GUIDANCE, ONBOARD SIGNAL PROCESSING AND ROBUST CONTROL OF AGILE FLEXIBLE REMOTE SENSING SPACECRAFT

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Abstract: Problems on guidance, onboard signal processing and robust gyromoment motion control of agile spacecraft (SC) for the Earth remote sensing and for the flexible payload transportation, are considered. Elaborated methods for dynamic research of the SC angular motion at principle modes under external and parametric disturbances, partial discrete measurement of the state, multiple filtering and digital control by the gyro moment cluster, are presented

Keywords: agile spacecraft, guidance, robust attitude control

1. INTRODUCTION

The dynamic requirements to the attitude control systems (ACSs) for remote sensing SC are:

- guidance the telescope's line-of-sight to a predetermined part of the Earth surface with the scan in designated direction;
- stabilization of an image motion at the onboard optical telescope focal plane.

Moreover, for the remote sensing spacecraft these requirements are expressed by rapid angular manoeuvering and spatial compensative motion with a variable vector of angular rate, see Fig. 1. Increased requirements to such information satellites (lifetime up to 10 years, spatial rotation manoeuvers with damping the SC flexible construction oscillations, robustness, fault-tolerance as well as to reasonable mass, size and energy characteristics) have motivated intensive development the gyro moment clusters (GMCs) based on excessive number of gyrodines (GDs) — single-gimbal control moment gyros.

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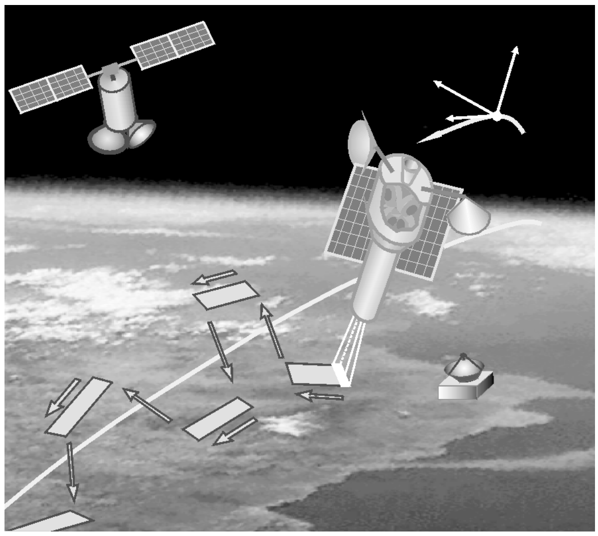

Fig. 1. The scanning pattern of given targets

Mathematical aspects of the SC nonlinear gyromoment control were represented in a number of research works (Junkins and Turner, 1986; Singh and Bossart, 1993; Hoelscher and Vadali, 1994; Schaub et al., 1998) including authors' papers. The paper suggests new results on guidance and nonlinear robust gyromoment attitude control of the agile SC.

2. MATHEMATICAL MODELS

Let us introduce the inertial reference frame (IRF) $\mathbf{I}_\oplus$ $(\mathrm{O}_\oplus \mathrm{X}_\mathrm{e}^\mathrm{I}\mathrm{Y}_\mathrm{e}^\mathrm{I}\mathrm{Z}_\mathrm{e}^\mathrm{I})$, the geodesic Greenwich reference frame (GRF) $\mathbf{E}_\mathrm{e}$ $(\mathrm{O}_\oplus \mathrm{X}^\mathrm{e}\mathrm{Y}^\mathrm{e}\mathrm{Z}^\mathrm{e})$ which is rotated with respect to the IRF by angular rate vector $\boldsymbol{\omega}_\oplus \equiv \boldsymbol{\omega}_\mathrm{e}$ and the geodesic horizon reference frame (HRF) $\mathbf{E}_\mathrm{e}^\mathrm{h}$ $(\mathrm{C}\,\mathrm{X}_\mathrm{c}^\mathrm{h}\mathrm{Y}_\mathrm{c}^\mathrm{h}\mathrm{Z}_\mathrm{c}^\mathrm{h})$ with origin in a point C and ellipsoidal geodesic coordinates altitude H_c, longitude L_c and latitude B_c, fig. 2. There are standard defined the SC

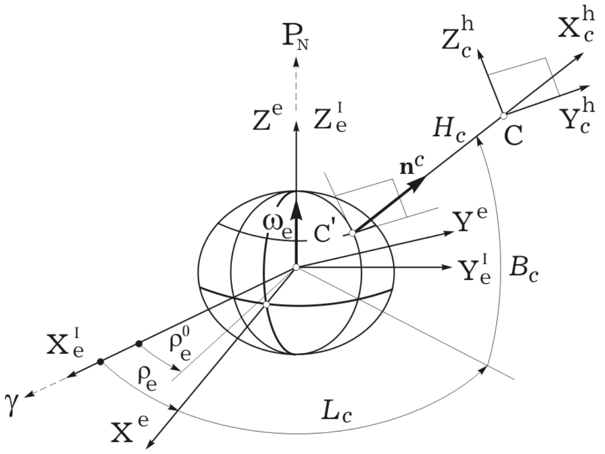

Fig. 2. The reference frames $\mathbf{I}_\oplus$, $\mathbf{E}_\mathrm{e}$ and $\mathbf{E}_\mathrm{c}^\mathrm{h}$

body reference frame (BRF) $\mathbf{B}$ $(\mathrm{O}xyz)$ with origin in the SC mass center O, the orbit reference frame (ORF) $\mathbf{O}$ $(\mathrm{O}x^\mathrm{o}y^\mathrm{o}z^\mathrm{o})$, the optical telescope (sensor) reference frame (SRF) $\boldsymbol{\mathcal{S}}$ $(\mathrm{O}x^\mathrm{s}y^\mathrm{s}z^\mathrm{s})$ and the image field reference frame (FRF) $\boldsymbol{\mathcal{F}}$ $(\mathrm{O}_i x^i y^i z^i)$ with origin in center O_i of the telescope focal plane $y^i\mathrm{O}_i z^i$.

The BRF attitude with respect to the IRF is defined by quaternion $\boldsymbol{\Lambda}_\mathrm{I}^b \equiv \boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda}), \boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$, and with respect to the ORF — by the column $\boldsymbol{\phi} = \{\phi_i, i = 1;3\}$ of *Euler-Krylov* elementary angles ϕ_i in the sequence $31'2''$. Let us vectors $\boldsymbol{\omega}(t)$, $\mathbf{r}_\mathrm{o}(t)$ and $\mathbf{v}_\mathrm{o}(t)$ are standard denotations of the SC body vector angular rate, the SC mass center position and progressive velocity with respect to the IRF, respectively. Further the symbols $\langle\cdot,\cdot\rangle$, $\times$, $\{\cdot\}$, $[\cdot]$ for vectors and $[\mathbf{a}\times]$, $(\cdot)^\mathrm{t}$ for matrixes are conventional denotations. For a fixed position of the SC *flexible* structures with some simplifying assumptions and $t \in \mathrm{T}_{t_0} = [t_0, +\infty)$ a SC angular motion model appears as:

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda}\circ\boldsymbol{\omega}/2; \quad \mathbf{A}^o\{\dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}, \ddot{\boldsymbol{\beta}}\} = \{\mathbf{F}^\omega, \mathbf{F}^q, \mathbf{F}^\beta\}, \qquad (1)$$

$$\mathbf{F}^\omega = \mathbf{M}^\mathrm{g} - \boldsymbol{\omega}\times\mathbf{G} + \mathbf{M}_d^o + \mathbf{Q}^o; \qquad \mathbf{M}^\mathbf{g} = -\dot{\boldsymbol{\mathcal{H}}} = -\mathbf{A}_\mathrm{h}\dot{\boldsymbol{\beta}};$$

$$\mathbf{F}^q = \{-a_{jj}^q((\delta^q/\pi)\Omega_j^q\dot{q}_j + (\Omega_j^q)^2 q_j) + \mathrm{Q}_j^q(\boldsymbol{\omega}, \dot{q}_j, q_j)\};$$

$$\mathbf{F}^\beta = \mathbf{A}_\mathrm{h}^\mathrm{t}\boldsymbol{\omega} + \mathbf{M}^g + \mathbf{M}_d^g + \mathbf{M}_f^g + \mathbf{Q}^g; \; \mathbf{A}_\mathrm{h} = [\partial\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})/\partial\boldsymbol{\beta}];$$

$$\mathbf{A}^o = \begin{bmatrix} \mathbf{J} & \mathbf{D}_q & \mathbf{D}_g \\ \mathbf{D}_q^\mathrm{t} & \mathbf{A}^q & \mathbf{0} \\ \mathbf{D}_g^\mathrm{t} & \mathbf{0} & \mathbf{A}^g \end{bmatrix}; \quad \begin{array}{l} \mathbf{G} = \mathbf{G}^o + \mathbf{D}_q\dot{\mathbf{q}} + \mathbf{D}_g\dot{\boldsymbol{\beta}}; \\ \mathbf{G}^o = \mathbf{J}\,\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}); \\ \boldsymbol{\omega} = \{\omega_i\}; \mathbf{q} = \{q_j\}; \boldsymbol{\beta} = \{\beta_p\}; \end{array}$$

the GMC's angular momentum (AM) vector $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = \mathrm{h}_g\sum\mathbf{h}_p(\beta_p)$, there $\mathrm{h}_g = \mathrm{const}$ is own AM value for each GD; a damping torque vector $\mathbf{M}_d^g$ is continuous function, and vector $\mathbf{M}_f^g$ of the friction torques in the GD's bearings is *discontinuous* function.

3. THE PROBLEM STATEMENT

Applied onboard measuring subsystem is based on initial gyro unit corrected by the fine fixed-head star trackers. This subsystem is intended for precise determination of the SC BRF $\mathbf{B}$ angular position with respect to the IRF $\mathbf{I}_\oplus$. Applied contemporary filtering & alignment calibration algorithms and a discrete astatic observer give finally a fine discrete estimating the SC angular motion coordinates by the quaternion $\boldsymbol{\Lambda}_s^\mathrm{m} = \boldsymbol{\Lambda}_s\circ\boldsymbol{\Lambda}_s^\mathrm{n}$, $s \in \mathbb{N}_0$, where $\boldsymbol{\Lambda}_s \equiv \boldsymbol{\Lambda}(t_s)$, $\boldsymbol{\Lambda}_s^\mathrm{n}$ is a "noise-drift" digital quaternion and a measurement period $T_q = t_{s+1} - t_s \leq T_u$ is multiply with respect to a control period T_u.

As for applied *2-SPE* scheme (Somov et al., 2005c), within precession theory of control moment gyros the GMC torque vector $\mathbf{M}^\mathbf{g}$ is presented as

$$\mathbf{M}^\mathbf{g} = -\dot{\boldsymbol{\mathcal{H}}} = -h_g\mathbf{A}_\gamma\mathbf{A}_h(\boldsymbol{\beta})\,\mathbf{u}; \;\; \dot{\boldsymbol{\beta}} = \mathbf{u}; \;\; \dot{\mathbf{u}} = \mathbf{v}. \qquad (2)$$

Here $\mathrm{u}_p^\mathrm{g}(t) = a^\mathrm{g}\,\mathrm{Zh}[\mathrm{Sat}(\mathrm{Qntr}(u_{pk}^g, b_u), B_u), T_u]$ with a constant a^g and a control period $T_u = t_{k+1} - t_k$, $k \in \mathbb{N}_0$; discrete functions $u_{pk}^g \equiv u_p^g(t_k)$ are outputs of nonlinear control law (CL), and functions $\mathrm{Sat}(x,a)$ and $\mathrm{Qntr}(x,a)$ are general-usage ones, while the holder model with the period T_u is such:

$$y(t) = \mathrm{Zh}[x_k, T_u] = x_k \, \forall t \in [t_k, t_{k+1});$$

matrix $\mathbf{A}_\gamma = \{[1,0,0], [0, s_\gamma, s_\gamma], [0, -c_\gamma, c_\gamma]\}$;

$$\mathbf{A}_h(\boldsymbol{\beta}) = \left[\frac{\partial\mathbf{h}(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}}\right] = \begin{bmatrix} -\mathsf{y}_1 & -\mathsf{y}_2 & -\mathsf{z}_3 & -\mathsf{z}_4 \\ \mathsf{x}_1 & \mathsf{x}_2 & 0 & 0 \\ 0 & 0 & -\mathsf{x}_3 & -\mathsf{x}_4 \end{bmatrix};$$

$\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = h_g\mathbf{h}^\mathbf{c}; \mathbf{h}^\mathbf{c} = \{x_c, y_c, z_c\} = \mathbf{A}_\gamma\mathbf{h}; \gamma = \mathrm{const};$ $\mathbf{h}(\boldsymbol{\beta}) = \{\mathsf{x}, \mathsf{y}, \mathsf{z}\} = \sum\mathbf{h}_p(\beta_p)$; $c_\alpha = \cos\alpha$; $s_\alpha = \sin\alpha$; $\mathsf{x} = \mathsf{x}_{12} + \mathsf{x}_{34}$; $\mathsf{y} = \mathsf{y}_1 + \mathsf{y}_2$; $\mathsf{z} = -(\mathsf{z}_3 + \mathsf{z}_4)$; $\mathsf{x}_p = c_{\beta_p}$; $\mathsf{y}_p = s_{\beta_p}$; $\mathsf{z}_p = s_{\beta_p}$; $\mathsf{x}_{12} = \mathsf{x}_1 + \mathsf{x}_2$; $\mathsf{x}_{34} = \mathsf{x}_3 + \mathsf{x}_4$; $\mathbf{u}(t) \equiv \{\mathrm{u}_p(t), p = 1:4\}$; $\mathbf{v}(t) \equiv \{\mathrm{v}_p(t), p = 1:4\}$; $\mathrm{v}_p = (\mathrm{Sat}(\bar{\mathrm{v}}, \mathrm{u}_p^\mathrm{c}(t)/T_u),\ if\ |\mathrm{u}_p| \leq \bar{\mathrm{u}}) \vee (0,\ if\ |\mathrm{u}_p| > \bar{\mathrm{u}})$.

Applied distribution law (Somov, 2002) of the normed AM $\mathbf{h}(\boldsymbol{\beta}) = \mathbf{A}_\gamma^{-1}\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})/h_g$ between GD's pairs ensures global maximum of *Grame* determinant $\mathsf{G} = \det(\mathbf{A}_h\mathbf{A}_h^\mathrm{t}) = 64/27$ and maximum module of the warranted control torque vector $\mathbf{M}^\mathbf{g}$ (2) in an arbitrary direction for the "park" state $\mathbf{h}(\boldsymbol{\beta}) = \mathbf{0}$, as well as large singularitiless central part inside of the GMC AM's variation domain, the singular set only at *separate* time moments (with *Lebesgue* zero measure) and *bijectively* connects the vector $\mathbf{M}^\mathbf{g}$ with vectors $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$. Problems consist in synthesis of the SC guidance laws for calculating the GMC control $\mathbf{u}_k^g = \{u_{pk}^g\}$ when the SC structure characteristics are uncertain and its damping is very weak.

4. SYNTHESIS OF FEEDBACK CONTROL

Applied general approach to synthesis of *nonlinear* control system (NCS) with a partial measurement of its state is presented, moreover the method of *vector Lyapunov functions* (VLF), which has a strong mathematical basis for analysis of stability and other dynamical properties of various nonlinear interconnected systems with the *discontinuous right-hand side*, is used in cooperation with the *exact feedback linearization* (EFL) technique. Let there be given a nonlinear controlled object

$$D^{+}\mathrm{x}(t)=\mathcal{F}(\mathrm{x}(t),\mathrm{u});\quad \mathrm{x}(t_0)=\mathrm{x}_0;\ t\in \mathrm{T}_{t_0},$$

where $\mathrm{x}(t)\in\mathcal{H}\subset\mathbb{R}^n$ is a state vector with an initial condition $\mathrm{x}_0\in\mathcal{H}_0\subseteq\mathcal{H}$; $\mathrm{u}=\{u_j\}\in\mathrm{U}\subset\mathbb{R}^r$ is a control vector. Let some *vector norms* $\rho(\mathrm{x})\in\overline{\mathbb{R}}_+^l$ and $\rho^0(\mathrm{x}_0)\in\overline{\mathbb{R}}_+^{l_0}$ also be *given*. For any control law (CL) $\mathrm{u}=\mathcal{U}(\mathrm{x})$ the closed-loop system has the form

$$D^{+}\mathrm{x}(t)=\mathcal{X}(t,\mathrm{x});\quad \mathrm{x}(t_0)=\mathrm{x}_0, \tag{3}$$

where $\mathcal{X}(t,\mathrm{x})=\mathcal{F}(\mathrm{x},\mathcal{U}(\mathrm{x})), \mathcal{X}:\mathrm{T}_{t_0}\times\mathcal{H}\to\mathcal{H}$ is a *discontinuous* operator. Assuming the existence and the non-local continuability of the *right-sided* solution $x(t)\equiv\mathrm{x}(t_0,\mathrm{x}_0;t)$ of the system (3) for its *extended definition* in the aspect of physics, the most important dynamic property is obtained, that is $\rho\rho^0$-*exponential invariance* of the solution $x(t)=0$ under the *desired* $\gamma\in\overline{\mathbb{R}}_+^l$:

$$(\exists\alpha\in\mathbb{R}_+)\,(\exists\mathcal{B}\in\overline{\mathrm{B}}_+^{l\times l_0})\,(\exists\delta\in\mathbb{R}_+^{l_0})\,(\forall\rho^0(\mathrm{x}_0)<\delta)$$
$$\rho(x(t))\le\gamma+\mathcal{B}\,\rho^0(\mathrm{x}_0)\,\exp(-\alpha(t-t_0))\quad\forall t\in\mathrm{T}_{t_0}.$$

For the VLF $\upsilon:\mathcal{H}\to\overline{\mathbb{R}}_+^k$ with components $\upsilon^s(\mathrm{x})\ge 0$, $\upsilon^s(0)=0$, $s=1:k$ and the norm $\|\upsilon(\mathrm{x})\|=\max\{\upsilon^s(\mathrm{x}),s=1:k\}$, defined are the scalar function $\overline{\upsilon}(\mathrm{x})=\max\{\upsilon^s(\mathrm{x}),s=1:l_k,1\le l_k\le k\}$ and a *lower right derivative* with respect to (3):

$$\underline{\upsilon}'(\mathrm{x})\equiv\varliminf_{\delta t\to 0+}\{(\upsilon(\mathrm{x}+\delta t\,\mathcal{X}(t,\mathrm{x}))-\upsilon(\mathrm{x}))/\delta t\}.$$

Theorem. *Let there exist the VLF υ, so that:*

1) $(\exists\mathrm{a}\in\mathbb{R}_+^l)\ (\forall\mathrm{x}\in\mathcal{H})\qquad \rho(\mathrm{x})\le\mathrm{a}\cdot\overline{\upsilon}(\mathrm{x})$;
2) $(\exists\mathrm{b}\in\mathbb{R}_+^{l_0})\ (\forall\mathrm{x}_0\in\mathcal{H}_0)\ \|\upsilon(\mathrm{x}_0)\|\le\langle\mathrm{b},\rho^0(\mathrm{x}_0)\rangle$;
3) $\exists\gamma_c\in\overline{\mathbb{R}}_+^k$ *and a function* $\varphi_\gamma(\cdot)$ *exists so that* $\gamma_c\le\varphi_\gamma(\mathrm{a},\gamma)$;
4) $\forall\ (t,\mathrm{x})\in(\mathrm{T}_{t_0}\times\mathcal{H})$ *the conditions are satisfied*:
   a) $\underline{\upsilon}'_\gamma(\mathrm{x})\dot{\le}\mathrm{f}_c(t,\upsilon_\gamma(\mathrm{x}))\equiv\mathrm{P}\upsilon_\gamma(\mathrm{x})+\tilde{\mathrm{f}}_c(t,\upsilon_\gamma(\mathrm{x}))$;
   b) Hurwitz condition for positive matrix P;
   c) Ważewski condition on quasi-monotonicity for the function $\tilde{\mathrm{f}}_c(t,\mathrm{y})$;
   d) Carateodory condition for the function $\tilde{\mathrm{f}}_c(t,\mathrm{y})$, *bounded in each domain* $\Omega_c^r=(\mathrm{T}_{t_0}\times\mathcal{S}_c^r)$, *where* $r>0$ *and* $\mathcal{S}_c^r=\{\mathrm{y}\in\mathbb{R}^k:\|\mathrm{y}\|_{\mathrm{E}}<r\}$;
   e) $(\tilde{\mathrm{f}}_c(t,\mathrm{y})/\|\mathrm{y}\|)\overset{t\in\mathrm{T}_{t_0}}{\Longrightarrow}0$ *for* $\mathrm{y}\to 0$ *uniformly with respect to time* $t\in\mathrm{T}_{t_0}$,

where $\upsilon_\gamma=\upsilon-\gamma_c$. *Then solution* $x(t)=0$ *of the system* (3) *is* $\rho\rho^0$*-exponential invariant and the matrix* $\mathcal{B}$ *has the form* $\mathcal{B}=c\cdot\mathrm{ab}^{\mathrm{t}}$ *with* $c\in\mathbb{R}_+$.

Proof. The basis of inequality for vector norm $\rho(x(t))$ is attained by the *comparison principle*, using the *maximum right-sided* solution $\overline{x}_c(t)\equiv\overline{\mathrm{x}}_c(t_0,\mathrm{x}_{c0};t)$ of a *comparison system* $\dot{\mathrm{x}}_c(t)=\mathrm{P}\,\mathrm{x}_c(t)+\tilde{\mathrm{f}}_c(t,\mathrm{x}_c(t))$, see Somov et al. (1999).

There is such an important problem: by what approach is it possible to create *constructive* techniques for constructing the VLF $\upsilon(\mathrm{x})$ and *simultaneous* synthesis of a nonlinear control law $\mathrm{u}=\mathcal{U}(\mathrm{x})$ for the close-loop system (3) with given vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x}_0)$? Recently, a pithy technique on constructing VLF at such synthesis has been elaborated. This method is based on a *nonlinear transformation* of the NCS model and solving the problem in two stages.

In stage 1, the right side $\mathcal{F}(\cdot)$ in (3) is transformed as $\mathcal{F}(\cdot)=\mathrm{f}(\mathrm{x})+\mathrm{G}(\mathrm{x})\,\mathrm{u}+\tilde{\mathcal{F}}(t,\mathrm{x}(t),\mathrm{u})$, some *principal variables* in a state vector $\mathrm{x}\in\tilde{\mathcal{H}}\subset\mathbb{R}^{\tilde{n}}\subseteq\mathbb{R}^n$ with $\tilde{n}\le n$, $\mathrm{x}_0\in\tilde{\mathcal{H}}_0\subseteq\tilde{\mathcal{H}}$ are selected and a *simplified nonlinear model* of the object (3) is presented in the form of an affine *quite smooth* nonlinear control system

$$\dot{\mathrm{x}}\ =\ \mathrm{F}(\mathrm{x},\mathrm{u})\ \equiv\ \mathrm{f}(\mathrm{x})+\mathrm{G}(\mathrm{x})\mathrm{u}\ \equiv\ \mathrm{f}(\mathrm{x})+\textstyle\sum\mathrm{g}_j(\mathrm{x})u_j,$$

which is structurally synthesized by the EFL technique. In this aspect, based on the structural analysis of *given* vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x})$, and also vector-functions $\mathrm{f}(\mathrm{x})$ and $\mathrm{g}_j(\mathrm{x})$, the *output vector-function* $\mathrm{h}(\mathrm{x})=\{\mathrm{h}_i(\mathrm{x})\}$ is carefully selected. Furthermore, the nonlinear invertible (one-to-one) coordinate transformation $\mathrm{z}=\Phi(\mathrm{x})\ \forall\mathrm{x}\in\mathcal{S}_{\mathrm{h}}\subseteq\tilde{\mathcal{H}}$ with $\Phi(0)=0$ is analytically obtained with *simultaneous* constructing the VLF. Finally, bilateral componentwise inequalities for the vectors $\mathrm{x},\mathrm{z},\upsilon(\mathrm{x}),\rho(\mathrm{x}),\rho^0(\mathrm{x}_0)$ are derived, it is most desirable to obtain the *explicit* form for the nonlinear transformation $\mathrm{x}=\Psi(\mathrm{z})$, inverse with respect to $\mathrm{z}=\Phi(\mathrm{x})$, and the VLF aggregation procedure is carried out with analysis of proximity for a singular directions in the *Jacobian* $[\partial\mathrm{F}(\mathrm{x},\mathcal{U}(\mathrm{x}))/\partial\mathrm{x}]$.

In stage 2, the problem of nonlinear CL synthesis for the *complete model* of the NCS (3), taking rejected coordinates, nonlinearities and restrictions on control, into account is solved by the VLF-method. If a forming control is digital, a measurement the model's state is discrete and incomplete, then a simplified nonlinear discrete object's model is obtained by *Teylor-Lie* series, a *nonlinear* digital CL is formed and its parametric synthesis is carried out with a simultaneously construct a *discrete sub-vector* VLF.

5. GUIDANCE AT A COURSE MOTION

The analytic matching solution have been obtained for problem of the SC angular guidance at a course motion (CM) when a space opto-electronic observation is executed at given time interval $t\in\mathrm{T}_n\equiv[t_0^n,t_f^n]$, $t_f^n\equiv t_0^n+T_n$. This problem consists in determination of quaternion $\mathbf{\Lambda}(t)$ by the SC BRF $\mathbf{B}$ attitude with respect to the IRF $\mathbf{I}_\oplus$, angular rate vector $\boldsymbol{\omega}(t)=\{\omega_i(t)\}$, vectors of angular acceleration $\boldsymbol{\varepsilon}(t)=\{\varepsilon_i(t)\}=\dot{\boldsymbol{\omega}}(t)$ and its derivative $\dot{\boldsymbol{\varepsilon}}(t)=\boldsymbol{\varepsilon}^*(t)+\boldsymbol{\omega}(t)\times\boldsymbol{\varepsilon}(t)$ in the form of *explicit* functions, proceed from principle requirement: optical image of the Earth given part must to move by desired way at focal plane $y^i\mathrm{O}_i z^i$ of the telescope. Solution is based on a vector composition of all elemental motions in the GRF $\mathbf{E}_\mathrm{e}$ with regard to initial coordinates, the scan azimuth, the Sun zenith angle and a current observation perspective, using next reference frames: the HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, the SRF $\boldsymbol{\mathcal{S}}$ and the FRF $\boldsymbol{\mathcal{F}}$. Vectors $\mathbf{r}_\mathrm{o}(t)$ and $\mathbf{v}_\mathrm{o}(t)$ are presented in the GRF $\mathbf{E}_\mathrm{e}$:

$$\mathbf{r}_\mathrm{o}^\mathrm{e} = \mathbf{T}_\mathrm{I}^\mathrm{e}\mathbf{r}_\mathrm{o}; \quad \mathbf{v}_\mathrm{o}^\mathrm{e} = \mathbf{T}_\mathrm{I}^\mathrm{e}(\mathbf{v}_\mathrm{o} - [\omega_\oplus \mathbf{i}_3 \times]\mathbf{r}_\mathrm{o}),$$

where $\mathbf{T}_\mathrm{I}^\mathrm{e} = [\rho_\mathrm{e}(t)]_3$; $\rho_\mathrm{e}(t) = \rho_\mathrm{e}^0 + \omega_\oplus(t - t_0)$, see fig. 2, and vectors $\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}$ and $\mathbf{v}_\mathrm{e}^\mathrm{s}$ are defined as $\boldsymbol{\omega}_\mathrm{e}^\mathrm{s} = \mathbf{T}_b^\mathrm{s}(\boldsymbol{\omega} - \tilde{\boldsymbol{\Lambda}}_\mathrm{I}^b \odot \omega_\oplus \mathbf{i}_3 \odot \boldsymbol{\Lambda}_\mathrm{I}^b)$; $\mathbf{v}_\mathrm{e}^\mathrm{s} = \tilde{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s} \odot \mathbf{v}_\mathrm{o}^\mathrm{e} \odot \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s}$, where $\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s} = \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{I} \odot \boldsymbol{\Lambda}_\mathrm{I}^b \odot \boldsymbol{\Lambda}_b^\mathrm{s}$; $\dot{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s} = \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s} \odot \boldsymbol{\omega}_\mathrm{e}^\mathrm{s}/2$ and constant matrix $\mathbf{T}_b^\mathrm{s}$ represents the telescope fixation on the SC body. The observation oblique range D is analytically calculated as $\mathrm{D} = |\mathbf{r}_\mathrm{c}^\mathrm{e} - \mathbf{r}_\mathrm{o}^\mathrm{e}|$.

If orthogonal matrix $\mathbf{C}_\mathrm{h}^\mathrm{s} \equiv \tilde{\mathbf{C}} = \| \tilde{c}_{ij} \|$ defines the SRF $\boldsymbol{\mathcal{S}}$ orientation with respect to the HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, then for any point $\mathrm{M}(\tilde{y}^i, \tilde{z}^i)$ at the telescope focal plane $y^i\mathrm{O}_i z^i$ the components $\tilde{V}_y^i$ and $\tilde{V}_z^i$ of normed vector by an image motion velocity appears as:

$$\begin{bmatrix} \tilde{V}_y^i \\ \tilde{V}_z^i \end{bmatrix} \equiv \begin{bmatrix} \dot{\tilde{y}}^i \\ \dot{\tilde{z}}^i \end{bmatrix} = \begin{bmatrix} \tilde{y}^i & 1 & 0 \\ \tilde{z}^i & 0 & 1 \end{bmatrix} \begin{bmatrix} q^i \tilde{\mathrm{v}}_\mathrm{e1}^\mathrm{s} - \tilde{y}^i \ \omega_\mathrm{e3}^\mathrm{s} + \tilde{z}^i \ \omega_\mathrm{e2}^\mathrm{s} \\ q^i \tilde{\mathrm{v}}_\mathrm{e2}^\mathrm{s} - \quad \omega_\mathrm{e3}^\mathrm{s} - \tilde{z}^i \ \omega_\mathrm{e1}^\mathrm{s} \\ q^i \tilde{\mathrm{v}}_\mathrm{e3}^\mathrm{s} + \quad \omega_\mathrm{e2}^\mathrm{s} + \tilde{y}^i \ \omega_\mathrm{e1}^\mathrm{s} \end{bmatrix}.$$

Here the normed focal coordinates $\tilde{y}^i = y^i/f_e$ and $\tilde{z}^i = z^i/f_e$, where f_e is the telescope equivalent focal distance; function $q^i \equiv 1 - (\tilde{c}_{21}\tilde{y}^i + \tilde{c}_{31}\tilde{z}^i)/\tilde{c}_{11}$, and normed velocity $\tilde{\mathrm{v}}_{\mathrm{e}i}^\mathrm{s} = \mathrm{v}_{\mathrm{e}i}^\mathrm{s}/\mathrm{D}, i = 1:3$. At conditions

$$\tilde{V}_y^i(0,0) = \tilde{\mathrm{W}}_y^i = -\tilde{\mathrm{W}}_y^s \equiv -\mathrm{W}_y^s/\mathrm{D} = \mathrm{const};$$
$$\tilde{V}_z^i(0,0) = 0; \quad \partial\tilde{V}_y^i(0,0)/\partial\tilde{z}^i = 0$$

calculation of vector $\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}$ is carried out by relations

$$\omega_\mathrm{e1}^\mathrm{s} = -\tilde{\mathrm{v}}_\mathrm{e2}^\mathrm{s}\tilde{c}_{31}/\tilde{c}_{11}; \ \omega_\mathrm{e2}^\mathrm{s} = -\tilde{\mathrm{v}}_\mathrm{e3}^\mathrm{s}; \ \omega_\mathrm{e3}^\mathrm{s} = -\tilde{W}_y^i + \tilde{\mathrm{v}}_\mathrm{e2}^\mathrm{s}. \quad (4)$$

By numerical solution of the quaternion differential equation $\dot{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s} = \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s} \odot \boldsymbol{\omega}_\mathrm{e}^\mathrm{s}/2$ with regard to (4) one can obtain values of vectors $\boldsymbol{\lambda}_{es}^\mathrm{s} \equiv \boldsymbol{\lambda}_\mathrm{e}^\mathrm{s}(t_s)$ for the discrete time moments $t_s \in \mathrm{T}_n$ with period $T_q = t_{s+1} - t_s$, $s = 0, 1, 2 ... n_q \equiv 0 : n_q$, $n_q = T_n/T_q$ when initial value $\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s}(t_0^n)$ is given. Further solution is based on *extrapolation* of the vector $\boldsymbol{\lambda}_{\mathrm{e}k}^\mathrm{s} \equiv \boldsymbol{\lambda}_\mathrm{e}^\mathrm{s}(t_k)$ values which are defined in the time moments $t_k \in \mathrm{T}_n$ with step $T_a = t_{k+1} - t_k$, $k = 0 : n$, $n \equiv T_n/T_a$. Extrapolation is carried out by set of n 3-degree *vector* splines $\mathbf{l}_k(\tau)$ at normed time $\tau = (t - t_k)/T_a \in [0, 1]$, with analytical obtaining a high-precise approximation the SRF $\boldsymbol{\mathcal{S}}$ guidance motion with respect to the GRF $\mathbf{E}_\mathrm{e}$ both on vector of angular rate and on vector of angular acceleration with its local derivative. At last stage, required functions $\boldsymbol{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$ is calculated by *explicit* formulas. These functions are applied at onboard computer for the time moments $t_s \in \mathrm{T}_n$.

6. GUIDANCE AT A ROTATION MANEUVER

Fast onboard algorithms for the SC guidance at a rotation maneuver (RM) with restrictions to $\boldsymbol{\omega}(t)$, $\dot{\boldsymbol{\omega}}(t)$ and $\ddot{\boldsymbol{\omega}}(t)$, corresponding restrictions to $\mathbf{h}(\boldsymbol{\beta}(t))$, $\dot{\boldsymbol{\beta}}(t)$ and $\ddot{\boldsymbol{\beta}}(t)$ in a class of the SC angular motions, were elaborated. Here for given time interval $t \in \mathrm{T}_p \equiv [t_0^p, t_f^p]$, $t_f^p \equiv t_0^p + T_p$ a problem consists in determination the *explicit* time functions — quaternion $\boldsymbol{\Lambda}(t)$, vectors $\boldsymbol{\omega}(t)$, $\varepsilon(t)$ and $\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$ for the boundary conditions on left ($t = t_0^p$) and right ($t = t_f^p$) trajectory ends:

$$\boldsymbol{\Lambda}(t_0^p) = \boldsymbol{\Lambda}_0; \boldsymbol{\omega}(t_0^p) = \boldsymbol{\omega}_0; \boldsymbol{\varepsilon}(t_0^p) = \boldsymbol{\varepsilon}_0; \quad (5)$$

$$\begin{aligned} &\boldsymbol{\Lambda}(t_f^p) = \boldsymbol{\Lambda}_f; \boldsymbol{\omega}(t_f^p) \equiv \boldsymbol{\omega}_f; \boldsymbol{\varepsilon}(t_f^p) = \boldsymbol{\varepsilon}_f; \\ &\dot{\boldsymbol{\varepsilon}}(t_f^p) = \dot{\boldsymbol{\varepsilon}}_f \equiv \varepsilon_f^* \ \mathbf{e}_f^{\varepsilon *} + \boldsymbol{\omega}_f \times \boldsymbol{\varepsilon}_f. \end{aligned} \quad (6)$$

In (6) last condition presents requirements for a *smooth conjugation* of guidance by a rotation maneuver with guidance at next the SC route motion. Developed approach to the problem is also based on *necessary* and *sufficient* condition for solvability of *Darboux* problem. At general case the solution is presented as result of composition by three ($k = 1:3$) *simultaneously* derived elementary rotations of *embedded* bases $\mathbf{E}_k$ about units $\mathbf{e}_k$ of *Euler* axes, which position is defined from the boundary conditions (5) and (6) for initial spatial problem. For all 3 elementary rotations with respect to units $\mathbf{e}_k$ the boundary conditions are analytically assigned. Into the IRF $\mathbf{I}_\oplus$ the quaternion $\boldsymbol{\Lambda}(t)$ is defined by the production

$$\boldsymbol{\Lambda}(t) = \boldsymbol{\Lambda}_0 \odot \boldsymbol{\Lambda}_1(t) \odot \boldsymbol{\Lambda}_2(t) \odot \boldsymbol{\Lambda}_3(t), \quad (7)$$

where $\boldsymbol{\Lambda}_k(t) = (\cos(\varphi_k(t)/2), \sin(\varphi_k(t)/2) \ \mathbf{e}_k)$, $\mathbf{e}_k$ is unit of *Euler* axis by k's rotation, and functions $\varphi_k(t)$ present the elementary rotation angles in analytical form. These functions are selected in class of splines by relative degree, moreover vectors of angular acceleration $\boldsymbol{\varepsilon}(t)$ and its derivative $\dot{\boldsymbol{\varepsilon}}(t)$ are *analytically* defined for each time moment $t \in \mathrm{T}_p$ by recurrent algorithm, and a module of a angular rate in a *position transfer* ($k = 3$) may be limited. The technique is based on the generalized integral's properties for the AM of the mechanical system "SC+GMC" and allows to evaluate vectors $\boldsymbol{\beta}(t)$, $\dot{\boldsymbol{\beta}}(t)$, $\ddot{\boldsymbol{\beta}}(t)$ in analytical form for a preassigned SC motion $\boldsymbol{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\dot{\boldsymbol{\omega}}(t)$, $\ddot{\boldsymbol{\omega}}(t) \, \forall t \in \mathrm{T}_p$.

Let be $\mathbf{g}(t) = \mathbf{k}(t) + \mathbf{h^c}(t) = \tilde{\boldsymbol{\Lambda}}(t) \circ \mathbf{g}_\mathrm{i}^\mathrm{I} \circ \boldsymbol{\Lambda}(t)$, where $\mathbf{k}(t) = \mathbf{J}\,\boldsymbol{\omega}(t)/h_g$ and $\mathbf{g}_\mathrm{i}^\mathrm{I} = \boldsymbol{\Lambda}(t_\mathrm{i}) \circ \mathbf{g}(t_\mathrm{i}) \circ \tilde{\boldsymbol{\Lambda}}(t_\mathrm{i})$. Derived the principle relation

$$\delta = d\,(1 - (1 - 2ac\rho - e\rho^2)^{1/2})/\rho,$$

$$a = \mathsf{x}/d; \ b = \mathsf{q_y q_z}/d^2; \ c = (\mathsf{q_y} - \mathsf{q_z})/d;$$

$$d = \mathsf{q_y} + \mathsf{q_z}; \quad e = 4b - a^2;$$

$\mathsf{r_s} = (\mathsf{a_s^2} + \mathsf{b_s^2})^{1/2}$, $\mathsf{s} = \mathsf{y}, \mathsf{z}$; $\mathsf{a_y} = (\mathsf{x} + \delta)/2$; $\mathsf{b_y} = \mathsf{y}$, and $\mathsf{a_z} = (\mathsf{x} - \delta)/2$; $\mathsf{b_z} = -\mathsf{z}$, results in

$$\mathsf{x}_{1,2}(t) = (\mathsf{a_y} \mp \mathsf{b_y}\,\mathsf{x_y})/2; \qquad \mathsf{y}_{1,2}(t) = (\mathsf{b_y} \pm \mathsf{a_y}\,\mathsf{x_y})/2$$

for the 1st GD's pair, and similarly for 2nd pair. Then there are analytically computed:

$$\mathbf{h^c}(t) = \mathbf{g}(t) - \mathbf{k}(t) \implies \boldsymbol{\beta}(t); \ \mathbf{g}^a(t) = -\boldsymbol{\omega}(t) \times \mathbf{g}(t);$$
$$\mathbf{g}^b(t) = -\dot{\boldsymbol{\omega}}(t) \times \mathbf{g}(t) - \boldsymbol{\omega}(t) \times \mathbf{g}^a(t);$$
$$\dot{\mathbf{h}}^\mathbf{c}(t) = \mathbf{g}^a(t) - \dot{\mathbf{k}}(t) \implies \dot{\boldsymbol{\beta}}(t);$$
$$\ddot{\mathbf{h}}^\mathbf{c}(t) = \mathbf{g}^b(t) - \ddot{\mathbf{k}}(t) \implies \ddot{\boldsymbol{\beta}}(t).$$

These algorithms ensure the *profile smoothness* for the SC motion with small level of its flexible structure oscillations. Fig. 3 presents the SC guidance by rotation maneuver with general boundary conditions.

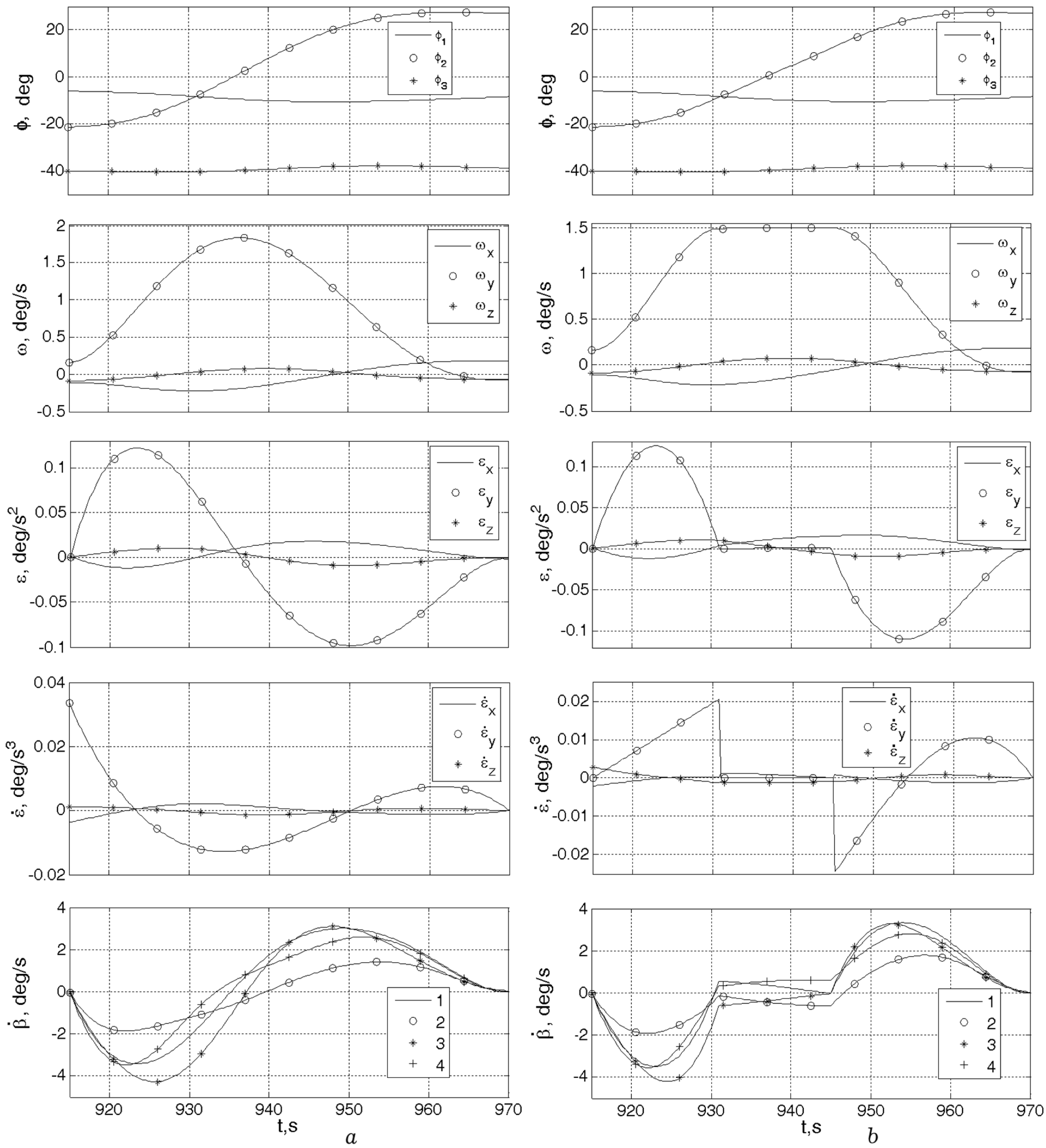

Fig. 3. Coordinates of the SC and the GMC at guidance by a rotation maneuver: a — without a limit of the SC angular rate in a *position transfer*; b — with such limit.

7. NONLINEAR ROBUST CONTROL

For control torque $\mathbf{M}^{\mathbf{g}}$ (2) and the SC model as a free rigid body the simplified controlled object is such:

$$\dot{\mathbf{\Lambda}} = \mathbf{\Lambda} \circ \boldsymbol{\omega}/2; \mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}\times]\mathbf{G}^o = \mathbf{M}^{\mathbf{g}}; \quad \dot{\boldsymbol{\beta}} = \mathbf{u}. \tag{8}$$

Assume that $\mathbf{\Lambda}^p(t)$ is a quaternion of the programmed SC body's motion in the IRF. The error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \tilde{\mathbf{\Lambda}}^p(t) \circ \mathbf{\Lambda}$, the *Euler* parameters' vector is $\boldsymbol{\mathcal{E}} = \{e_0, \mathbf{e}\}$, and the attitude error's matrix is $\mathbf{C}_e \equiv \mathbf{C}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 - 2[\mathbf{e}\times]\mathbf{Q}_e$, where $\mathbf{Q}_e \equiv \mathbf{Q}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 e_0 + [\mathbf{e}\times]$ with $\det(\mathbf{Q}_e) = e_0$.

If error $\delta\boldsymbol{\omega} \equiv \tilde{\boldsymbol{\omega}}$ in the rate vector $\boldsymbol{\omega}$ is defined as $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \mathbf{C}_e \boldsymbol{\omega}^p(t)$, and the GMC's required control torque vector $\mathbf{M}^{\mathbf{g}}$ is formed as

$$\mathbf{M}^{\mathbf{g}} = \boldsymbol{\omega} \times \mathbf{G}^o + \mathbf{J}(\mathbf{C}_e \dot{\boldsymbol{\omega}}^p(t) - [\boldsymbol{\omega}\times]\mathbf{C}_e \boldsymbol{\omega}^p(t) + \tilde{\mathbf{m}}),$$

then the simplest nonlinear model of the SC's attitude error is as follows:

$$\dot{e}_0 = -\langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle/2; \quad \dot{\mathbf{e}} = \mathbf{Q}_e \tilde{\boldsymbol{\omega}}/2; \quad \dot{\tilde{\boldsymbol{\omega}}} = \tilde{\mathbf{m}}. \tag{9}$$

By the relations $\mathbf{Q}_e^{-1}\mathbf{Q}_e^{\mathrm{t}} = \mathbf{C}_e$; $\mathbf{Q}_e^{-1} = \mathbf{Q}_e^{\mathrm{t}} + \mathbf{e} \cdot \mathbf{e}^{\mathrm{t}}/e_0$; $\mathbf{Q}_e^{-1}\mathbf{e} = \mathbf{e}/e_0$; $\mathbf{I}_3 - e_0 \mathbf{Q}_e^{-1} = \mathbf{Q}_e^{\mathrm{t}}[\mathbf{e}\times]$, which are used for $e_0 \neq 0$ (Somov, 1997), for model (9) a *non-local nonlinear* coordinate transformation is defined and used at analytical synthesis by the *exact feedback linearization.* This results in the NCL

$$\tilde{\mathbf{m}}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}) = -\mathbf{A}_0 \cdot \mathbf{e} \cdot \mathrm{Sgn}(e_0) - \mathbf{A}_1 \cdot \tilde{\boldsymbol{\omega}}, \tag{10}$$

where $\mathbf{A}_0 = ((2a_0^* - \tilde{\omega}^2/2)/e_0)\mathbf{I}_3$; $\mathbf{A}_1 = a_1^* \mathbf{I}_3 - \mathbf{R}_{e\omega}$, $\mathrm{Sgn}(e_0) = (1,\ \text{if}\ e_0 \geq 0) \vee (-1,\ \text{if}\ e_0 < 0)$, matrix $\mathbf{R}_{e\omega} = \langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle \mathbf{Q}_e^{\mathrm{t}}[\mathbf{e}\times]/(2e_0)$, and constants a_0^*, a_1^* are analytically calculated on spectrum $\mathrm{S}_{ci}^* = -\alpha_c \pm j\omega_c$.

Simultaneously using the *Vandermonde* matrix the *vector Lyapunov function* (VFL) $\boldsymbol{\upsilon}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}})$ is analyti-

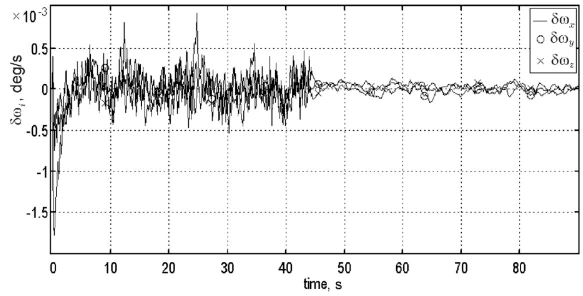

Fig. 4. The rate errors for consequence of the SC rotational maneuver and course motion

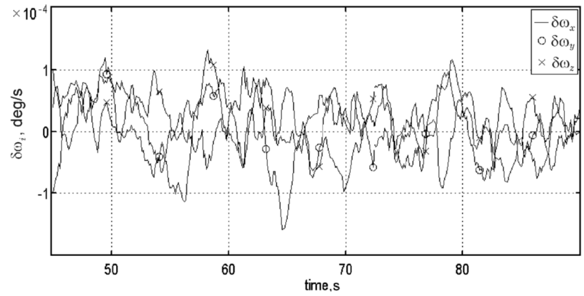

Fig. 5. The rate errors at the course motion

cally constructed for close-loop system (9) and (10). Taking into account restrictions on the GMC control, special nonlinear functions of the type *"division of variables with scaling"* were introduced in Somov (1997). In result the *nonlinear control law* was obtained for model (2) and (8), details see in Somov et al. (1999).

In stage 2, the problems of synthesising nonlinear control law were solved for model of the *flexible* spacecraft (1). Furthermore, the selection of parameters in the structure of the GMC *nonlinear robust* control law (which optimizes the main quality criterion for given restrictions) is fulfilled by a multistage numerical analysis and *parametric* optimization of the *comparison system* for the VLF. Thereto, the VLF has the structure derived above for the *error coordinates* $\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}$ and the structure of other VLF components in the form of *sublinear norms* for vector variables $\mathbf{q}(t)$, $\dot{\mathbf{q}}(t)$, $\dot{\boldsymbol{\beta}}(t)$ using the vector $\boldsymbol{\beta}(t)$.

8. COMPUTER SIMULATION

Fig. 4 and Fig. 5 present some results on computer simulation of a ACS for Russian remote sensing SC by the *Resource-DK* type. Here the rate errors are represented at consequence of the SC spatial rotational maneuver for time $t \in [0, 45)$ sec and the SC course motion for time $t \in [45, 90]$ sec with a nearly-constant vector of acceleration $\boldsymbol{\varepsilon}(t)$. Applied digital nonlinear control law is flexible switched at the time $t = 45$ sec on astatic ones with respect to the acceleration.

9. CONCLUSIONS

Contemporary approaches and some new results in progress of Somov et al. (2005a) and Somov et al. (2005b) were presented on onboard signal processing by multiple discrete filtering, attitude guidance and nonlinear robust gyromoment control applied for the agile remote sensing spacecraft, including for newest Russian spacecraft *Resource-DK 1*.

These results were also successfully applied for a space free-flying robot at transportation the flexible large-scale mechanical payload (Somov, 2006) .

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