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DECENTRALIZED GENERALIZED PREDICTIVE CONTROL BASED ON THE EQUIVALENT SUBSYSTEMS METHOD

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Abstract
The paper deals with the decentralized model predictive controller design. The Generalized Predictive Control (GPC) design approach is considered to design local controllers within the Equivalent Subsystems Methodology (ESM). According to ESM, the original multivariable plant is diagonalized by generating so-called equivalent subsystems, for which local controllers are tuned independently. Resulting local controllers constituting a decentralized controller are implemented on the real plant. The closed-loop stability and performance under the decentralized controller are guaranteed if local controllers provide stability and required performance of equivalent subsystems. The proposed approach has been verified on a case study - decentralized GPC design for a two-input two-output laboratory plant.

Key words
Decentralized control, generalized predictive control, frequency domain, equivalent subsystems method.

1 Introduction

Complex systems with multiple inputs and multiple outputs (MIMO) are usually controlled using multi-loop or decentralized controllers. Compared to centralized control, decentralized control is characterized by several important benefits such as hardware simplicity, operation simplicity as well as design simplicity and improved reliability; due to them, decentralized control design techniques still remain popular among control strategies. The Equivalent Subsystems Method (ESM) [Kozáková, Veselý and Osuský, 2009; Rosinová and Kozáková, 2012] is a Nyquist based frequency domain technique for decentralized controller design. According to it, interactions are taken into account through a chosen characteristic locus of the matrix of interactions used to modify frequency responses of decoupled subsystems; these modified frequency responses are frequency models of equivalent subsystems. Local controllers are designed for individual equivalent subsystems independently using any single-input single-output (SISO) method, frequency-domain methods are preferred (Bode design, Neymark D-partition and the recently developed Sine wave method [Bucz, Kozáková and Veselý, 2012]). It has been proved [Kozáková, Veselý and Osuský, 2009; Rosinová and Kozáková, 2012] that if closed-loop stability of individual equivalent subsystems under the respective local controllers is guaranteed then the full closed-loop system is stable as well.

Implementation of the ESM for decentralized controller (DC) design using local GPC controllers as proposed in this paper provides new perspectives to further development of the ESM-based approach. Generalized Predictive Control e.g. [Camacho and Bordons, 2004; Rossiter, 2004] is one of the most popular and successfully implemented Model Predictive Control (MPC) algorithms. There are many papers on decentralized MPC design and implementation. In [Richards and How, 2004] a decentralized MPC algorithm is presented for systems consisting of multiple subsystems with independent dynamics and disturbances but with coupled constraints. A plug-and-play MPC based on linear programming is proposed in [Riverso, Farina and Ferrari-Trecate, 2013]. An extension of the GPC algorithm to a multivariable case by designing several SISO controllers and compensation for interactions is presented in [Linkens and Mahfouf, 1992], in [Veselý and Osuský, 2013] a frequency-domain robust model predictive controller with hard input constraints is addressed. The paper by [Shah and Engell, 2010] presents a systematic approach that relates MPC tuning to linear control theory; in particular a systematic tuning of the prediction horizon and the cost function weights are provided to achieve desired closed-loop pole and zero locations for the unconstrained case. Results presented in this paper can be extended for the MIMO case and be applied in the decentralized GPC design proposed in this paper as well.

In this paper, the GPC algorithm is applied within the ESM framework. Obtained polynomial controller structure [Landau, 1998] is useful for implementation of local unconstrained GPCs. In case of considering constraints on input and output variables, the optimization problem has to be solved in each sampling instant.

The paper is organized as follows: Section 2 presents the preliminaries and problem formulation. Novel approach to decentralized predictive control design is developed in Section 3. In Section 4, the case study illustrating effectiveness of the proposed approach is provided. Conclusions are given at the end of the paper.

2 Preliminaries and Problem Formulation

Consider a continuous-time multivariable system modelled by a discrete-time transfer function matrix obtained using a suitably chosen sampling time T_s

$$y(t) = G(z)u(t) \tag{1}$$

The discrete-time transfer function matrix $G(z) \in \mathbb{R}^{m\times m}$ with $z = e^{sT_s}$ is interconnected with a decentralized controller $R(z) \in \mathbb{R}^{m\times m}$ in the standard feedback loop (Fig. 1). The variables w, u, y, d, e are vectors of reference, control, output, disturbance and control error of compatible dimensions. By analogy with the continuous-time case, the discrete frequency responses can be depicted either in the complex plane (discrete Nyquist plot) or in logarithmic coordinates (discrete Bode plots) considering $z = e^{sT_s} = e^{j\omega T_s}$ by analogy with the continuous-time case.

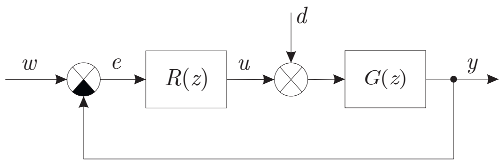

Figure 1. Standard feedback loop configuration.

Assume, that $G(z)$ can be split into diagonal and off-diagonal parts (Fig. 2) describing respective models of decoupled subsystems $G_d(z)$ and interactions $G_w(z)$

$$G(z) = G_d(z) + G_w(z) \tag{2}$$

where

$$G_d(z) = \text{diag}\{G_i(z)\}_{m\times m}, \;\; \det G_d(z) \neq 0$$
$$G_w(z) = G(z) - G_d(z), \;\;\; \forall s \in D$$

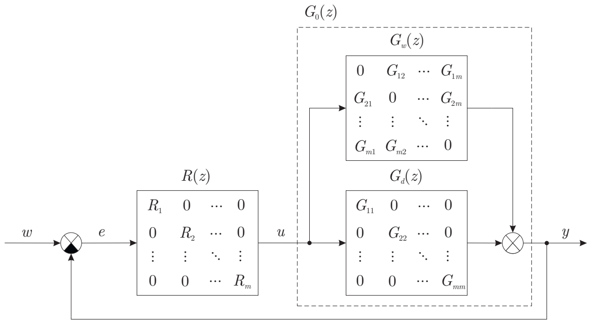

Figure 2. Standard feedback configuration under decentralized controller.

Besides properly choosing the sampling time T_s with respect to the plant dynamics it is necessary to keep in mind that a discrete frequency response is periodic with respect to the sampling frequency $\omega_s = 2\pi/T_s$. The frequency distortion does not occur if the frequency response is represented only for frequencies up to half of the sampling frequency, i.e. $\omega \in \langle 0; \omega_s/2\rangle$; otherwise higher frequencies would be wrapped to some other frequency in the range. A proper choice of sampling time is crucial for achievable bandwidth and feasibility of the required phase margin [Lewis, 1992].

For the system (2) a diagonal controller

$$R(z) = \text{diag}\{R_i(z)\}_{m\times m} \tag{3}$$

is to be designed to guarantee specified performance of the full system. In the proposed approach the overall system is viewed as composed by several subsystems. The Equivalent Subsystems Method (ESM) [Kozáková, 2012] which allows independent design of local controllers for the equivalent subsystems generated based on the full system transfer function matrix is used. In the following, the basics of the discrete-time ESM version are revisited.

2.1 Equivalent Subsystem Method

Consider the following factorization of the return difference matrix $F(z) = I + G(z)R(z)$ in terms of the split system (2) and the decentralized controller (3) in the form

$$\begin{aligned} F(z) &= I + R(z)[G_d(z) + G_w(z)] = \\ &= R(z)[R^{-1}(z) + G_d(z) + G_w(z)] \end{aligned} \tag{4}$$

Since $R(z)$ and $G_d(z)$ are diagonal matrices involving information about closed-loop dynamics of individual subsystems, we can include them into one diagonal matrix $H(z)$

$$H(z) = R^{-1}(z) + G_d(z) \tag{5}$$

where $H(z) = \text{diag}\{\eta_i(z)\}_{i=1}^m$. In addition, by a simple manipulations of (5) we obtain

$$I + R(z)[G_d(z) - H(z)] = I + R(z)G^{eq}(z) = 0 \quad (6)$$

where

$$\begin{aligned} G^{eq}(z) &= [G_d(z) - H(z)] = \\ &= \text{diag}\{G_{di}(z) - \eta_i(z)\}_{m\times m} = \\ &= \text{diag}\{G_i^{eq}(z)\}_{m\times m} \end{aligned}$$

is the diagonal matrix of equivalent subsystems. On the subsystem level, we obtain

$$1+R_i(z)[G_{di}(z)-\eta_i(z)] = 1+R_i(z)G_i^{eq}(z) = 0 \quad (7)$$

for $i = 1, ..., m$.

By appropriate choice of $H(z)$ it is possible to affect the dynamics of individual subsystems. Therefore, the entries $\eta_i(z)$ are chosen so as to appropriately take into account interactions between subsystems given by $G_w(z)$. The problem of generating diagonal equivalent subsystems models $G_{eq}(z)$ is then reduced to finding a diagonal matrix $H(z) = \text{diag}\{\eta_k(z)\}_{i=1,2,...,m}$.

Characteristic functions $g_i(z),\ i = 1, ..., m$ of matrix $G_w(z)$ are defined as follows

$$\det[g_i(z)I - G_w(z)] = 0, \quad i = 1, 2, ..., m \quad (8)$$

The system $G(z)$ under the decentralized controller $R(z)$ is stable if

$$\begin{aligned} &1.\ \det[H(z) + G_w(z)] \det R(z) \neq 0 \\ &2.\ N[0, \det(H(z) + G_w(z)) \det R(z)] = n_c \end{aligned} \quad (9)$$

where n_c is the number of unstable poles of open-loop system, and $z = \exp^{j\omega T_s}$, $\omega \in \langle 0; \omega_s/2\rangle$.

Assuming $\text{diag}R(z) \neq 0$, the overall system is at the limit of instability if

$$\det[H(z) + G_w(z)] = 0 \quad (10)$$

Since $H(z) = \text{diag}\{\eta_i(z)\}_{i=1,2,...,m}$ we obtain m characteristic functions $\eta_i(z)$ which actually defines m characteristic functions $g_i(z)$ of the matrix $-[G_w(z)]$

$$\det[\eta_i(z)I + G_w(z)] = 0, \quad i = 1, 2, ..., m \quad (11)$$

Then, $H(z)$ in (11) "compensates" interactions given by $G_w(z)$ if the matrix $H(z) = \text{diag}\{\eta_i(z)\}I$ is chosen such that diagonal entries of $H(z)$ are identical and equal to any of m characteristic functions of $[G_w(z)]$, i.e. if

$$H(z) = -g_k(z)I \quad (12)$$

where $k \in \{1, 2, ..., m\}$ is fixed. It can be easily shown that by substituting (12) into (10) results in

$$\det[g_k(z) - G_w(z)] = \prod_{i=1}^{m}[g_k(z) - g_i(z)] = 0 \quad (13)$$

Considering (12) in (6), related diagonal matrix of equivalent subsystems is obtained

$$G_k^{eq}(z) = G_d(z) + g_k(z)I = \text{diag}\{G_{ik}^{eq}(z)\}_{m\times m} \quad (14)$$

Individual entries of $G_k^{eq}(z)$ are then generated as follows

$$G_{ik}^{eq}(z) = G_{di}(z) + g_k(z) \quad (15)$$

where $i = 1, 2, ..., m$, $k \in \{1, ..., m\}$. Hence, for $H(z)$ satisfying (12) we obtain the closed-loop characteristic equation in the form of product of equivalent characteristic equations

$$\det[I + G_k^{eq}(z)R(z)] = \prod_{i=1}^{m}[1 + G_{ik}^{eq}(z)R_i(z)] \quad (16)$$

for $k \in \{1, ..., m\}$.

Based on these results, stability of the overall closed-loop system under a decentralized controller is guaranteed if and only if all equivalent closed-loop subsystems are stable, i.e. if there exists $g_k(z)$ such that

$$\begin{aligned} &1.\ \det[\eta_k(z)I + G_w(z)] = 0, \quad k \in \{1, ..., m\} \\ &2.\ N[\det(I + G_k^{eq}(z)R(z))] = n_c \end{aligned} \quad (17)$$

and the equivalent characteristic equations
$1 + R_i(z)\big(G_{di}(z) - \eta_k(z)\big), \quad i = 1, ..., m,$
for $k \in \{1, ..., m\}$ are stable.

Decentralized controller design is performed by independent design of local controllers for equivalent subsystems. In the sequel, local controllers will be designed as predictive ones based on the GPC framework.

3 Decentralized Generalized Predictive Control

GPC incorporates all major features of the predictive controllers in an unified framework and provides several advantages such as ability to control non-minimum phase and open-loop unstable processes and also plants with unknown orders. In this section the whole control design procedure is proposed involving identification of generated equivalent subsystems, and local predictive controllers design for individual equivalent subsystems considering both unconstrained and constrained input and output variables of the plant.

3.1 Identification of Equivalent Subsystems

To apply a predictive control design discrete-time linear models of individual equivalent subsystems have to be identified.

Equivalent subsystems are identified as polynomial models. The best result has been obtained using Output-Error (OE) models in the form

$$y(t) = \frac{B(z)}{F(z)}u(t) + e(t) \tag{18}$$

To estimate the model, a cut curve fitting approach has been used. It is based on least-squares method and provided by available software tools [Ljung, 2008]. Identified discrete-time transfer function models can be of arbitrary order.

The diagonal transfer function matrix of the plant is composed of identified equivalent subsystems obtained in the form of discrete-time transfer function models

$$G^{eq}(z) = \begin{bmatrix} G_1^{eq}(z) & 0 & \dots & 0 \\ 0 & G_2^{eq}(z) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & G_m^{eq}(z) \end{bmatrix} \tag{19}$$

where

$$G_i^{eq}(z) = \frac{B_i^{eq}(z^{-1})}{A_i^{eq}(z^{-1})} = \frac{b_0 + b_1 z^{-1} + ... + b_{nb} z^{-nb}}{1 + a_1 z^{-1} + ... + a_{na} z^{-na}} \tag{20}$$

for $i = 1, ..., m$.

Now, it is possible to design local SISO predictive controllers for individual m equivalent subsystems. In the following, the design procedure developed using Generalized Predictive Control (GPC) methodology is described.

3.2 Generalized Predictive Control

For the controlled process with m inputs and m outputs described by a diagonal transfer function matrix comprising equivalent subsystems $G_i^{eq}(z)$, $i = 1, .., m$, a set of m CARIMA models [Camacho and Bordons, 2004; Levine, 1999] is utilized.

$$\begin{aligned} A_i^{eq}(z^{-1})y_i(t) = B_i^{eq}(z^{-1})z^{-d}u_i(t-1) + \\ + \frac{C_i^{eq}(z^{-1})}{\Delta}\xi_i(t), \quad i = 1, ..., m \end{aligned} \tag{21}$$

where

$$\begin{aligned} A_i^{eq}(z^{-1}) &= 1 + a_{1(i)}z^{-1} + ... + a_{na(i)}z^{-na}, \\ B_i^{eq}(z^{-1}) &= b_{0(i)} + b_{1(i)}z^{-1} + ... + b_{nb(i)}z^{-nb}, \\ C_i^{eq}(z^{-1}) &= 1 + c_{1(i)}z^{-1} + ... + c_{nc(i)}z^{-nc} \end{aligned} \tag{22}$$

$u_i(t), y_i(t)$ are the plant input and output, d is time delay, $\xi_i(t)$ represents the effect of disturbances, and $\Delta = 1 - z^{-1}$. If $\xi(t)$ is a zero-mean white noise, the polynomials $C_i^{eq}(z^{-1})$ representing a noise filter can be set to $C_i(z^{-1}) = 1$.

To derive a prediction model, (21) can be rewritten as follows:

$$\hat{A}_i^{eq}(z^{-1})y_i(t) = B_i^{eq}(z^{-1})\Delta u_i(t-d-1) + \xi_i(t) \tag{23}$$

where

$$\hat{A}_i^{eq}(z^{-1}) = \Delta A_i^{eq}(z^{-1}) \tag{24}$$

Since only derivative of the control input $\Delta u(k)$ is used, an integrator has to be introduced in the form

$$u_i(t) = \frac{1}{1 - z^{-1}}\Delta u_i(t) \tag{25}$$

The parameter $\xi(t)$ is a zero-mean, thus it can be neglected. The respective cost function with the following structure is defined

$$\begin{aligned} J_i(t) = \sum_{l=1}^{N_{y,i}} |\tilde{y}_i(t+l|t) - \tilde{w}_i(t+l|t)|^2_{\Pi_{y,i}} + \\ + \sum_{l=1}^{N_{u,i}} |\Delta\tilde{u}_i(t+l-1|t)|^2_{\Pi_{u,i}} \end{aligned} \tag{26}$$

where Π_y, Π_u denote appropriate symmetric positive (semi)definite weighting matrices, and N_y, N_u denote prediction and control horizons, respectively. The output prediction model can be obtained in several ways [Rossiter, 2004].

One of them is via a set of diophantine equations [Camacho and Bordons, 2004] solved independently for each subsystem

$$1 = E_{i,l}(z^{-1})\hat{A}_i^{eq}(z^{-1}) + z^{-l}F_{i,l}(z^{-1}) \tag{27}$$

Multiplying (23) with $E_{i,l}(z^{-1})z^{-l}$ we obtain

$$\begin{aligned} \hat{A}_i^{eq}(z^{-1})E_{i,l}(z^{-1})y_i(t+l) = \\ = B_i^{eq}(z^{-1})E_{i,l}(z^{-1})\Delta u_i(t+l-d-1) \end{aligned} \tag{28}$$

Finally, substituting (27) into (28) the prediction for y_i is obtained in the form

$$\begin{aligned} \tilde{y}_i(t+l|t) = G_{i,l}(z^{-1})\Delta u_i(t+l-d-1) + \\ + F_{i,l}(z^{-1})y_i(t) \end{aligned} \tag{29}$$

where

$$G_{i,l}(z^{-1}) = E_{i,l}(z^{-1})B_i^{eq}(z^{-1}) = \\ = g_0 + g_1 z^{-1} + ... + g_n g z^{-ng} \quad (30)$$

The polynomials $E_{i,l}(z^{-1})$ and $F_{i,l}(z^{-1})$ can be calculated recursively as follows

$$E_{i,l+1}(z^{-1}) = E_{i,l}(z^{-1}) + f_{l,0}^i z^{-l} \\ f_{l+1,j}^i = f_{l,j+1}^i - f_{l,0}^i \tilde{a}_{i,j+1}, \quad j = 0, ..., n_a - 1 \quad (31)$$

where $\tilde{a}_j$ denotes the coefficient of the $j + 1^{th}$ term in the polynomial $\hat{A}_i^{eq}(z^{-1})$. Since the control input $u_i(t)$ influences the system output after l sampling periods, the values of horizons $N_{y,i}, N_{u,i}$ can be defined as $N_{y,i} = N_{u,i} = N_i$.

Thus, the equations for prediction steps are obtained in the form:

$$\begin{aligned} \tilde{y}_i(t+d+1|t) &= G_{i,d+1}(z^{-1})\Delta u_i(t) + \\ &+ F_{i,d+1}(z^{-1})y_i(t) \\ \tilde{y}_i(t+d+2|t) &= G_{i,d+2}(z^{-1})\Delta u_i(t) + \\ &+ F_{i,d+1}(z^{-1})y_i(t) \\ &\vdots \\ \tilde{y}_i(t+d+N_i|t) &= G_{i,d+N_i}(z^{-1})\Delta u_i(t+N_i-1) + \\ &+ F_{i,d+N_i}(z^{-1})y_i(t) \end{aligned} \quad (32)$$

or in the vector form

$$\tilde{\mathbf{y}}_i(t) = G_i \tilde{u}_i(t) + \tilde{G}_i(z^{-1})\Delta u_i(t+l-1) + \\ + \tilde{F}_i(z^{-1})y_i(t) \quad (33)$$

$$\tilde{y}_i(t) = \begin{bmatrix} \tilde{y}_i(t+d+1|t) \\ \vdots \\ \tilde{y}_i(t+d+N_i|t) \end{bmatrix} \\ \tilde{u}_i(t) = \begin{bmatrix} \Delta u_i(t) \\ \vdots \\ \Delta u_i(t+N_i-1) \end{bmatrix} \quad (34)$$

$$\tilde{F}_i = \begin{bmatrix} F_{i,d+1}(z^{-1}) \\ \vdots \\ F_{i,d+N_i}(z^{-1}) \end{bmatrix} \\ G_i = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N_i-1} & g_{N_i-2} & \cdots & g_0 \end{bmatrix} \quad (35)$$

$$\tilde{G}_i(z^{-1}) = \begin{bmatrix} (G_{i,d+1}(z^{-1}) - g_0)z \\ (G_{i,d+1}(z^{-1}) - g_0 - g_1 z^{-1})z^2 \\ \vdots \\ (G_{i,d+1}(z^{-1}) - g_0 - g_1 z^{-1} \cdots - g_{N_i} z^{N_i})z^{N_i} \end{bmatrix} \quad (36)$$

The last two terms in (33) depend only on the previous values of the inputs and the outputs, therefore the prediction equation can be rewritten as follows:

$$\tilde{\mathbf{y}}_i(t) = G_i \tilde{u}_i(t) + f_i \quad (37)$$

where f_i denotes the *free response* and the rest with the constant matrix G_i denotes the *forced response*.

Using (33) the cost function (26) can be simplified to

$$J_i(t) = \Big(G_i\tilde{u}_i(t) + f_i - \tilde{w}_i(t)\Big)^T \Pi_{y,i} \Big(G_i\tilde{u}_i(t) + f_i - \tilde{w}_i(t)\Big) + \\ + \tilde{u}_i^T \Pi_{u,i} \tilde{u}_i = \frac{1}{2}\tilde{u}_i^T H \tilde{u}_i(t) + b_i^T \tilde{u}_i(t) + f_{0,i} \quad (38)$$

where

$$\begin{aligned} H_i &= 2\Big(G_i^T \Pi_{y,i} G_i + \Pi_{u,i}\Big) \\ b_i^T &= 2\Big(f_i - \tilde{w}_i(t)\Big)^T \Pi_{y,i} G_i \\ f_{0,i} &= \Big(f_i - \tilde{w}_i(t)\Big)^T \Pi_{y,i} \Big(f_i - \tilde{w}_i(t)\Big) \end{aligned} \quad (39)$$

3.3 Formulation of Unconstrained Control Law

Having computed the optimal prediction, it is now possible to calculate future control inputs, which minimizes the cost function (38) with respect to the known set-point, chosen control and predictive horizons and the weighting factors. Using some algebra [Camacho and Bordons, 2004] we obtain a control law in the form

$$u_i(t) = \left(G_i^T G_i + \Pi_{u,i} I\right)^{-1} G_i^T \left(\tilde{w}_i(t) - f_i\right) \quad (40)$$

According to a receding horizon approach [Maciejowski, 2002], only the first element of the control signal is applied at the plant input

$$\Delta u_i(t) = K_i(\tilde{w}_i(t) - f_i) \quad (41)$$

with K representing the first row of $\left(G_i^T G_i + \Pi_{u,i} I\right)^{-1} G_i^T$. The above equation with substituted (33) and (37) leads to

$$\Delta u_i(t) = \sum_{l=1}^{N_{y,i}} k_{i,l}(\tilde{w}_i(t+l) - f_i(t+l)) = \\ = \sum_{l=1}^{N_{y,i}} k_{i,l}\tilde{w}_i(t+l) - \sum_{l=1}^{N_{y,i}} k_{i,l} G_{i,l}(z^{-1})\Delta u_i(t-1) - \\ - \sum_{l=1}^{N_{y,i}} k_{i,l}\tilde{F}_i(z^{-1})y_i(t) \quad (42)$$

If the reference trajectory is considered to be constant over the prediction horizon, i.e. $w(t+l) = w(t)$, (42) can be rewritten as follows:

$$\begin{aligned} &1 - z^{-1}\sum_{l=1}^{N_{y,i}} k_{i,l}G_{i,l}(z^{-1})\Delta u_i(t) = \\ &= \sum_{l=1}^{N_{y,i}} k_{i,l}(\tilde{w}_i(t+l) - \sum_{l=1}^{N_{y,i}} k_{i,l}\tilde{F}_i(z^{-1})y_i(t) \end{aligned} \quad (43)$$

Comparing (43) with the pole-placement structure [Landau, 1998] with the control law given as

$$R_i(z^{-1})\Delta u(t) = T_i z^{-1} w(t) - S_i(z^{-1})y(t) \quad (44)$$

the polynomials of GPC controller can be obtained in the following form:

$$R_i(z^{-1}) = 1 + z^{-1}\sum_{l=1}^{N_{y,i}} k_{i,l}G_{i,l}(z^{-1}) \quad (45)$$

$$S_i(z^{-1}) = \sum_{l=1}^{N_{y,i}} k_{i,l}F_{i,l}(z^{-1}) \quad (46)$$

$$T_i(z^{-1}) = \sum_{l=1}^{N_{y,i}} k_{i,l} \quad (47)$$

The polynomial representation of the controller is used according to Fig. 3 where the local predictive controllers are described in the RST form [Landau, 1998].

In the unconstrained case when the control law and the controlled plant are linear, it is possible to derive the closed-loop characteristic equation, find its poles, and possibly to examine various frequency domain characteristics.

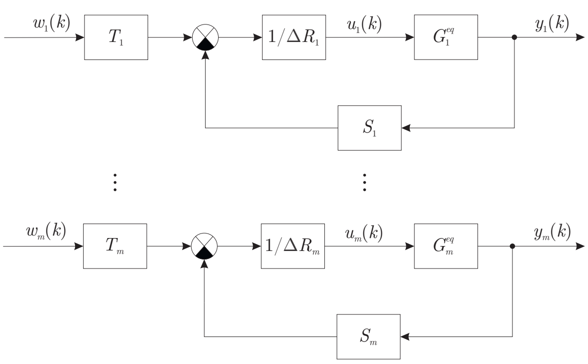

Figure 3. Polynomial structure of the decentralized controller for MIMO system represented by equivalent subsystems.

3.4 Formulation of Constrained Control Law

When considering constraints on inputs $\langle \underline{u}_i, \overline{u}_i \rangle$, outputs $\langle \underline{y}_i, \overline{y}_i \rangle$ or input rates $\langle \underline{du}_i, \overline{du}_i \rangle$, it is no longer possible to express the solution explicitly in an analytical form. Instead, an optimization procedure has to be performed at each sampling period, solving a constrained minimization problem defined by (38) and constraints transformed so as to be related to $\tilde{u}(t)$

$$\begin{aligned} \underline{u}_i &\leq \tilde{u}_i(t) \leq \overline{u}_i \\ \underline{du}_i &\leq Z_i d\tilde{u}_i(t) - L_i u_i(t-1) \leq \overline{du}_i \\ \underline{y}_i &\leq G_i\tilde{u}_i(t) + f_i \leq \overline{y}_i \end{aligned} \quad (48)$$

Since $\tilde{u}_i(t) = L_i u_i(t-1) + Z_i d\tilde{u}_i(t)$ the above equations can be rewritten into the following form

$$\mathbf{\Theta}_i \tilde{u}_i(t) \leq \boldsymbol{r}_i \quad (49)$$

where

$$\mathbf{\Theta}_i = \begin{bmatrix} I_i \\ -I_i \\ Z_i \\ -Z_i \\ G_i \\ -G_i \end{bmatrix}, \quad \boldsymbol{r}_i = \begin{bmatrix} \overline{u}_i \\ -\underline{u}_i \\ \overline{du}_i - L_i u_i(t-1) \\ -\underline{du}_i + L_i u_i(t-1) \\ \overline{y}_i - f_i \\ -\underline{y}_i + f_i \end{bmatrix} \quad (50)$$

and

$$Z_i = \begin{bmatrix} 1\ 0\ 0 \dots 0 \\ 1\ 1\ 0 \dots 0 \\ \vdots\ \vdots\ \ \ddots\ \ \vdots \\ 1\ 1\ 1 \dots 1 \end{bmatrix}, \quad L_i = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad (51)$$

In order to find solution to this optimization problem, usually a quadratic programming solver is used. Formulation of the quadratic programming problem for $i = 1, ..., m$ subsystems is in the form

$$\begin{aligned} \underset{\Delta u_i}{\text{minimize}} \quad & \frac{1}{2}\Delta u_i^T H_i \Delta u_i + b_i^T \Delta u_i \\ \text{subject to} \quad & \mathbf{\Theta}_i \tilde{u}_i(t) \leq \boldsymbol{r}_i \end{aligned} \quad (52)$$

The final solution is a trajectory consisting from the control input differences; according to the receding horizon strategy just the first element is applied at the plant input in each sampling period.

3.5 Controller Design Procedure

The proposed decentralized predictive control strategy is based on designing local predictive controllers for m individual equivalent subsystems. By finding m predictive controllers that guarantee specified performance of equivalent closed-loops, closed-loop stability and performance for the overall system is guaranteed.

The decentralized predictive control design procedure consists of the following steps [Kozáková and Krasňanský, 2015]:

1. Plant model discretization:
   - Discretization of the continuous-time plant $G(s) \in \mathbb{R}^{m\times m}$ using an appropriately chosen sampling time T_s.
   - Specification of the sampling frequency and of the feasible frequency range.

$$\omega_s = 2\pi/T_s, \quad \omega \in \langle 0;\omega_s/2\rangle$$

2. Generating equivalent subsystems
   - Partition of obtained discrete-time transfer function matrix $G(z)$ into the diagonal $G_d(z)$ and off-diagonal part $G_w(z)$ according to (2).
   - Calculation and plotting of characteristic loci $g_i(z)$, $i = 1, ..., m$ of the matrix $G_w(z)$ for $z = e^{j\omega T_s}$, where $\omega \in \langle 0, \omega_s/2\rangle$.
   - Choosing characteristic function $g_k(z)$ for a fixed $k \in \{1, ..., m\}$.
   - Generating and plotting discrete frequency responses of independent equivalent subsystems for selected $g_k(z)$

$$G^{eq}_{ik}(z) = G_{di}(z) + g_k(z),\ i = 1, ..., m,$$

   for $k \in \{1, ..., m\}$.

3. Identification of linear models (of appropriate order) of equivalent subsystems using the frequency responses data (moduli and phases of equivalent subsystems).
4. Independent design and tuning of m local SISO GPC controllers with specified performance requirements for each equivalent subsystem G^{eq}_i, $i = 1, ..., m$ using GPC design methodology according to Section 3.2.
5. Assessment of achieved performance in individual equivalent closed-loops, and of the overall system respectively. Equivalent closed-loop characteristic equations are easily obtained by substituting the analytic solutions (44) into (23).

4 Case Study

A laboratory plant consisting of two interconnected DC motors (a MIMO plant with two inputs and two outputs) has been used to demonstrate practical application of the proposed control design technique. System inputs u_1, u_2 are armature voltages, measured outputs y_1, y_2 are the angular velocities of DC motors converted to voltages. It is possible to adjust the load of each motor in the range of the input voltage. Measurement of these signals have been preformed using the data acquisition card Advantech PCI 1711. Interconnection of the DC motors brings about interactions among plant subsystems. Functional block diagram of the system is in Fig. 4.

The main objective is to control the angular velocities of individual DC motors to track a time-varying reference signal. Mathematical model of the plant was identified experimentally from step responses measured in the operating point specified by $u_1 = u_2 = 3.5V$, and the loads $u_{L1} = 4V, u_{L2} = 3V$.

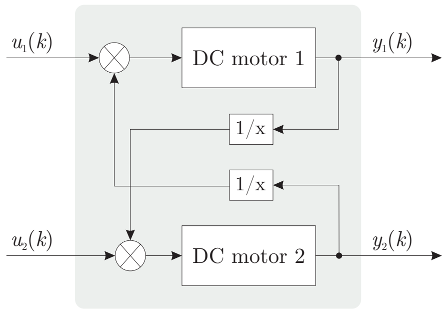

Figure 4. Schematic of interconnected DC motors system.

Discrete-time transfer function model obtained using sampling time $T_s = 0.1$s is in the form:

$$G(z^{-1}) = \begin{bmatrix} G_{11}(z^{-1}) & G_{12}(z^{-1}) \\ G_{21}(z^{-1}) & G_{22}(z^{-1}) \end{bmatrix} \tag{53}$$

where

$$G_{11}(z^{-1}) = \frac{-0.006647z^{-1} + 0.01394z^{-2}}{1 - 1.855z^{-1} + 0.861z^{-2}}$$

$$G_{12}(z^{-1}) = \frac{-0.0001223z^{-1} + 0.0002967z^{-2}}{1 - 1.971z^{-1} + 0.9717z^{-2}}$$

$$G_{21}(z^{-1}) = \frac{-0.0001501z^{-1} + 0.0003441z^{-2}}{1 - 1.972z^{-1} + 0.9725z^{-2}}$$

$$G_{22}(z^{-1}) = \frac{-0.006151z^{-1} + 0.01607z^{-2}}{1 - 1.821z^{-1} + 0.8292z^{-2}}$$

To generate equivalent subsystems, characteristic loci $g_k(z), k = 1, 2$ of the off-diagonal transfer function matrix $G_w(z)$ have been calculated; $g_1(z)$ was chosen to generate equivalent subsystems according to (15).

Subsequently, discrete-time linear models have been obtained by identification from the frequency response data using fourth-order Output-Error models.

$$G^{eq}(z^{-1}) = \begin{bmatrix} G^{eq}_1(z^{-1}) & 0 \\ 0 & G^{eq}_2(z^{-1}) \end{bmatrix} \tag{54}$$

where

$$G_i^{eq}(z^{-1}) = \frac{B_i^{eq}}{A_i^{eq}}, \ \ i = 1, 2$$

and

$$B_1^{eq}(z^{-1}) = -0.006782z^{-1} + 0.02761z^{-2} - \\ - 0.03465z^{-3} + 0.01382z^{-4}$$
$$B_2^{eq}(z^{-1}) = -0.006287z^{-1} + 0.02876z^{-2} - \\ - 0.03835z^{-3} + 0.01589z^{-4}$$
$$A_1^{eq}(z^{-1}) = 1 - 3.826z^{-1} + 5.49z^{-2} - \\ - 3.5z^{-3} + 0.8369z^{-4}$$
$$A_2^{eq}(z^{-1}) = 1 - 3.793z^{-1} + 5.392z^{-2} - \\ - 3.405z^{-3} + 0.806z^{-4}$$

Bode diagrams of generated equivalent subsystems and their polynomial models are compared in Fig. 5.

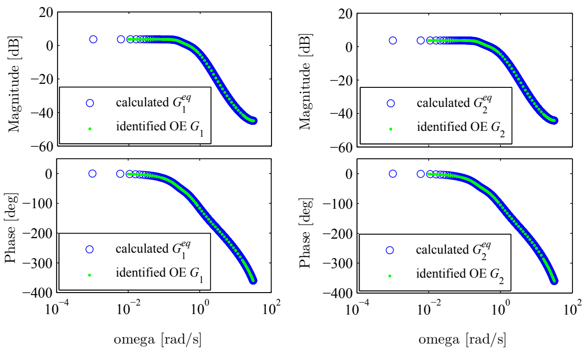

Figure 5. Bode plots of calculated equivalent subsystems (circle-marked line) and identified OE models (dotted line).

Local MPC controllers in the polynomial form (44) were designed using the above GPC design procedure with the following parameters: $N_y = 7, N_u = 7$, weights $\Pi_{y1} = 0.57, \Pi_{y2} = 1.58$ and sampling time $T = 0.1$.

Resulting RST controller polynomials for the equivalent subsystem $G_1^{eq}(z^{-1})$:

$$R_1(z^{-1}) = 1 - 0.9992z^{-1} - 0.0007846z^{-2}$$
$$S_1(z^{-1}) = 3.477 - 11.33z^{-1} + 14.15z^{-2} \\ - 7.993z^{-3} + 1.718z^{-4}$$
$$T_1(z^{-1}) = 0.0191$$

Controller polynomials for the equivalent subsystem $G_2^{eq}(z^{-1})$:

$$R_2(z^{-1}) = 1 - 0.9991z^{-1} - 0.0008891z^{-2}$$
$$S_2(z^{-1}) = 2.638 - 8.405z^{-1} + 10.32z^{-2} \\ - 5.754z^{-3} + 1.222z^{-4}$$
$$T_2(z^{-1}) = 0.0252$$

For a comparison consider now different design parameters: $N_y = 7, N_u = 7, \Pi_{y1} = 0.85, \Pi_{y2} = 3.50$.

The resulting RST controller polynomials for equivalent subsystem $G_1^{eq}(z^{-1})$ are:

$$R_1(z^{-1}) = 1 - 0.9995z^{-1} - 0.000512z^{-2}$$
$$S_1(z^{-1}) = 2.3326 - 7.6011z^{-1} + 9.4908z^{-2} \\ - 5.3617z^{-3} + 1.1522z^{-4}$$
$$T_1(z^{-1}) = 0.0128$$

Controller polynomials for the equivalent subsystem $G_2^{eq}(z^{-1})$:

$$R_2(z^{-1}) = 1 - 0.9996z^{-1} - 0.000422z^{-2}$$
$$S_2(z^{-1}) = 1.1914 - 3.7961z^{-1} + 4.6635z^{-2} \\ - 2.5991z^{-3} + 0.5518z^{-4}$$
$$T_2(z^{-1}) = 0.0114$$

Closed-loop stability has been examined from the closed-loop equivalent characteristic polynomials as well as from closed-loop poles of the full system under the decentralized controller.

For the weights $\Pi_{y1} = 0.57$, $\Pi_{y2} = 1.58$ the closed-loop pole with a maximum modulus of the full system is $\|p\| = 0.9861$. For the individual equivalent subsystems, maximal moduli are $\|p_1^{eq}\| = 0.9882$ and $\|p_2^{eq}\| = 0.9872$. Considering selected weights $\Pi_{y1} = 0.85$, $\Pi_{y2} = 3.50$ the maximum closed-loop pole moduli are $\|p\| = 0.9861$, $\|p_1^{eq}\| = 0.9886$ and $\|p_2^{eq}\| = 0.9886$.

4.1 Experimental Results

Experimental results on the real plant under the designed decentralized controller are shown in Fig. 6 and Fig. 7.

Obtained theoretical and experimental results prove that the local GPC controllers independently designed for each equivalent subsystem which constitute the resulting decentralized controller guarantee the closed-loop stability.

Considering constraints on input and output variables, the design procedure has been performed similarly but considering on-line computation of the control laws according to Section 3.4. The design parameters have been set as follows: $N_u = N_y = 7$, $\Pi_{y1} =$

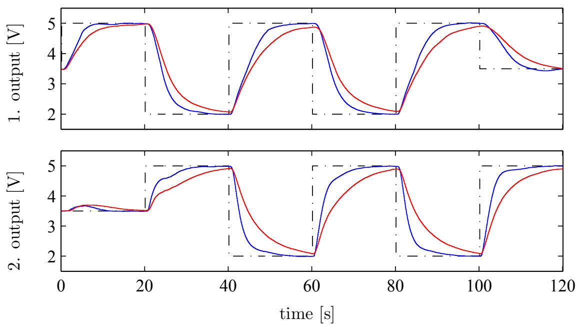

Figure 6. Measured time responses of the system outputs for different values of weighting parameters.

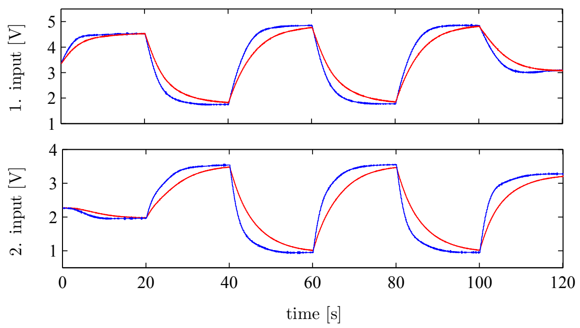

Figure 7. Measured time responses of the control inputs for different values of weighting parameters.

9, $\Pi_{y2} = 15$ and the constraints $u_{min} = [0,0]^T$, $u_{max} = [3.7, 3.4]^T$, $\Delta u_{min} = [0,0]^T$, $\Delta u_{max} = [0.025, 0.025]^T$, $y_{min} = [0,0]^T$, $y_{max} = [4.5, 5]^T$. The simulation results depicted in Fig. 8 demonstrate

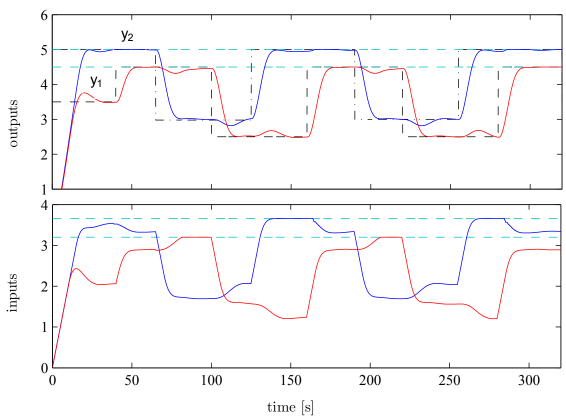

Figure 8. Closed-loop responses of constrained plant inputs/outputs (solid red/blue lines).

good performance of the proposed control design. In this case, closed-loop stability can be guaranteed via choosing a reasonably long prediction horizon.

5 Conclusion

In this paper a novel approach to the decentralized GPC controller design within the ESM design framework has been proposed. The main advantage of this approach is a diagonalization of the original plant by generating a diagonal matrix of equivalent subsystems. Thus, local predictive controllers of individual equivalent subsystems can be designed and tuned independently; stability and achieved performance of equivalent closed-loops are guaranteed for the full system. Important points in the design procedure are model identification from frequency responses of equivalent subsystems.

Stability can easily be analyzed based on the polynomial control structure of the unconstrained GPC control algorithm. Considering constraints on system variables, optimization has to be performed at each sampling instant. In this case, stability is guaranteed via choosing a reasonably long prediction horizon.

The proposed decentralized GPC design approach was verified on a real laboratory plant. Presented theoretical and experimental results have proved effectiveness of the proposed control design strategy.

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