(x, \zeta, z+\theta), \end{aligned} \tag{8}$$

$$\dot{z} = \omega(x, \zeta, z+\theta) + \frac{\partial \beta}{\partial x} f(x, u(x, \zeta, z+\theta), \theta), \tag{9}$$

$$\dot{x} = f(x, u(x, \zeta, z+\theta), \theta) \tag{10}$$

are bounded and satisfy

$$\lim_{t\to\infty} \zeta(t) = 0, \tag{11}$$

$$\lim_{t\to\infty} [\phi(x(t), z(t)+\theta) - \phi(x(t), \theta)] = 0. \tag{12}$$

Then all trajectories of the closed-loop system

$$\begin{aligned} \dot{x} &= f(x, u(x, \phi(x, \hat{\theta}+\beta(x)), \hat{\theta}+\beta(x)), \theta) \\ \dot{\hat{\theta}} &= w(x, \phi(x, \hat{\theta}+\beta(x)), \hat{\theta}+\beta(x)) \end{aligned} \tag{13}$$

are bounded and satisfy (4).

Finally,

$$\lim_{t\to\infty} x(t) - \pi(\xi(t), \theta) = 0,$$

and, if $\phi(x(0), \hat{\theta}(0)) = 0$, $\hat{\theta}(0) - \theta + \beta(x(0)) = 0$ and $\xi(0) = \pi(x(0), \theta)$ $x(t) = \pi(\xi(t), \theta)$, for all $t \geq 0$.

*Proof.* Define the variables

$$\zeta = \phi(x, \hat{\theta}), \qquad z = \hat{\theta} - \theta + \beta(x)$$

and note that the dynamics of $\zeta$ and $z$ are given by (8) and (9). From (H4) and the fact that $z_2$ is globally diffeomorphic to $\hat{\theta}$ it follows that all trajectories of the closed-loop system are bounded and are such that (11) and (12) hold. This implies, by (H3), that $x(t)$ converges to $\pi(\xi, \theta)$, hence by (H1) all trajectories of the closed-loop system are such that (4) holds.

Finally, the last claim is a consequence of (H2), boundedness of solutions of system (10)-(11)-(12) and conditions (11) and (12). □

*Remark 1.* The condition (12) is strictly weaker than the requirement that $z$ converges to zero. However, if the latter holds, then $\hat{\theta}(t) + \beta(x(t))$ is an asymptotically converging estimate of the unknown parameter vector $\theta$.

In the original formulation of adaptive I&I, as given in (Astolfi and Ortega, 2003), the target dynamics are selected as the closed-loop system that would result from applying a stabilizing known-parameters controller. This implies that one can select $\pi(\xi, \theta) = \xi$ and that the variable $\zeta$ is not needed. Therefore, Theorem 1 provides a nontrivial extension of the results in (Astolfi and Ortega, 2003), and it leads to the following definition.

*Definition 1.* The system (2) is said to be adaptively I&I stabilisable with target dynamics (5) if (H1)–(H4) of Theorem 1 are satisfied.

*Example 1.* Consider the first-order nonlinear system

$$\dot{x} = \theta x^2 + u\,, \tag{14}$$

where $\theta \in \mathbb{R}$ is an unknown constant parameter. A stabilizing certainty-equivalence controller for the system (14) is given by $u = -kx - \hat{\theta}x^2$, with $k > 0$ and $\dot{\hat{\theta}} = \gamma x^3$. The above adaptive law is chosen to cancel the parameter-dependent terms from the time-derivative of the Lyapunov function $V(x, \hat{\theta}) = \frac{1}{2}x^2 + \frac{1}{2\gamma}\big(\hat{\theta} - \theta\big)^2$, which is given by

$$\dot{V}(x, \hat{\theta}) = -kx^2 + \left(\frac{\dot{\hat{\theta}}}{\gamma} - x^3\right)\left(\hat{\theta} - \theta\right) = -kx^2.$$

This establishes boundedness of $x$ and $\hat{\theta}$, and convergence of $x$ to zero. However, the result relies on a (possibly non-robust) cancellation of terms in the derivative of the Lyapunov function. Moreover, no conclusion can be drawn on the behaviour of the *estimation error* $\hat{\theta} - \theta$, except that it converges to a constant value.

I&I provides an alternative approach which avoids the cancellation and provides a mean of shaping the dynamic response of the estimation error. To illustrate this property, consider the augmented system

$$\dot{x} = \theta x^2 + u, \qquad \dot{\hat{\theta}} = w, \tag{15}$$

and the target system $\dot{\xi} = -k\xi$, with $k > 0$.

Following the proof of Theorem 1, consider, in the space $(x, \hat{\theta})$, the manifold

$$z = \hat{\theta} - \theta + \beta(x) = 0\,, \tag{16}$$

where $\beta(\cdot)$ is a continuous function yet to be specified. The dynamics of the system (15) restricted to the manifold (16) (provided it is invariant) are described by the equation

$$\dot{x} = \big(\hat{\theta} + \beta(x)\big)x^2 + u = \big(\theta + z\big)x^2 + u,$$

hence they are completely known and the equilibrium $x = 0$ is asymptotically stabilised by the control law

$$u = -kx - \big(\hat{\theta} + \beta(x)\big)x^2, \tag{17}$$

which yields a closed-loop system which is a copy of the target system.

For the above design to be feasible, the first step in the proposed approach consists in finding an update law $w$ that renders the manifold (16) *invariant*. To this end, consider the dynamics of $z$, which are given by the equation

$$\dot{z} = w + \frac{\partial \beta}{\partial x}\left[\big(\hat{\theta} + \beta(x) - z\big)x^2 + u\right],$$

and note that the update law

$$w = -\frac{\partial \beta}{\partial x}\left[\big(\hat{\theta} + \beta(x)\big)x^2 + u\right] = \frac{\partial \beta}{\partial x}kx \tag{18}$$

is such that the manifold (16) is invariant and the *off-the-manifold* dynamics are described by the equation

$$\dot{z} = -\frac{\partial \beta}{\partial x}x^2 z.$$

Selecting the function $\beta(\cdot)$ as

$$\beta(x) = \gamma\frac{x^3}{3}, \tag{19}$$

with $\gamma > 0$, yields the system

$$\dot{z} = -\gamma x^4 z. \tag{20}$$

Consider now the Lyapunov function $V = \frac{1}{2}z^2$, and note that $\dot{V} = -\gamma x^4 z^2 \leq 0$, hence $z = 0$ is a (uniformly) stable equilibrium and therefore $z(t) \in \mathcal{L}_\infty$. By integrating $\dot{V}$ it follows that $V(\infty) - V(0) = -\gamma\int_0^\infty |x^2(t)z(t)|^2 dt$, hence $x^2(t)z(t) \in \mathcal{L}_2$. From (14)-(17) this implies that $x(t)$ converges to zero. Moreover, the extra term $\beta(x)x^2$ in the control law (17) renders the closed-loop system (15)-(17)-(18) ISS with respect to $\hat{\theta} - \theta$. Note finally, that the closed-loop system can be regarded as the cascaded interconnection between two stable systems whose *gains* can be tuned via the arbitrary constants $k$ and $\gamma$. This *modularity* property is one of the prominent features of the I&I approach.

## 3. AIRCRAFT WING ROCK

We consider the problem of wing rock elimination in high-performance aircrafts. Wing rock is a limit cycle oscillation which appears in the rolling motion of slender delta wings at high angles of attack, see, *e.g.,* (Guglieri and Quagliotti, 2001; Hsu and Lan, 1985; Monahemi and Krstić, 1996) and the references in (Krstić *et al.*, 1995, Section 4.6) for more detail. This example has been adopted from (Monahemi and Krstić, 1996) and (Krstić *et al.*, 1995, Section 4.6), where a controller, based on the adaptive backstepping method, has been proposed.

Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 + \psi(x_1, x_2)^\top \theta \\ \dot{x}_3 &= \frac{1}{\tau}u - \frac{1}{\tau}x_3, \end{aligned} \tag{21}$$

where the states $x_1$, $x_2$ and $x_3$ represent the roll angle, roll rate and aileron deflection angle respectively, $\tau$ is the aileron time constant, $u$ is the control input, $\theta \in \mathbb{R}^5$ is an unknown constant vector and

$$\psi(x_1, x_2)^\top = \left[\,1,\ x_1,\ x_2,\ |x_1|x_2,\ |x_2|x_2\,\right].$$

The control objective is to regulate $x_1$ to zero. To begin with we augment the system (21) with the update law

$$\dot{\hat{\theta}} = w, \tag{22}$$

with $\hat{\theta} \in \mathbb{R}^5$ and $w \in \mathbb{R}^5$, and define the target system

$$\dot{\xi}_1 = \xi_2, \qquad \dot{\xi}_2 = -k_1\xi_1 - k_2\xi_2,$$

with $k_1 > 0$ and $k_2 > 0$. Setting $\pi_1(\xi, \theta) = \xi_1$ and $\pi_2(\xi, \theta) = \xi_2$ ensures that condition (H1) of Theorem 1 holds. Let $\beta(x) = \beta(x_1, x_2)$. Then from the first equation in (6) we obtain

$$\pi_3(\xi, \theta) = -k_1\xi_1 - k_2\xi_2 - \psi(\xi)^\top\theta,$$

hence the manifold $\mathcal{M}$ in (7) is described by the function

$$\begin{aligned}\phi(x, \theta) &= x_3 - \pi_3([x_1, x_2]^\top, \theta)\\ &= x_3 + k_1x_1 + k_2x_2 + \psi([x_1, x_2]^\top)^\top\theta.\end{aligned}$$

Consider now the variables

$$\zeta = \phi(x, \hat{\theta}), \qquad z = \hat{\theta} - \theta + \beta(x_1, x_2),$$

and note that the system (21)-(22) can be expressed in the $\zeta, z, x_1, x_2$ co-ordinates as

$$\begin{aligned}\dot{\zeta} &= \frac{1}{\tau}u - \frac{1}{\tau}x_3 + \delta_1(x, \hat{\theta})x_2 +\\ &\qquad \delta_2(x, \hat{\theta})\left(x_3 + \psi(x_1, x_2)^\top\theta\right) + \psi(x_1, x_2)^\top w,\\ \dot{z} &= w + \frac{\partial\beta}{\partial x_1}x_2 + \frac{\partial\beta}{\partial x_2}\left(x_3 + \psi(x_1, x_2)^\top\theta\right),\\ \dot{x}_1 &= x_2,\\ \dot{x}_2 &= -k_1x_1 - k_2x_2 - \psi(x_1, x_2)^\top z + \zeta,\end{aligned}$$

where

$$\delta_i(x, \hat{\theta}) = k_i + \frac{\partial\psi^\top(x_1, x_2)}{\partial x_i}\hat{\theta} + \frac{\partial(\psi(x_1, x_2)^\top\beta(x))}{\partial x_i}.$$

By an appropriate definition of the control laws $w$ and $u$ the above system can be rewritten as

$$\begin{aligned}\dot{\zeta} &= -\sigma(x, \hat{\theta})\zeta - \delta_2(x, \hat{\theta})\psi(x_1, x_2)^\top z,\\ \dot{z} &= -\frac{\partial\beta}{\partial x_2}\psi(x_1, x_2)^\top z,\\ \dot{x}_1 &= x_2,\\ \dot{x}_2 &= -k_1x_1 - k_2x_2 - \psi(x_1, x_2)^\top z + \zeta,\end{aligned} \tag{23}$$

for some function $\sigma(x, \hat{\theta})$, which clearly highlights the role of the perturbation term $\psi(x_1, x_2)^\top z$. In particular, when this term is zero, the $(x_1, x_2, \zeta)$ system has a globally asymptotically stable equilibrium at zero, provided $\sigma(x, \hat{\theta})$ is properly selected.

The design of the adaptive control law is thus completed by selecting the functions $\sigma(\cdot)$ and $\beta(\cdot)$ in (23) to satisfy condition (H4) of Theorem 1. The function $\beta(\cdot)$ can be selected as

$$\beta(x_1, x_2) = \gamma\left[x_2,\ x_1x_2,\ \frac{1}{2}x_2^2,\ \frac{1}{2}|x_1|x_2^2,\ \frac{1}{3}|x_2|x_2^2\right],$$

with $\gamma > 0$, yielding the error dynamics

$$\dot{z} = -\gamma\psi(x_1, x_2)\psi(x_1, x_2)^\top z, \tag{24}$$

for which $z = 0$ is a globally stable equilibrium, and $\psi(x_1, x_2)^\top z \in \mathcal{L}_2$. Finally, the function $\sigma(\cdot)$ can be selected as

$$\sigma(x, \hat{\theta}) = \lambda + \epsilon\delta_2(x, \hat{\theta})^2,$$

with $\lambda > 0$ and $\epsilon > 0$, which guarantees boundedness of all trajectories of the system (23).

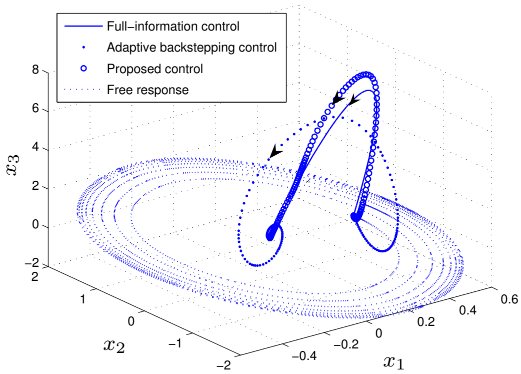


Fig. 1. State-space trajectory of the aircraft wing rock system for initial conditions $x_1(0) = 0.4$, $x_2(0) = x_3(0) = 0$.

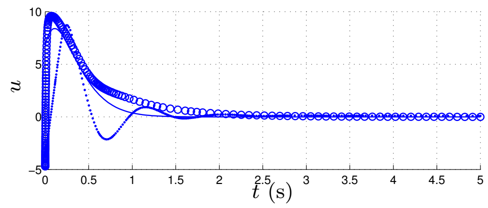


Fig. 2. Time history of the control input $u$. Dotted line: full-information controller. Dashed line: adaptive backstepping controller. Solid line: proposed controller.

The controlled aircraft wing rock system (21) has been simulated using the data provided in (Krstić *et al.*, 1995, Section 4.6). The design parameters are $k_1 = 25$, $k_2 = 10$, $\gamma = 100$, $\lambda = 5$, $\epsilon = 0.0001$. Note that the target dynamics have been selected to match the behaviour of the adaptive backstepping controller of (Krstić *et al.*, 1995, Section 4.6), hence the responses are directly comparable. In addition the full-information controller, which is obtained by assuming the parameters are known and applying feedback linearisation, has also been implemented. Figure 1 shows the trajectory of the controlled system and of the uncontrolled system ($u = 0$) for the initial conditions $x_1(0) = 0.4$, $x_2(0) = x_3(0) = 0$, $\hat{\theta}(0) = 0$, and Figure 2 shows the corresponding control efforts. Observe that the proposed adaptive scheme recovers the performance of the full-information controller and is considerably faster than the adaptive backstepping controller. The speed of response can be further increased (or reduced) by tuning the parameter $\gamma$. Note that, due to the form of the error dynamics (24), which is imposed by the selection of the function $\beta(\cdot)$, the speed of adaptation is directly related to the gain $\gamma$. This is in contrast with adaptive backstepping.

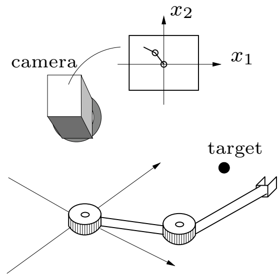


Fig. 3. Diagram of the visual servoing problem.

## 4. A VISUAL SERVOING PROBLEM

We illustrate, by means of a visual servoing problem, how adaptive I&I stabilisation can be applied in the nonlinearly parameterised case. Consider the visual servoing of a planar two-link robot manipulator in the so-called fixed-camera configuration, where the camera orientation and scale factor are unknown (Hutchinson *et al.*, 1996; Kelly, 1996; Astolfi *et al.*, 2002). The control goal is to place the robot end-effector in some desired constant position, or to make it track a (slowly moving) trajectory, by using a vision system equipped with a fixed camera that is perpendicular to the plane where the robot evolves, as depicted in Figure 3. We model the action of the camera as a static mapping from the joint positions $q \in \mathbb{R}^2$ to the position (in pixels) of the robot tip in the image output, denoted $x \in \mathbb{R}^2$. This mapping is described by

$$x = ae^{J\theta}\left(k(q) - \vartheta_1\right) + \vartheta_2, \tag{25}$$

where $\theta$ is the orientation of the camera with respect to the robot frame, $a \geq a_m > 0$ and $\vartheta_1$, $\vartheta_2$ denote intrinsic camera parameters (scale factors, focal length and centre offset, respectively), $k(\cdot) : \mathbb{R}^2 \to \mathbb{R}^2$ defines the robot direct kinematics, and

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \qquad e^{J\theta} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

Invoking standard time-scale separation arguments and assuming an inner fast loop for the robot velocity control, we concentrate on the kinematic problem of generating the references for the robot velocities. The robot dynamics are then described by a simple integrator $\dot{q} = v$, where $v \in \mathbb{R}^2$ are the joint velocities. The direct kinematics yield $\dot{k} = \mathcal{J}(q)\dot{q}$, where $\mathcal{J}(q) = \partial k/\partial q$ is the analytic robot Jacobian, which is assumed nonsingular. Differentiating (25) and replacing the latter expression yields the dynamic model of the overall system of interest, namely

$$\dot{x} = ae^{J\theta}u, \tag{26}$$

where $u = \mathcal{J}(q)v$ is a new input. The problem is to find a control law $u$ such that $x(t)$ asymptotically tracks a reference trajectory $x^*(t)$ in spite of the lack of knowledge of $a$ and $\theta$.

Note that, if $\theta$ were known, a stabilising control law for system (26) could be obtained without the knowledge of the uncertain parameter $a$. Indeed, the feedback

$$v(x,\theta) = -\frac{1}{a_m}e^{-J\theta}\left(\tilde{x} - \dot{x}^*\right),$$

where $\tilde{x} = x - x^*$, yields the target closed-loop dynamics

$$\dot{\tilde{x}} = -\frac{a}{a_m}\left(\tilde{x} - \dot{x}^*\right) - \dot{x}^*,$$

whose trajectories converge to zero if either $a/a_m = 1$ or $|\dot{x}^*(t)| \to 0$. Motivated by this property, the adaptive I&I methodology yields the following result.

*Proposition 1.* Consider the system (26) and a bounded reference trajectory $x^*$, with bounded first and second order derivatives $\dot{x}^*$, $\ddot{x}^*$, and assume that a positive lower bound on the scale factor $a$ is known, *i.e.,* $a \geq a_m > 0$. Then the adaptive I&I controller

$$u = -\frac{e^{-J\left(\hat{\theta}+\frac{1}{2}|s|^2\right)}}{a_m}s, \; \dot{\hat{\theta}} = s^\top\left(s + \dot{x}^* + \ddot{x}^*\right), \tag{27}$$

where $s = \tilde{x} - \dot{x}^*$, is such that all trajectories of the closed-loop system (26)–(27) are bounded and the tracking error either satisfies

$$\lim_{t\to\infty}|\tilde{x}(t) - w(t)| = 0, \tag{28}$$

where $w(t)$ is the solution of

$$\begin{aligned} \dot{w} = &-\frac{a}{a_m}e^{-J\arccos(a_m/a)}w \\ &+\Big(\frac{a}{a_m}e^{-J\arccos(a_m/a)} - I\Big)\dot{x}^*, \end{aligned} \tag{29}$$

with initial conditions $w(0) = \tilde{x}(0)$, or

$$\lim_{t\to\infty}|s(t)| = 0, \quad \lim_{t\to\infty}\tilde{x}(t) = 0. \tag{30}$$

In particular, if either $a_m = a$ or $\lim_{t\to\infty}|\dot{x}^*(t)| = 0$, then $\lim_{t\to\infty}\tilde{x}(t) = 0$.

*Sketch of the proof.* Let $z = \hat{\theta} - \theta + \beta(s)$, where $\beta(s) = \frac{1}{2}|s|^2$, and note that the closed-loop dynamics are given by

$$\dot{\tilde{x}} = -\frac{a}{a_m}e^{-Jz}\tilde{x} - \left(-\frac{a}{a_m}e^{-Jz} + I\right)\dot{x}_*. \tag{31}$$

Comparing with (29), it is clear that the control objective is achieved if $z(t) \to \arccos(a_m/a)$, which is established from the dynamics of $z$ which are given by (note that $s^\top e^{-Jz}s = |s|^2\cos(z)$)

$$\dot{z} = -|s|^2\left(\frac{a}{a_m}\cos(z) - 1\right).$$

□

The adaptive I&I controller (27) has been tested through simulations [2]. The simulations have been carried out in the same conditions as (Hsu and Aquino, 1999). A case of large disorientation is

[2] See (Zachi *et al.*, 2004) for an experimental study.

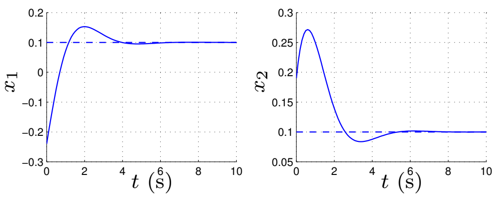


Fig. 4. Behaviour of the normalised image output signals $x_1(t)$ and $x_2(t)$ for $x_1^* = x_2^* = 0.1$.

taken into consideration: $\theta = 1$ rad, with $\hat{\theta}(0) = 0$ and we have used $a = 0.7$ and $a_m = 0.5$. The initial conditions of the manipulator are $q_1(0) = 1.3$ rad and $q_2(0) = -1.3$ rad. Figure 4 shows the evolution of the states for the set-point control case, with $x_1^* = x_2^* = 0.1$.

## 5. ADAPTIVE OUTPUT FEEDBACK CONTROL OF THE DC–DC BOOST CONVERTER

Consider the problem of designing an adaptive (output feedback) voltage regulator for the DC–DC boost converter with the topology shown in Figure 5. The problem of regulating these devices by PWM control schemes has been treated in (Kassakian *et al.*, 1991) and, in the framework of nonlinear control theory, in (Ortega *et al.*, 1998*a*) and related references, see also (Escobar *et al.*, 1999) for an experimental comparative study and (Rodríguez *et al.*, 2000) for a design example.

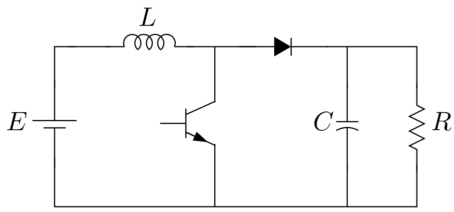


Fig. 5. Diagram of the DC–DC boost converter circuit.

The averaged model of the boost converter is given by the equations

$$L\dot{x} = -uy + E, \quad C\dot{y} = -\frac{1}{R}y + ux, \tag{32}$$

where $x$ is the input (inductor) current, $y$ is the output (capacitor) voltage, $u$ is the duty ratio of the transistor switch and $L$, $C$, $R$ and $E$ are positive constants representing the inductance, capacitance, load resistance and input voltage, respectively. It is assumed that only the state $y$ is available for feedback, the constant $E$ is unknown, and $u \in [\epsilon, 1]$ with $0 < \epsilon < 1$. The control problem is to regulate the output $y$ to a positive value $y^*$.

The system (32) can be rewritten as [3]

$$\begin{aligned} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L}uy \end{bmatrix} \\ \dot{y} &= -\frac{1}{RC}y + \frac{1}{C}u\theta_2, \end{aligned} \tag{33}$$

where $\theta_1 = E$ and $\theta_2 = x$ are unmeasured. A solution to the output voltage regulation problem is given by the following proposition.

*Proposition 2.* Consider the DC–DC boost converter described by the system (33) and the constant reference voltage $y^*$. Then, for any $0 < \epsilon < 1$, and any $y^*$ such that

$$\epsilon \le \frac{\theta_1}{y^*} \le 1, \tag{34}$$

there exists a dynamic output feedback control law such that all trajectories of the closed-loop system are bounded and $\lim_{t\to\infty} y(t) = y^*$.

*Proof.* To begin with note that the constraint (34) is a consequence of the control constraint $u \in [\epsilon, 1]$, *i.e.,* if $u \in [\epsilon, 1]$ then all equilibria of the system (33) are such that (34) holds. Consider now the full-information control law [4]

$$u^* = \mathrm{sat}_{[\epsilon,1]}(\frac{\theta_1}{y^*}), \tag{35}$$

which has been shown in (Rodríguez *et al.*, 2000) to yield asymptotic regulation. Moreover, using the function $V(\theta_2, y) = \frac{1}{2}L\theta_2^2 + \frac{1}{2}Cy^2 - a\theta_2 y$, with $0 < a < \min(\sqrt{LC}, \frac{2RLC\epsilon}{2R^2C\epsilon+L})$, it is straightforward to prove that, for any $u \in [\epsilon, 1]$, the trajectories of system (33) are bounded.

Consider now the error variable $z = \hat{\theta} - \theta + \beta(y)$, with

$$\beta(y) = \begin{bmatrix} \lambda_1 y \\ \lambda_2 y \end{bmatrix},$$

where $\lambda_1$ and $\lambda_2$ are positive constants, and define the dynamic output feedback controller

$$\begin{aligned} \dot{\hat{\theta}}_1 &= -\frac{\lambda_1}{C}\left(u\left(\hat{\theta}_2 + \lambda_2 y\right) - \frac{1}{R}y\right) \\ \dot{\hat{\theta}}_2 &= -\frac{\lambda_2}{C}\left(u\left(\hat{\theta}_2 + \lambda_2 y\right) - \frac{1}{R}y\right) \\ &\quad + \frac{1}{L}\left(\hat{\theta}_1 + \lambda_1 y - uy\right), \\ u &= \mathrm{sat}_{[\epsilon,1]}\left(\frac{\hat{\theta}_1 + \lambda_1 y}{y^*}\right). \end{aligned} \tag{36}$$

As a result, the dynamics of $z$ are given by the equation

$$\dot{z} = \begin{bmatrix} 0 & -\frac{\lambda_1}{C}u \\ \frac{1}{L} & -\frac{\lambda_2}{C}u \end{bmatrix} z\,. \tag{37}$$

[3] This example highlights the fact that the I&I approach allows to deal with unknown parameters and unmeasured states in a unified way.

[4] The saturation function is not required in the definition of $u^*$ since, from (34), the argument of the saturation is always in the interval $[\epsilon, 1]$. It is, however, required in the construction of the output feedback controller.

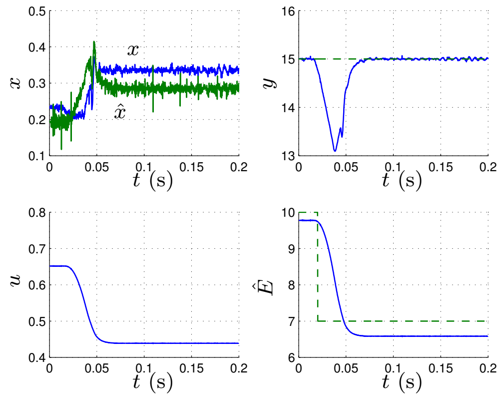


Fig. 6. State and control histories for a change in the input voltage.

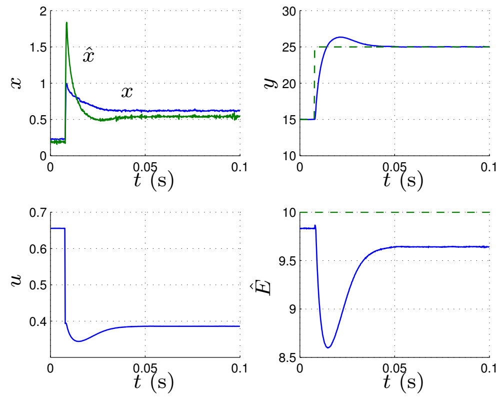


Fig. 7. State and control histories for a change in the output voltage reference.

The proof is completed by noting that, for any $0 < \epsilon < 1$, it is possible to select $\lambda_1 > 0$ and $\lambda_2 > 0$ such that the zero equilibrium of system (37) is uniformly asymptotically stable. □

Laboratory experiments have been carried out on the DC–DC boost converter described in detail in (Becherif *et al.*, 2002). The physical parameters of the converter are: $C = 94\ \mu$F, $L = 1.36\ m$H and $R = 120\ \Omega$. The switching frequency of the PWM is 20 $k$Hz. The desired output voltage is $y^* = 15$ V. The control lower bound $\epsilon$ has been fixed to 0.1 and $\lambda_1 = 0.2$ and $\lambda_2 = 1$ have been set to ensure uniform asymptotic stability of the zero equilibrium of system (37). The first experiment consists of a change in the input voltage $E$ from 10 V to 7 V, at $t = 0.02$ s. Figure 6 shows the inductor current $x$, together with its estimate $\hat{x} = \hat{\theta}_2 + \lambda_2 y$, the output voltage $y$, the control signal $u$, and the input voltage $E$ together with its estimate $\hat{E} = \hat{\theta}_1 + \lambda_1 y$. Observe that the output voltage tracks the desired value, despite partial state measurement and the change in the input voltage. In addition, the inductor current and the input voltage estimates approach the true values with a static error, which is due to unmodeled parasitic elements. Note that, even in the presence of this error, the (static) error in the output voltage is zero. This very useful property is due to the inherent integral action in the controller (36) [5]. Figure 7 shows the response of the system to a voltage reference change from $y^* = 15$ V to $y^* = 25$ V at $t = 0.01$ s, when $E = 10$ V. The output voltage tracks the desired reference value with a small overshoot.

In contrast with the full-information control law (35), the controller (36) requires knowledge of the parameter $R$. This is a potential drawback, since in most applications the load is not known precisely. Surprisingly, it can be shown that it is not necessary to know the precise value of $R$, but any constant estimate $\hat{R}$ within a certain range can be used in the implementation of the control law (36), *i.e.,* output voltage tracking is (locally) achieved despite the uncertainty in the parameter $R$. To validate the above argument select $\hat{R} = 120\ \Omega$. Figure 8 shows the response of the system to a step change in the load from $R = 120\ \Omega$ to $R = 60\ \Omega$ at $t = 0.01$ s, for $E = 10$ V and $y^* = 15$ V. Observe that the estimate of $E$ is not affected significantly; as a result, the output voltage is almost insensitive to load changes.

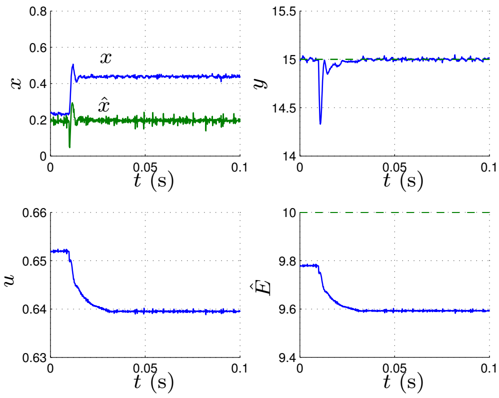


Fig. 8. State and control histories for a change in the load from $R = 120\ \Omega$ to $R = 60\ \Omega$.

## 6. ADAPTIVE FLIGHT CONTROL

Flight control systems are traditionally designed based on *linear* approximations of the aircraft dynamics around a large number of operating points (Stengel, 2004). These local designs are then combined using gain scheduling in order to cover the entire flight envelope, a procedure that is time-consuming and difficult to iterate. This fact has led in recent years to the wider use of nonlinear control techniques such as feedback lin-

[5] Note that from (36), ignoring the saturation, we obtain $\dot{\hat{\theta}}_1 - \frac{\lambda_1}{\lambda_2}\dot{\hat{\theta}}_2 = \frac{\lambda_1\left(\hat{\theta}_1 + \lambda_1 y\right)}{\lambda_2 L y^*}\left(y - y^*\right).$

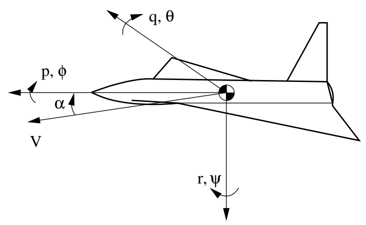


Fig. 9. Diagram of the aircraft body axes system.

earisation (Isidori, 1995; Enns *et al.*, 1994; Morton *et al.*, 1996), and backstepping (Krstić *et al.*, 1995; Härkegård and Glad, 2001). The main drawback of these methods is that they rely on *exact* knowledge of the aircraft dynamics and are therefore particularly sensitive to modelling errors. This problem can be alleviated by representing the uncertainties in the model as nonlinear functions with *unknown* coefficients and applying nonlinear adaptive control techniques, see, *e.g.,* (Sastry and Bodson, 1989; Krstić *et al.*, 1995; Singh and Steinberg, 1996; Calise and Rysdyk, 1998; Karagiannis and Astolfi, 2006).

We consider the problem of controlling a rigid-body aircraft so that it follows a predefined path in space with desired speed and attitude. The control actuation is assumed to be represented by three surface deflection angles (corresponding to ailerons, elevators and rudder) and the thrust. The variables that are assumed to be measurable are position, velocity, Euler angles and angular rates. The mathematical model used in the design is that of a six-degrees-of-freedom rigid body of constant mass, whose attitude is expressed in standard Euler angles (roll, pitch and yaw) based on a conventional body axes system, depicted in Figure 9. The proposed control scheme consists of the following modules.

1. An adaptive *attitude controller* that achieves asymptotic tracking of the roll, pitch and yaw angles, in the presence of aerodynamic moments with unknown coefficients.
2. An adaptive *airspeed controller* that achieves asymptotic regulation of the airspeed, in the presence of unknown drag and lift coefficients.
3. A *trajectory tracking controller* that produces the required references for the roll, pitch and yaw, in order to asymptotically follow a predefined geometric path.

In addition, the adaptive scheme generates *asymptotic* estimates of the unknown aerodynamic forces and moments acting upon the aircraft.

## 6.1 *Attitude control*

The equations of rotational motion are given by (Etkin and Reid, 1996)

$$\dot{\varphi} = R(\varphi)\omega \qquad I\dot{\omega} = S(\omega)I\omega + \mathcal{M}, \tag{38}$$

where $\varphi = [\phi, \theta, \psi]^\top$ are the Euler angles, $\omega = [p, q, r]^\top$ is the angular velocity vector, $I = [I_1^\top, I_2^\top, I_3^\top]^\top$ is the inertia matrix,

$$R(\varphi) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix},$$

$$S(\omega) = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix},$$

and $\mathcal{M} = [L, M, N]^\top$ are the (unknown) aerodynamic moments acting on the body.

We first consider the problem of forcing the aircraft attitude $\varphi$ to follow a smooth reference signal $\varphi_d$ by controlling the flaps. For simplicity (and to ensure that the adaptive control problem is solvable) we assume that the aerodynamic moments can be described by equations of the form

$$\mathcal{M} = \begin{bmatrix} \vartheta_1^\top \rho_1(I_1\omega) \\ \vartheta_2^\top \rho_2(I_2\omega) \\ \vartheta_3^\top \rho_3(I_3\omega) \end{bmatrix} + B\delta_f, \tag{39}$$

where $\delta_f = [\delta_a, \delta_e, \delta_r]^\top$ are the control inputs, corresponding to the aileron, elevator and rudder deflection angles, respectively, $\rho_i(\cdot)$ are continuous functions, $B \in \mathbb{R}^{3\times 3}$ is a known matrix function and $\vartheta_i$ are *unknown* parameter vectors.

Define the *tracking errors* $\tilde{\varphi} = \varphi - \varphi_d$, and $\tilde{\omega} = \omega - R_1(\varphi)^{-1}\dot{\varphi}_d$, where

$$R_1(\varphi)^{-1} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix},$$

and the energy function

$$H(\tilde{\varphi}, \tilde{\omega}) = \frac{1}{2}\tilde{\varphi}^\top K_1 \tilde{\varphi} + \frac{1}{2}\tilde{\omega}^\top I^2 \tilde{\omega}, \tag{40}$$

where $K_1 = K_1^\top > 0$ is a constant matrix, and note that in the new co-ordinates the system (38) can be written in the perturbed *port-controlled Hamiltonian* form (Ortega *et al.*, 2002)

$$\begin{bmatrix} \dot{\tilde{\varphi}} \\ I\dot{\tilde{\omega}} \end{bmatrix} = \begin{bmatrix} 0 & R_1(\varphi)I^{-1} \\ -I^{-1}R_1(\varphi)^\top & S_1(\omega) \end{bmatrix} \begin{bmatrix} \dfrac{\partial H^\top}{\partial \tilde{\varphi}} \\ \dfrac{\partial H^\top}{\partial (I\tilde{\omega})} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathcal{M} + \kappa(\varphi, \varphi_d) + \star \end{bmatrix}, \tag{41}$$

where

$$\begin{aligned} \kappa(\varphi, \varphi_d) &= I^{-1}R_1(\varphi)^\top K_1 \tilde{\varphi}, \\ \star &= S_1(\omega) I R_1(\varphi)^{-1}\dot{\varphi}_d - I\dot{R}_1(\varphi)^{-1}\dot{\varphi}_d \\ &\quad - IR_1(\varphi)^{-1}\ddot{\varphi}_d. \end{aligned}$$

Consider now the parameter estimation errors

$$z_i = \hat{\vartheta}_i - \vartheta_i + \beta_i(I_i\omega), \tag{42}$$

for $i = 1, 2, 3$, where $\beta_i(\cdot)$ are continuous functions to be defined. A control law $\delta_f$, which drives to zero the Hamiltonian function (40) along the trajectories of (41) when $z_i = 0$, is given by

$$\delta_f = -B^{-1}\begin{bmatrix} \left(\hat{\vartheta}_1 + \beta_1(I_1\omega)\right)^\top \rho_1(I_1\omega) \\ \left(\hat{\vartheta}_2 + \beta_2(I_2\omega)\right)^\top \rho_2(I_2\omega) \\ \left(\hat{\vartheta}_3 + \beta_3(I_3\omega)\right)^\top \rho_3(I_3\omega) \end{bmatrix} - B^{-1}\left(\kappa(\varphi,\varphi_d) + \star + K_2 I\tilde{\omega}\right),$$

where $K_2$ is a positive definite matrix function. The resulting closed-loop system can be written in the *perturbed* Hamiltonian form

$$\begin{bmatrix} \dot{\tilde{\varphi}} \\ I\dot{\tilde{\omega}} \end{bmatrix} = \begin{bmatrix} 0 & R_1(\varphi)I^{-1} \\ -I^{-1}R_1(\varphi)^\top & S_1(\omega) - K_2 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H^\top}{\partial\tilde{\varphi}} \\ \dfrac{\partial H^\top}{\partial(I\tilde{\omega})} \end{bmatrix} - \begin{bmatrix} 0 \\ \Delta \end{bmatrix}, \tag{43}$$

where each element of the perturbation vector $\Delta$ is given by

$$\Delta_i = z_i^\top \rho_i(I_i\omega). \tag{44}$$

Note that the system (43) is *converging-input converging-state* stable with respect to $\Delta$, for any $K_2 > 0$, and has an asymptotically stable equilibrium at the origin when $\Delta = 0$. We now design the functions $\beta_i(\cdot)$ so that the perturbation (44) is driven asymptotically to zero. To this end, consider the estimation errors (42) and the dynamic update laws

$$\dot{\hat{\vartheta}}_i = -\frac{\partial\beta_i}{\partial(I_i\omega)}\left[I_i\dot{\omega} + \Delta_i\right]. \tag{45}$$

Note that the term in brackets is a function of $\varphi$, $\omega$ and the first and second derivative of $\varphi_d$, therefore it is measurable. Using (45), the dynamics of the variables $z_i$ along the trajectories of (43) are given by

$$\dot{z}_i = -\frac{\partial\beta_i}{\partial(I_i\omega)}\rho_i(I_i\omega)^\top z_i.$$

Following the result in (Karagiannis and Astolfi, 2006) we select the functions $\beta_i(\cdot)$ according to

$$\beta_i(I_i\omega) = \gamma_i\int_0^{I_i\omega}\rho_i(s)\mathrm{d}s \Rightarrow \frac{\partial\beta_i}{\partial(I_i\omega)} = \gamma_i\rho_i(I_i\omega),$$

with $\gamma_i > 0$, yielding the system

$$\dot{z}_i = -\gamma_i\rho_i(I_i\omega)\rho_i(I_i\omega)^\top z_i, \tag{46}$$

which has a uniformly globally stable equilibrium at zero. In particular, the Lyapunov function $W(z_1, z_2, z_3) = \sum_{i=1}^3 \frac{1}{\gamma_i}|z_i|^2$ is such that $\dot{W} = -2|\Delta|^2$, which implies that the perturbation signal $\Delta$ is square-integrable. Asymptotic stability of the zero equilibrium of system (43)-(46) follows by considering the Lyapunov function $H(\tilde{\varphi}, \tilde{\omega}) + W(z_1, z_2, z_3)$ and invoking similar arguments as in (Karagiannis and Astolfi, 2006).

### 6.2 *Airspeed control*

The translational dynamics of the aircraft are given by

$$m\dot{\nu} = S(\omega)m\nu + mg\varepsilon(\phi,\theta) + \mathcal{F} + \mathcal{T}, \tag{47}$$

where $\nu = [u, v, w]^\top$ is the velocity vector, $m$ is the aircraft mass, $g$ is the gravitational acceleration, $\varepsilon(\phi,\theta) = [-\sin\theta, \sin\phi\cos\theta, \cos\phi\cos\theta]^\top$, $\mathcal{F} = [X, Y, Z]^\top$ are the aerodynamic forces, and $\mathcal{T} = [T_x, 0, 0]^\top$ is the thrust (along the $x$ body axis).

Consider now the problem of finding a control law for the thrust $T_x$ that regulates the total airspeed $V = |\nu|$ to a desired constant $V_d$. We assume that the aerodynamic forces acting on the aircraft are constant and unknown

To begin with define the *kinetic energy* error

$$E = \frac{1}{2}m\left(V^2 - V_d^2\right) \tag{48}$$

and note that, from (47), the dynamics of $E$ are given by

$$\dot{E} = \nu^\top mg\varepsilon(\phi,\theta) + \nu^\top\mathcal{F} + uT_x. \tag{49}$$

Define now the estimation error $\tilde{z} = \hat{F} - \mathcal{F} + \tilde{\beta}(\nu)$, where $\beta(\cdot)$ is a continuous functions to be defined, and the control law

$$T_x = -\frac{1}{u}\left[\nu^\top\left(\hat{F} + \tilde{\beta}(\nu)\right) + \nu^\top mg\varepsilon(\phi,\theta) + \kappa(V)E\right],$$

where $\kappa(V)$ is a positive function. Selecting the update law

$$\dot{\hat{F}} = -\frac{\partial\tilde{\beta}}{\partial\nu}\left[S(\omega)\nu + g\varepsilon(\phi,\theta) + \frac{1}{m}\left(\hat{F} + \tilde{\beta}(\nu) + \mathcal{T}\right)\right]$$

yields the closed-loop system

$$\dot{E} = -\kappa(V)E - \nu^\top\tilde{z}, \qquad \dot{\tilde{z}} = -\frac{1}{m}\frac{\partial\tilde{\beta}}{\partial\nu}\tilde{z}. \tag{50}$$

Finally, an appropriate selection of $\kappa(V)$ and $\tilde{\beta}(\nu)$ ensures that the *cascaded* system (50) has a globally stable equilibrium at the origin with $E$ converging to zero. Two such selections are $\kappa(V) = k$, $\tilde{\beta}_i(\nu) = \gamma_i\nu_i^3$ and $\kappa(V) = kV^2$, $\tilde{\beta}_i(\nu) = \gamma_i\nu_i$, with $k > 0$, $\gamma_i > 0$. Note that the first selection, similarly to the attitude control case, ensures that the *perturbation* signal $\nu^\top\tilde{z}$ in (50) is square-integrable.

### 6.3 *Trajectory tracking*

Consider now the problem of finding a reference $\varphi_d$ such that, when $\varphi = \varphi_d$, the aircraft converges to a predefined geometric path. For simplicity, we assume that the path to be tracked consists of *straight lines* or *arcs of circles* in the $x$-$y$ plane, while the desired altitude $h$ is constant. Note, however, that the proposed approach can also be used to track more complex paths.

The translational motion of the aircraft's centre of gravity is described by the equations

$$\begin{aligned}\dot{x} &= (u\cos\theta + v\sin\phi\sin\theta + w\cos\phi\sin\theta)\cos\psi \\ &\quad - (v\cos\phi - w\sin\phi)\sin\psi, \\ \dot{y} &= (u\cos\theta + v\sin\phi\sin\theta + w\cos\phi\sin\theta)\sin\psi \\ &\quad + (v\cos\phi - w\sin\phi)\cos\psi, \\ \dot{h} &= u\sin\theta - (v\sin\phi - w\cos\phi)\cos\theta.\end{aligned}$$

Using the identity

$$\begin{aligned}a\sin\psi + b\cos\psi &= \sqrt{a^2+b^2}\sin(\psi + \arctan(\frac{b}{a})) \\ &= \sqrt{a^2+b^2}\cos(\psi - \arctan(\frac{a}{b}))\end{aligned} \qquad (51)$$

the equations of motion can be rewritten as

$$\begin{aligned}\dot{x} &= V_1\cos\gamma\cos\psi - (v\cos\phi - w\sin\phi)\sin\psi, \\ \dot{y} &= V_1\cos\gamma\sin\psi + (v\cos\phi - w\sin\phi)\cos\psi, \\ \dot{h} &= V_1\sin\gamma,\end{aligned}$$

where $V_1 = \left(u^2 + (v\sin\phi + w\cos\phi)^2\right)^{1/2}$ and

$$\gamma = \theta - \arctan(\frac{v\sin\phi + w\cos\phi}{u}). \qquad (52)$$

For wings-level, non-sideslipping flight (*i.e.,* $\phi = 0$, $v = 0$), the variable $\gamma$ represents the *path angle*, while $V_1$ is equal to the total airspeed $V$. Applying the identity (51) once more yields the system

$$\dot{x} = V_2\cos\lambda, \qquad (53)$$

$$\dot{y} = V_2\sin\lambda, \qquad (54)$$

$$\dot{h} = V_1\sin\gamma, \qquad (55)$$

where $V_2 = \left((V_1\cos\gamma)^2 + (v\cos\phi - w\sin\phi)^2\right)^{1/2}$ and

$$\lambda = \psi + \arctan(\frac{v\cos\phi - w\sin\phi}{V_1\cos\gamma}). \qquad (56)$$

Note that for wings-level, non-sideslipping flight, the variable $\lambda$ represents the *heading*, while $V_2$ is equal to $V\cos\gamma$.

The trajectory tracking problem for the planar system (53)–(54), *i.e.,* the problem of tracking a given time-varying reference $x_d$, $y_d$ by controlling $\lambda$, has been studied extensively, see, *e.g.,* (Jiang *et al.*, 2001; Ren and Beard, 2004). In this work we are interested instead in tracking a *geometric* path defined by equations of the form

$$e_s = ax + by + c = 0 \qquad (57)$$

or

$$e_c = (x-a)^2 + (y-b)^2 - c^2 = 0, \qquad (58)$$

for some $a$, $b$ and $c$. Note that, in the case of a circular path, we must have $c \neq 0$. In addition, we require that the altitude $h$ is regulated around a constant reference $h_d$, *i.e.,* $\lim_{t\to\infty} h(t) = h_d$.

We first design $\lambda$ so that the lines (57) and (58) are invariant and globally attractive. To this end consider the dynamics of $e_s$ and $e_c$ which are given respectively by

$$\dot{e}_s = V_2\sqrt{a^2+b^2}\sin(\lambda + \arctan(\frac{a}{b})) \qquad (59)$$

and

$$\dot{e}_c = 2V_2\sqrt{e_c + c^2}\sin(\lambda + \arctan(\frac{x-a}{y-b})), \qquad (60)$$

where we have used the identity (51). Selecting $\lambda$ equal to

$$\lambda_s = -\arctan(\frac{a}{b}) - \arctan(k_p e_s),$$

or

$$\lambda_s = -\arctan(\frac{a}{b}) + \pi + \arctan(k_p e_s),$$

with $k_p > 0$, ensures that $e_s$ converges asymptotically to zero, while selecting $\lambda$ equal to

$$\lambda_c = -\arctan(\frac{x-a}{y-b}) - \arctan(k_p e_c),$$

or

$$\lambda_c = -\arctan(\frac{x-a}{y-b}) + \pi + \arctan(k_p e_c),$$

ensures convergence of $e_c$. Note that for each line there are two different control laws that drive the system onto the desired path in opposite directions.

Consider now the system (55) and the *control law*

$$\gamma = -\arctan(k_h(h - h_d))$$

with $k_h > 0$, yielding the closed-loop dynamics

$$\dot{h} = -V_1\frac{k_h(h-h_d)}{\left(1 + (k_h(h-h_d))^2\right)^{1/2}}.$$

The above system has a globally asymptotically stable equilibrium at $h = h_d$, hence the altitude converges to its reference value. Note finally that, given $\lambda$ and $\gamma$, it is possible to select $\theta$ and $\psi$ so that (52) and (56) hold, with $\phi$ a free variable. For straight flight we typically choose $\phi = 0$ (or, more generally, $\phi = n\pi$, where $n$ is an integer), while in the case of a steady coordinated turn along the path (58) we select $\phi$ according to[6] $\phi = \arctan(\frac{G}{\cos\alpha})$, where $G = V^2/(gc)$ is the centripetal acceleration (in $g$'s).

### 6.4 *Simulations*

The proposed controller has been implemented in a MATLAB/SIMULINK environment and combined with a detailed model of the Eclipse flight demonstrator developed at Cranfield University. Note that the model includes also actuator dynamics, which have been neglected in the control design. Moreover, we have assumed that the

[6] This equation gives the condition for turn coordination when the sideslip and path angle are zero. For a more general condition, see (Stevens and Lewis, 2003, p. 190).

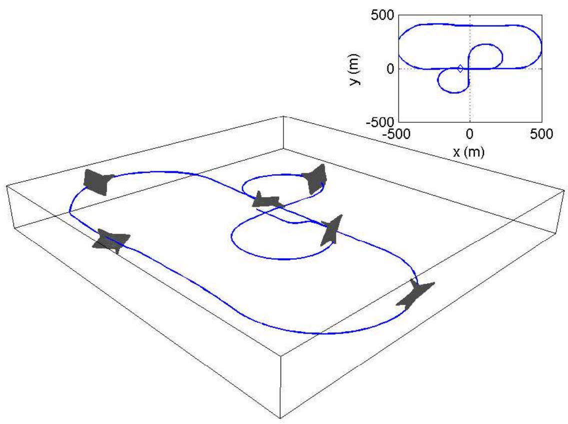


Fig. 10. Trajectory of the aircraft.

aerodynamic moments are linear functions, *i.e.,* $\rho_i(I_i\omega) = [1, I_i\omega]^\top$ in (39), and all parameter estimates are initialised at zero. The purpose of the simulations is to verify the asymptotic tracking properties of the controller, both during standard demonstration manoeuvres (climb, steady turn, dive), and non-standard ones (such as a 180-degree roll). The flight plan includes the following sectors.

(1) Initial speed 26 m/s. Climb from 30 m to 60 m at a rate of 12 m/s.
(2) Accelerate to 42 m/s.
(3) Perform a 180-degree turn of radius 200 m (at roll angle 43 deg).
(4) Perform a 180-degree roll.
(5) Perform a 180-degree turn of radius 200 m (at roll angle 137 deg).
(6) Resume level flight.
(7) Perform two successive 270-degree turns of radius 110 m (at roll angle 60 deg).

The trajectory of the aircraft is shown in Figure 10. The aircraft follows the desired trajectory with zero tracking error for the airspeed and attitude and zero steady-state error for the altitude, despite the lack of information on the aerodynamics and despite the fact that we have used a simplified aerodynamic model for the design.

## 7. CONCLUSION

Adaptive control of linear time invariant systems is a well–established discipline whose major theoretical issues have been fully sorted out and the main plant structural obstacles clearly identified. Many successful applications of adaptive control for linear plants have also been reported in the literature. Adaptive control of nonlinear systems, on the other hand, is a active research area where many questions remain to be answered. The main difference with respect to the case of linear plants is that the system in closed–loop with the known parameter controller still depends (in general) on the unknown parameters. (A notable exception to this rule are robot manipulators and, more generally, Euler–Lagrange systems, see e.g. (Ortega *et al.*, 1998*b*).) Consequently, the Lyapunov function, which is typically used to establish stability, is also a function of the parameters, and is hence unknown. The *classical* procedures to address the nonlinear adaptive control problem have been reviewed in (Astolfi and Ortega, 2003; Astolfi *et al.*, 2007)–we refer the interested reader to these references–where it is indicated that there is a clear predominance in the literature of the so–called adaptive backstepping approach, which assumes that the effect of the parameters can be *rejected* when considered as disturbances with known derivative. This leads to the so-called *extended matching* condition that, as is well known (Krstić *et al.*, 1995), is (essentially) satisfied only by systems in triangular forms. Furthermore, the requirement of linear parameterization is essential for the application of backstepping techniques. The I&I approach proposed adopts a robustness– instead of disturbance rejection–perspective that does not rely on matching nor linear parametrization. The fact that several physical systems are not in triangular form nor depend linearly on the unknown parameters provides a clear practical motivation for adaptive I&I. The proposed technique yields non-certainty equivalent controllers and does not postulate a-priori the existence of a Lyapunov function for the closed-loop system. However, if a Lyapunov perspective is adopted for the stability analysis, I&I naturally incorporates in the Lyapunov function cross terms between the plant states and the estimated parameters.

We have presented the basic theoretical tools of adaptive I&I and illustrated its applicability with some physically motivated problems. Even though adaptive I&I has been introduced to deal with nonlinear systems it is interesting to note that it has been instrumental also for the solution of some adaptive problems for linear systems. Namely, robust stabilization of relative degree one MIMO plants (Ortega *et al.*, 2003) and performance improvement of model reference schemes (Astolfi *et al.*, 2007).

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