

SYMBS - SYMBOLICAL ANALYSIS AND OPTIMIZATION OF MULTIBODY SYSTEMS IN MATLAB

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Abstract

For many applications of multibody systems the symbolic equations of motion are useful, but in most commercial multibody programs the dynamic behavior is not formulated as an explicit function and not available symbolically for the user. Within this paper the Matlab program SYMBS is introduced. By using the *Matlab Symbolic Toolbox* the modelling of the multibody system is done symbolically, while offering different numerical possibilities for the analysis and optimization of the system. After an explanation of the structure of SYMBS, the modelling and some possibilities for the analysis and optimization of a mechanical system are presented.

1 Introduction

In the modelling step of a multibody system the dynamic behavior has to be described. The presented program, SYMBS, does this by calculating the equations of motion symbolically with the *Matlab Symbolic Toolbox*. Multibody systems can be divided in two groups, systems with tree structure and systems with kinematic loops. For systems with tree structure all bodies can be associated schematically to one parent each, starting from the inertial system *ISYS*. By adding additional constraints to a system with tree structure, a system with kinematic loops can be obtained. With d'Alembert's Principle the equations of motion can be calculated, see [Schiehlen and Eberhard, 2004], which are for non-holonomic systems with tree structure

$$\dot{\mathbf{y}} = \mathbf{z}, \quad (1)$$

$$\mathbf{M} \cdot \dot{\mathbf{z}} + \mathbf{k} = \mathbf{q}. \quad (2)$$

The vector \mathbf{y} describes the generalized coordinates, while \mathbf{z} are the generalized velocities. The mass matrix \mathbf{M} , the vector containing the Coriolis- and gyroscopic forces \mathbf{k} and the vector of applied forces \mathbf{q} have to be calculated. For systems with kinematic loops the addi-

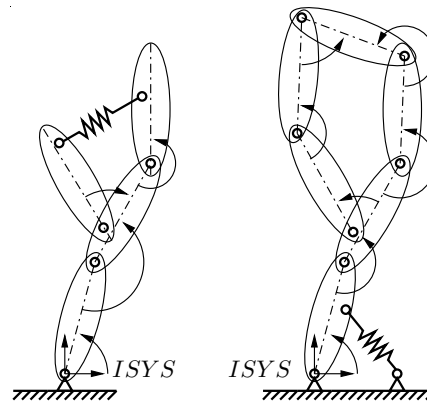


Figure 1. System with tree (left) and loop structure (right).

tional constraint equations \mathbf{c} have to be added and the equations have the form

$$\dot{\mathbf{y}} = \mathbf{z}, \quad (3)$$

$$\mathbf{M} \cdot \dot{\mathbf{z}} + \mathbf{k} - \mathbf{C} \cdot \boldsymbol{\lambda} = \mathbf{q}, \quad (4)$$

$$\mathbf{c} = \mathbf{0}. \quad (5)$$

All information describing the system and its simulation is stored in the Matlab data structure *sys*, see [Henninger, 2007]. It contains for example

Entry	Content
name	name of the system
userVar	userdefined parameters
body	bodies
ksys	coordinate systems
felem	force elements
genCoords	generalized coordinates
eqc	constraint equations
eqm	nonlinear equations of motion
results	simulation results
par	parameters for simulation
graphics	graphical representations

2 Modelling of Multibody Systems

For a given body called 'Body1' the data stored in the structure `sys` looks like

id:	'Body1'
name:	'Body 1'
idx:	1
refsys:	'ISYS'
r_rel:	[3x1 sym]
phi_rel:	[3x1 sym]
r_cg:	[3x1 sym]
phi_cg:	[3x1 sym]
m:	[1x1 sym]
I:	[3x3 sym]
M:	[6x6 sym]
ksys:	'Body1' 'Body1_cg'
geo:	156.0015

This data contains the identification of the body with 'id', 'name' and an index. Every body consists of at least two coordinate systems, one used to describe the position and orientation of the body in the reference system, here 'ISYS', denoted by 'r_rel' and 'phi_rel', and one coordinate system defining the center of gravity. All coordinate systems belonging to this body are stored in 'ksys', while 'geo' contains an object handle in the animation window. In the fields 'm', 'I' and 'M' all mass and inertia properties are stored.

To evaluate the symbolic expressions the numeric values have to be inserted. The easiest and fastest way to achieve this task is to automatically create files which accomplish only numerical evaluations. All files used and created by SYMBS are separated in problem specific data in a model folder and problem independent routines. In the model folder a directory is created called `sysFunctions/`, containing all automatically created functions for numerical evaluations, e.g. the position 'Body1_r.m', or the velocity 'Body1_v.m'. The nonlinear equations of motion are stored symbolically in 'sys.eqm', containing all quantities from Eq. (2), again with functions for numerical evaluation, e.g. 'eqm_nonlin_M.m'. Systems with kinematic loops are treated in the same way, with the additional informations of the constraint equations stored in 'sys.eqc'.

3 Analysis and Optimization of mechanical systems

Once the system modelling is finished, the symbolical expressions are calculated and the functions for numerical evaluation are created, the potential of numerical simulation of Matlab can be used. Time integration with a wide choice of integration algorithms is possible. For the kinematic analysis, functions for all generalized coordinates can be given. All results are stored in a similar way in 'sys.results', enabling a standardized animation of the results. Therefore, the graphics library `geom3d` is used, see [Legland, 2005]. This provides an easy way to assign simple graphical objects like cubes and cylinders to coordinate systems moved in the figure window, see Fig. 2.

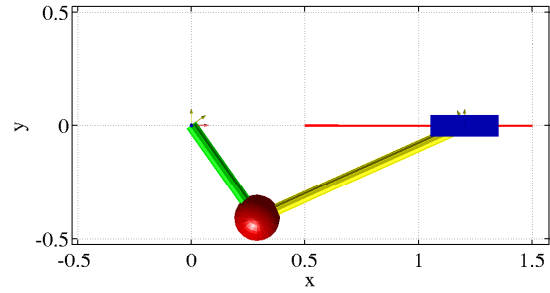


Figure 2. Animation window with a slider crank mechanism.

SYMBS also contains a module to run a parameter optimization of the mechanical system and to calculate sensitivities. For the optimization the algorithms by Matlab are available. For the sensitivity analysis two semi-analytical methods are implemented, the direct method and the adjoint variable method, see [Bestle, 1994; Kurz, 2007]. They are alternatives to the finite differences, which are often used. These semi-analytical methods have the advantage of a higher efficiency, especially for large systems, and much higher precision. To improve the usability of text based input files, a graphical user interface has been created, using the Matlab provided environment, see [Lutz, 2007].

4 Example

To verify the precision of the gradients calculated with the semi-analytical methods among others a simple example has been investigated, see [Bestle, 1994]. For this one mass oscillator the gradient can be obtained symbolically as a reference. The equations of motion are

$$\dot{y} = z, \quad (6)$$

$$m\dot{z} = -cy \quad (7)$$

with the initial conditions $y(0) = 0$ and $z(0) = 0.5$. The functional criterion for the gradient is

$$\psi := y^1, \quad (8)$$

evaluated at the final time $t^1 = \pi/2$. The mass is chosen as $m = 1$, then the gradient with respect to the spring constant $c = 1$ can be calculated analytically to be $\nabla\psi = -0.25$. The same example has been calculated with the two semi-analytical methods mentioned above with the errors shown in Fig. 3. For this investigation the absolute and relative error tolerance of the integration has been varied. Matlab restricts the relative error tolerance for numerical reasons not to be smaller than $2.2\text{e-}14$. These results show that by adjust-

ing the integration tolerances the semi-analytical methods are able to reach machine accuracy, see [Eberhard, 1996].

Schiehlen, W. and Eberhard, P. (2004). *Technische Dynamik*. B.G. Teubner. Wiesbaden.

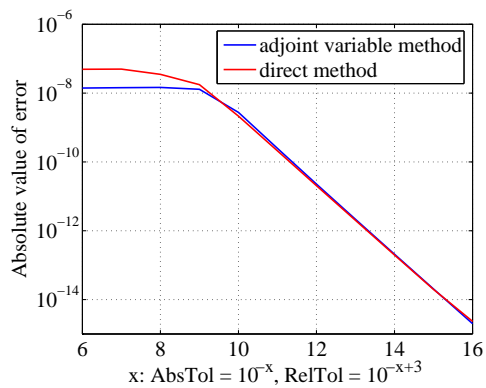


Figure 3. Error of the gradient calculation.

5 Conclusion

The program SYMBS is a Matlab based program to obtain symbolic equations of motion and kinematic expressions for given multibody systems. It offers a wide range of possibilities for the analysis and optimization of the systems based on the powerful Matlab environment. The expressions are stored symbolically in an internal data structure and in automatically created files for the numerical evaluation.

References

- Bestle, D. (1994). *Analyse und Optimierung von Mehrkörpersystemen*. Springer-Verlag. Berlin.
- Eberhard, P. (1996). *Zur Optimierung von Mehrkörpersystemen*. Dissertation. VDI Fortschritt-Berichte, Reihe 11, Nr. 227. VDI Verlag. Düsseldorf.
- Henninger, C. (2007). *SYMBS eine Matlab-Umgebung zur Simulation von Mehrkörpersystemen*. Zwischenbericht ZB-149. Institut für Technische und Numerische Mechanik, Universität Stuttgart.
- Kurz, T. (2007). *Entwicklung eines Optimierungsmoduls mit Sensitivitätsanalyse für die symbolische Mehrkörpersimulationsumgebung SYMBS*. Diplomarbeit DIPL-122. Institut für Technische und Numerische Mechanik, Universität Stuttgart.
- Legland, D. (2005). Graphics Library geom3d <http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=8002&objectType=file>.
- Lutz, M. (2007). *Erstellung einer symbolischen Mehrkörpersimulationsumgebung in Matlab mit grafischer Benutzeroberfläche und verschiedenen Schnittstellen*. Studienarbeit STUD-263. Institut für Technische und Numerische Mechanik, Universität Stuttgart.