ON THE ROLE OF META-ACCELERATIONS IN EVOLUTIONAL DYNAMICS AND STABILITY OF WEAKLY-DISSIPATIVE SOLIDS

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Abstract
The paper deals with the comparative analysis of stability of the torque-free rotation of a rigid body and a solid with internal dissipation. It is shown that the dissipation-induced instability has a critical influence on stability of the rotating body. Lyapunov's approach to stability is demonstrated to be very sensitive to the mechanical model considered. This weak side of Lyapunov's approach can be overcome by means of introducing the concept of the dissipative rigid body. On some examples we demonstrate as to how the dissipation in a rigid body is introduced and the equations of motion are derived. These equations allow one to study both stability and the evolutional dynamics of weakly-damped mechanical systems. The meta-accelerations (the third time derivatives of displacements and rotation angles) are shown to be of a crucial importance for the entire analysis.

Key words
Stability, evolutional dynamics, dissipative rigid body, meta-acceleration.

1 Introduction: statement of the problem

The majority of problems of analytical mechanics and stability of motion deals with the mathematical models of the conservative and non-conservative problems. Such a division of the systems into two classes is caused by the goal of the analysis and the time interval of the dynamic process. For instance, in the mechanical theory of vibration the duration of the oscillational process is usually estimated at the number of oscillation n of the fundamental frequency within the limits $1 < n < 10^2$. Within these limits the role of the dissipative factors is secondary and it can be often neglected when constructing an appropriate conservative model. Even if the damping is taken into account then the damping is modeled by an external (surface) friction which is proportional to the first, second or zeroth power of the relative velocity.

For the problems of stability of motion $10^2 < n < 10^6 - 10^9$ is typical. For such considerable time intervals the very usage of the conservative models becomes rather problematical since even arbitrary small dissipative forces have a "secular" influence on the system motion due to a permanent reduction of the mechanical energy of the system. This influence is especially clear observable in the problems of celestial mechanics and astronautics. In these problems there is no external friction and the only dissipative factor is the internal damping resulting in the evolutional effect upon the dynamic behaviour of the celestial objects and stability of their stationary motion. First of all such an "evolutional dynamics" is typical for the rotational motion of the celestial objects. Therefore consideration of the internal damping becomes the governing factor in the stability problems and requires developing the adequate mathematical models. This is the main objective of the present paper.

2 Stability of rotation of a torque-free dissipative solid

It is well-known that rotation of a rigid body about the axes of minimum and maximum moment of inertia is. The engineering experience contradicts the above theoretical results. The first American satellite Explorer 1 launched in 1958 was designed as a minor axis spinner. Nevertheless the minor axis spin became unstable and the satellite went into a flat rotation after a few hours. It is clear that in the case of no external forces and no external dissipation, only internal energy dissipation affects the theoretical results. A qualitative explanation for this effect is as follows. The angular momentum is conserved during the whole rotation, i.e. for both minor and major axis rotation we have

$$L^2 = \Theta_{\min}{}^2\Omega_1{}^2 = \Theta_{\max}{}^2\Omega_2{}^2 . \qquad (2.1)$$

However the major axis rotation has a lower kinetic energy than the minor axis rotation since

$$2T_2 = \Theta_{\max}\Omega_2{}^2 = \frac{L^2}{\Theta_{\max}} < 2T_1 = \Theta_{\min}\Omega_1{}^2 = \frac{L^2}{\Theta_{\min}}. \quad (2.2)$$

Hence any dissipation brings the rotating body with the same angular momentum into the rotation with the minimum energy which is the major axis spin. Nowadays this design regulation is referred to as the major-axis-rule and the instability of Explorer 1 is considered as a classical example of the internal-dissipation-induced instability.

Let us consider a solid of volume V and surface s. We study stability of rotation about the third axis, that is, the unperturbed motion is $\omega_1 = \omega_2 = 0, \omega_3 = \Omega = const$ whereas the unperturbed motion is $\omega_1 = \varepsilon_1, \omega_2 = \varepsilon_2, \omega_3 = \Omega + \varepsilon_3$. The differential form of conservation of the angular momentum is given by

$$\dot{\mathbf{L}} = \frac{d}{dt}\int_V \mathbf{r}\times\mathbf{v}\rho dV = \int_V \mathbf{r}\times\mathbf{k}dV + \int_s \mathbf{r}\times\mathbf{f}ds = 0. \quad (2.3)$$

Here ρ denotes the mass density, $\mathbf{k}$ and $\mathbf{f}$ are the volume and surface forces, $\mathbf{r}$ and $\mathbf{v}$ are the position vector and the velocity vector. However the kinetic energy is not invariant under rotation. The dissipation in the solid is governed by the law of change in the kinetic energy

$$\begin{aligned}\dot{T} &= \frac{d}{dt}\frac{1}{2}\int_V \mathbf{v}\cdot\mathbf{v}\rho dV = \\ &= \int_V \mathbf{v}\cdot\mathbf{k}dV + \int_s \mathbf{v}\cdot\mathbf{f}dS - \int_V \boldsymbol{\tau} : \dot{\boldsymbol{\varepsilon}}dV \neq 0,\end{aligned} \quad (2.4)$$

where $\boldsymbol{\tau}$ is the Cauchy (true) stress tensor and $\dot{\boldsymbol{\varepsilon}}$ is the tensor of the strain rate.

Let us prove that the rotation about the axis of the maximum moment of inertia ($\Theta_3 = \Theta_{\max}$) is stable. An appropriate Lyapunov function is as follows

$$\begin{aligned}V &= \frac{\kappa^2}{\Omega^2}\left[T_R + T_D + U_D - T_R(\Omega)\right]^2 + 2\Theta_3 T_R - L^2 = \\ &= \frac{\kappa^2}{\Omega^2}\left[T_R + T_D + U_D - T_R(\Omega)\right]^2 + \\ &\quad + \Theta_1(\Theta_3 - \Theta_1)\varepsilon_1^2 + \Theta_2(\Theta_3 - \Theta_2)\varepsilon_2^2.\end{aligned} \quad (2.5)$$

Here the subscripts D and R refer to the deformable and rigid body parts of the corresponding quantity, for instance T_R denotes the kinetic energy of the rigid body motion of the solid and U_D is the internal energy of the solid. This Lyapunov function is positive definite as it immediately follows from eq. (2.5). The time derivative

$$\begin{aligned}\dot{V} &= \frac{\kappa^2}{\Omega^2}\left[\Theta_1\varepsilon_1{}^2 + \Theta_2\varepsilon_2{}^2 + \Theta_3\varepsilon_3(\varepsilon_3 + 2\Omega) + \right. \\ &\quad \left. + 2(T_D + U_D)\right](\dot{T}_D + \dot{U}_D)\end{aligned} \quad (2.6)$$

is negative semidefinite for any solid with dissipation. The inequality $(\dot{T}_D + \dot{U}_D) = -d \le 0$ ($d \ge 0$ is the power of dissipated mechanical energy) holds by virtue of the first and second laws of thermodynamics for a load-free and thermally insulated solid with the internal dissipation. Then according to the dissipativity principle for the systems with distributed parameters, [Khalil, 2002; van der Schaft, 2000], this rotation is stable.

In order to show that rotation about the axis of minimum moment of inertia ($\Theta_3 = \Theta_{\min}$) is unstable we apply Chetaev's theorem of instability. Let us take the following Lyapunov function

$$\begin{aligned}V &= -\frac{\kappa^2}{\Omega^2}\left[T_R + T_D + U_D - T_R(\Omega)\right]^2 + L^2 - 2\Theta_3 T_R = \\ &= -\frac{\kappa^2}{\Omega^2}\left[T_R + T_D + U_D - T_R(\Omega)\right]^2 + \\ &\quad + \Theta_1(\Theta_1 - \Theta_3)\varepsilon_1^2 + \Theta_2(\Theta_2 - \Theta_3)\varepsilon_2^2.\end{aligned} \quad (2.7)$$

The time derivative is

$$\begin{aligned}\dot{V} &= -\frac{\kappa^2}{\Omega^2}\left[\Theta_1\varepsilon_1{}^2 + \Theta_2\varepsilon_2{}^2 + \varepsilon_3(\varepsilon_3 + 2\Omega) + \right. \\ &\quad \left. + 2(T_D + U_D)\right](\dot{T}_D + \dot{U}_D) > 0.\end{aligned} \quad (2.8)$$

The conditions of Chetaev's theorem are fulfilled for small values of κ, since total time derivative is positive definite along every trajectory (i.e. $\dot{V} > 0$ for $(\dot{T}_D + \dot{U}_D) = -d \le 0$) and the function itself assumes positive values (i.e. $V > 0$) for some arbitrary small values of the state variables. Thus the minor axis spin is unstable and the solid under consideration tends to the only stable invariant set which is the major axis spin. It is well known from the astronomy that any planet rotates about the axis of the maximum moment of inertia. In particular, the Earth is a major axis spinner because it is a geoid, i.e. a spheroid with the oblateness [Lurie, 2002]

$$\frac{R_{equator} - R_{polar}}{R_{equator}} = \frac{21.5\,km}{6378.4\,km} = 0{,}3375\%.$$

This example demonstrates the strong and weak sides of Lyapunov's approach. The merit is that there is no need to solve the system of nonlinear differential equations in order to judge the system stability. The shortcoming is also evident. The Lyapunov approach to stability provides one with a very categorical judgment: stable or unstable. This judgment is extremely sensitive to the model chosen. Some changes in the model may have no influence on the judgment, however some other changes may affect the stability judgment and lead to the opposite conclusion. The example with the internal dissipation has clearly demonstrated this drastic change.

Another way of judging the stability is obtaining the solution for free rotation of the deformable body with internal dissipation. The initial-value problem should be solved in the framework of the geometrically nonlinear continuum mechanics for arbitrary initial conditions and the solution obtained should be analysed from a perspective of stability of the rigid body rotation. This is a challenging problem and the very interpretation of the result presents a problem. An alternative approach accounting for a much simpler analysis is suggested in what follows.

3 Concept of the dissipative rigid body

Chronologically, the first model of the rigid body was constructed by L. Euler in 1765. The model of the deformable body was developed by O. Cauchy in the middle of XIX century. Both models are conservative. The first non-conservative models (Kelvin-Voigt body, Prandtl body etc. referred to as the rheological bodies) appeared only in XX century. These models are schematically presented in Table 1.

Table 1. Models of rigid and deformable bodies

	rigid body	deformable body (solid)
conservative model	Euler's model	Cauchy's model
non-conservative model		rheological body

The empty cell of this table puts a thought about a possible existence of such a model of the rigid body which differs from Euler's model only in presence of internal dissipation. Natural and artificial space objects can serve as real prototypes of this model. The instability of Explorer 1 discussed above shows that such objects do not exhibit the conservative torque-free rotation about the center of mass.

A qualitative difference of the permanent rotation of a rigid body from any other rotation (for example, a precession) consists in that the permanent rotation with a stationary field of the relative acceleration causes a stationary field of internal forces. Any other rotation results in a nonstationary field of internal forces, this field yielding internal microdisplacements and thus the internal dissipation of energy. Clearly, for an adequate description of the process one can make use of the viscoelastic model by introducing internal degrees of freedom. This way is rather broadly presented in the literature, see e.g. [Chernousko, 1968], however it complicates the mathematical model of the object. A possible alternative is the concept of the dissipative rigid body which implies using Euler's dynamic equations with the dissipative terms. A similar concept known as the "dissipative material particle" (a particle with the restitution coefficient) is widely used in mechanics. The dissipative rigid body possesses a more complicated structure and thus more complex dynamic behaviour. For example it can be treated as a massless rigid framework filled by a granular medium whose particles execute microdisplacements relative to the rigid framework. These microdisplacements can result in microscopic changes in the inertial properties of the body, e.g. the displacement of the center of mass in the field of external gravitational forces. As shown above this subtle effect can play the key role in the problem of stability of the relative motion of the celestial bodies. However this effect can also be observed on example of the simple pendulum provided that the corresponding function of internal dissipation is constructed.

4 Construction of the function of internal dissipation

The Rayleigh dissipative function

$$R = \frac{1}{2}\int_s \dot{\mathbf{q}}^T \mathbf{B}\dot{\mathbf{q}}\, ds \tag{4.1}$$

is widely used in analytical mechanics [Lurie, 2002] and is the generating function for the viscous forces of the external (surface) friction. Here $\dot{\mathbf{q}}$ denotes the column of the generalised relative velocities, $\mathbf{B}$ is the matrix of coefficients of viscous resistance and the integration is carried out over the frictional surface s. From a mechanical perspective R expresses the power of the dissipative forces of external friction.

Let us now proceed to construction of the dissipative function for the force of internal (volumetric) friction which would reflect the dependence on the general character of the body motion. To this end, we select an elementary volume dV in the body. Let dS denote the power of the dissipative force in this volume. If the dissipative force N (or the corresponding stresses) is constant in the local reference frame then the relative microdisplacements of volume dV corresponding to these stresses are also constant and thus no dissipation of the mechanical energy takes place. This is the case of the permanent rotation without external gravitational forces with a constant axipetal acceleration. If the relative acceleration in volume dV is time-dependent, then force $\mathbf{N}_r$ and, in turn, displacement $\mathbf{u}_r$ are also time-dependent. Expressing the elementary power dS as a quadratic form of the relative velocities $\dot{\mathbf{u}}_r$ and assuming that $\dot{\mathbf{u}}_r \sim \dot{\mathbf{N}}_r$ we arrive at the following result

$$dS = \frac{1}{2}\dot{\mathbf{N}}_r{}^T \mathbf{B}\dot{\mathbf{N}}_r dV\,. \tag{4.2}$$

Here $\mathbf{B}$ is a positive definite (or positive semidefinite) matrix of the dissipative coefficients. Integration over the dissipative volume yields

$$S = \frac{1}{2}\int_V \dot{\mathbf{N}}_r{}^T \mathbf{B}\dot{\mathbf{N}}_r dV\ , \tag{4.3}$$

where $\dot{\mathbf{N}}_r$ denotes the column of the time-derivative of the internal forces caused by the inertia forces of the rotating body and the forces of external gravitational field (if it exists).

As one can see from eq. (4.2) the internal dissipation is absent in three cases: (i) permanent rotation, (ii) translation with a constant acceleration and (iii) spiral motion with a constant angular velocity directed along the vector of uniformly accelerated motion. Only the first regime is of the practical importance since it is the limiting case of rotation for majority of the celestial bodies.

To illustrate constructing and using the dissipative function S we consider a number of simple problems of classical mechanics of pendulum systems.

5 Evolutional dynamics of the spherical pendulum

The spherical pendulum exhibits rich dynamics. In order to construct the evolutional mathematical model we use the dissipative function (4.3) and present the force of the thread tension N as a function of the phase variables $x, y, \dot{x}, \dot{y}$. Restricting ourselves by the quadratic approximation we can write

$$N_r = \frac{m}{l}\left[\left(\dot{x}^2+\dot{y}^2\right)-k^2\left(x^2+y^2-l^2\right)\right],$$

$$\dot{N}_r = \frac{2m}{l}\left[\left(\dot{x}\ddot{x}+\dot{y}\ddot{y}\right)-k^2\left(x\dot{x}+y\dot{y}\right)\right], \qquad (5.1)$$

$$S = \frac{1}{2}\beta\dot{N}_r^2 = 2\frac{mb}{l}\left[\left(\dot{x}\ddot{x}+\dot{y}\ddot{y}\right)-k^2\left(x\dot{x}+y\dot{y}\right)\right]^2.$$

The evolutional dynamic equations (without conservative nonlinearities) take the form

$$\ddot{x}+k^2x = -\frac{\partial S}{\partial \dot{x}} = -\nu\left(\ddot{x}-k^2x\right)\left[\left(\dot{x}\ddot{x}+\dot{y}\ddot{y}\right)-k^2\left(x\dot{x}+y\dot{y}\right)\right],$$

$$\ddot{y}+k^2y = -\frac{\partial S}{\partial \dot{y}} = -\nu\left(\ddot{y}-k^2y\right)\left[\left(\dot{x}\ddot{x}+\dot{y}\ddot{y}\right)-k^2\left(x\dot{x}+y\dot{y}\right)\right],$$

$$\nu = 4\beta\frac{m^2}{l^2}. \qquad (5.2)$$

It is easy to prove that the angular momentum is conserved. To this aim, we multiply the first and second equations by x and y respectively. The subtraction yields the following result

$$\left(\dot{x}y-\dot{y}x\right)^{\bullet} = -\nu\left(\dot{x}y-\dot{y}x\right)^{\bullet}\left[\left(\dot{x}\ddot{x}+\dot{y}\ddot{y}\right)-k^2\left(x\dot{x}+y\dot{y}\right)\right] \qquad (5.3)$$

which means $\left(\dot{x}y-\dot{y}x\right)^{\bullet}=0$ or $L=\dot{x}y-\dot{y}x=const$. Instead of the energy integral we obtain the dissipative relation

$$2\dot{E} = \left[\dot{x}^2+k^2x^2\right]^{\bullet}+\left[\dot{y}^2+k^2y^2\right]^{\bullet} = -4S \le 0, \qquad (5.4)$$

which determines the rate of the thread „heating“.

In order to construct the simpler evolutional equations one should calculate function S on the unperturbed motion. The result is

$$S = 8\frac{mb}{l}k^4\left[x\dot{x}+y\dot{y}\right]^2 \qquad (5.5)$$

and the equations of motion take the form

$$\begin{aligned}\ddot{x}+k^2x &= -4\nu k^4 x\left(x\dot{x}+y\dot{y}\right),\\ \ddot{y}+k^2y &= -4\nu k^4 y\left(x\dot{x}+y\dot{y}\right).\end{aligned} \qquad (5.6)$$

For constructing the solution let us introduce a complex variable $w=x+iy$. This allows us to set the solution of the later equations in the form

$$\frac{d}{dt}|A|^2 = \kappa|A|^2\left(|A|^2-\frac{L}{k}\right), \frac{d}{dt}|B|^2 = \kappa|B|^2\left(|B|^2+\frac{L}{k}\right).$$

Integration gives the following expressions

$$\begin{aligned}|A|^2 &= \frac{L}{k}\frac{D\exp\left(-\kappa Lt/k\right)}{D\exp\left(-\kappa Lt/k\right)-1},\\ |B|^2 &= \frac{L}{k}\frac{1}{D\exp\left(-\kappa Lt/k\right)-1},\end{aligned} \qquad (5.7)$$

with D being the integration constant. We see that $|A|$ tends to zero whereas $|B|$ tends to a finite value as time passes. At $|A|\to 0$ the ellipse is transformed into a circle of radius a, the area being equal to the area of osculating ellipse. The evolutional motion of the apex of the spherical pendulum is portrayed in Fig. 1. The process of evolution is seen to have irreversible character. A peculiar „nonlinear superposition“ of the processes of precession and evolution occurs which results in that the osculating ellipse precesses and slowly transforms into a circle.

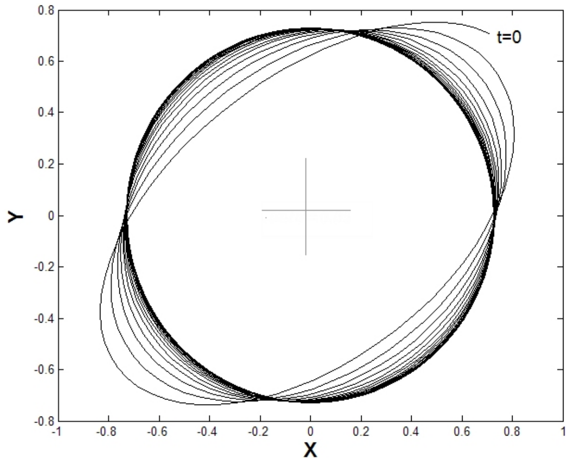

Figure 1 Evolutional motion of the apex of the spherical pendulum.

6 Evolutional equations of Euler's motion of the rigid body

Let us now proceed to the classical problem of the rigid body dynamics which is a torque-free rotation. The field of the acceleration $\mathbf{w}$ in the rotating rigid body is known to be given by the matrix formula [Lurie, 2002]

$$\mathbf{w} = \mathbf{Gr}, \qquad (6.1)$$

where $\mathbf{r}=\left[x,y,z\right]^T$ is the position vector of a point about the centre of mass and $\mathbf{G}$ is the quadratic matrix of acceleration at this point

$$\mathbf{G} = \begin{Vmatrix} -\omega_2^2-\omega_3^2 & \omega_1\omega_2-\dot{\omega}_3 & \omega_1\omega_3+\dot{\omega}_2 \\ \omega_1\omega_2+\dot{\omega}_3 & -\omega_3^2-\omega_1^2 & \omega_2\omega_3-\dot{\omega}_1 \\ \omega_3\omega_1-\dot{\omega}_2 & \omega_3\omega_2+\dot{\omega}_1 & -\omega_1^2-\omega_2^2 \end{Vmatrix}. \qquad (6.2)$$

Indices 1,2,3 denote the principal axes of inertia. The dissipative function is convenient to be written as a quadratic form in $\dot{\mathbf{w}}$, which we refer to as the meta-acceleration,

$$\begin{aligned}S &= \frac{1}{2}\int_V \dot{\mathbf{w}}^T\mathbf{B}\dot{\mathbf{w}}\rho dV = \frac{1}{2}\int_V \mathbf{r}^T\dot{\mathbf{G}}^T\dot{\mathbf{G}}\mathbf{r}\rho dV\\ &= \frac{1}{2}\beta\int_V \mathbf{r}^T\mathbf{D}\mathbf{r}\rho dV = \\ &= \frac{1}{2}\beta\left(D_{11}\int_V x^2\rho dV+D_{22}\int_V y^2\rho dV+D_{33}\int_V z^2\rho dV\right).\end{aligned} \qquad (6.3)$$

Here $\mathbf{B}=\beta\mathbf{E}, \mathbf{D}=\dot{\mathbf{G}}^T\dot{\mathbf{G}}$ ($\mathbf{E}$ is the unity matrix) and

$$\begin{aligned}D_{11} &= 4\left(\omega_2\dot{\omega}_2+\omega_3\dot{\omega}_3\right)^2+\left(\ddot{\omega}_3+\omega_1\dot{\omega}_2+\omega_2\dot{\omega}_1\right)^2+\\ &+\left(-\ddot{\omega}_2+\omega_1\dot{\omega}_3+\omega_3\dot{\omega}_1\right)^2\end{aligned} \qquad (6.4)$$

and D_{22}, D_{33} are obtained from the latter equation by a circular permutation of the indices. Taking time derivative of $\mathbf{G}$ yields the following formula for the dissipative functions

$$S = S_1 + S_2 + S_3 \ , \tag{6.5}$$

where

$$\begin{aligned} S_1 = \Theta_1 \frac{\beta}{2}\Big[& 4\omega_1 \dot{\omega}_1 \left(\omega_1 \dot{\omega}_1 + \omega_2 \dot{\omega}_2\right) + \\ & + 4\omega_3 \dot{\omega}_3 \left(\omega_1 \dot{\omega}_1 - \omega_2 \dot{\omega}_2\right) + 2\ddot{\omega}_2 \left(\omega_1 \dot{\omega}_3 + \omega_3 \dot{\omega}_1\right) - \\ & - 2\ddot{\omega}_3 \left(\omega_1 \dot{\omega}_2 + \omega_2 \dot{\omega}_1\right) + \ddot{\omega}_1^2 + \left(\omega_2 \dot{\omega}_3 + \omega_3 \dot{\omega}_2\right)^2 \Big] \end{aligned} \tag{6.6}$$

and S_2, S_3 are obtained from the latter equation by a circular permutation of the indices. In order to derive the dynamic equations for the dissipative Euler motion we substitute S, eq. (6.5), in the system

$$\begin{aligned} \Theta_1\dot{\omega}_1 + \left(\Theta_3 - \Theta_2\right)\omega_2\omega_3 &= -\partial S / \partial\omega_1 \ , \\ \Theta_2\dot{\omega}_2 + \left(\Theta_1 - \Theta_3\right)\omega_1\omega_3 &= -\partial S / \partial\omega_2 \ , \\ \Theta_3\dot{\omega}_3 + \left(\Theta_2 - \Theta_1\right)\omega_1\omega_2 &= -\partial S / \partial\omega_3 \end{aligned} \tag{6.7}$$

and obtain the first equation

$$\begin{aligned} & \Theta_1\dot{\omega}_1 + \left(\Theta_3 - \Theta_2\right)\omega_2\omega_3 = -\beta\Big\{\omega_1 \left(4\Theta_1\dot{\omega}_1^2 + \Theta_2\dot{\omega}_3^2 + \Theta_3\dot{\omega}_2^2\right) \\ & +\dot{\omega}_1 \left[\omega_3 \dot{\omega}_3 \left(2\Theta_3 + 2\Theta_1 - \Theta_2\right) + \omega_2 \dot{\omega}_2 \left(2\Theta_1 + 2\Theta_2 - \Theta_3\right)\right] \\ & +\left(\Theta_2 - \Theta_1\right)\dot{\omega}_2\ddot{\omega}_3 + \left(\Theta_1 - \Theta_3\right)\dot{\omega}_3\ddot{\omega}_2\Big\}, \end{aligned} \tag{6.8}$$

with the second and third equations being obtained from it by a circular permutation of the indices.

The equations are seen to contain the meta-accelerations, which are the second derivatives of the angular velocities or the third derivatives of the angle of rotation. They are normally absent in the equations of dynamics and their appearance is connected with that the internal degrees of freedom are not explicitly introduced. The dissipative coefficient β is very small and this fact gives rise to considerable complications for numerical simulation of the evolutional regimes. Nevertheless the correct study of stability of the weakly-dissipative solids requires account for the evolutional effects.

7 Conclusions

Stability and evolutional dynamics of rigid and deformable bodies are studied. We suggested the Lyapunov functions, which ensure stability of the free-torque rotation of a rigid body about the axes of the minimum and maximum moments of inertia. The Lyapunov approach to description of stability of a mechanical system is shown to be very sensitive to the model chosen. If one assumes that the system under consideration is deformable and dissipative, then the minor axis spin is no longer stable. This sensitivity represents a weak side of Lyapunov's approach and can be overcome by means of introducing the concept of the dissipative rigid body. By virtue of some examples we demonstrated as to how the dissipation in a rigid body is introduced and the equations of motion for these systems are derived. These equations stress the role of meta-acceleration in stability and the evolutional dynamics of the weakly-damped mechanical systems.

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