# MATHEMATICAL MODELS OF RADIO-TELESCOPE METALLIC STRUCTURE AS A CONTROLLED PLANT

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**Abstract**

In the article mathematical and computer modeling of metallic structure of the big fully rotating millimetre-wave radio-telescope is considered. The technique and algorithms of development of support-rotary mechanism mathematical models as nonlinear controlled plants and their reduction are given.

**Key words**

Identification, modeling, nonlinear systems, Numerical methods.

## 1 Introduction

Researches of deep space cosmic ray sources require developing big and fully rotating radio-telescopes (RT), which main dish diameters are about 100 meters. These radio-telescopes able to receive millimetric waves (1-10 mm) with wave intensity 10-30 W/m2Hz, which is a thousand times smaller than for a modern centimetre-wave RT.

If a wave-length is fixed, a beam width is known to decrease with increasing main dish diameter. In millimetric waves it leads to toughen the requirements for the pointing accuracy and nonlinear effects take place, therefore it is necessary to develop more adequate models of the RT metallic structure (MS), which can provide pointing RT with high accuracy.

The radio-telescope metallic structure is used to support and move the reflecting surfaces system (the dish system), in order that radio-waves accepted from the Universe are focused in the special point, where the receiver registering the characteristics of waves is installed. The radio-wave intensity distribution as a function of the wave frequency and the relative angular coordinates between the cosmic source and the receiver is a main characteristic of the RT pointing accuracy.

The metallic structure has a mass of about a thousand tons, and a size of up to hundred meters. It is a complex engineering construction loaded by gravitation, wind and temperature. There are close tolerance requirements of dish system. The deviations of the deformed dish surface from the theoretical shape must be less than several tens of microns, and the accuracy of angular pointing musts be of about 1 seconds of angle.

In spite of the small angular velocity of pointing, such as a daily rotation of the Earth (15 second of angle / second), the problem of assurance of the high pointing accuracy substantially depends on conformity of RT MS mathematical models to the real RT MS.

The main purpose of RT MS models is to estimate in real time the state-vector components, which are not measured but essentially influence on the radio-telescope pointing accuracy and the radio-wave reception, and, hence, on the radio-telescope instrumental error.

The more precisely it is required to estimate, the estimator has the greater order of differential equations. Obviously there is a compromise between the required accuracy of estimations and the order of differential equation of estimator. This compromise can be reached by solving of a properly formulated problem.

## 2 The full-scale finite element model of the radio-telescope metallic structure

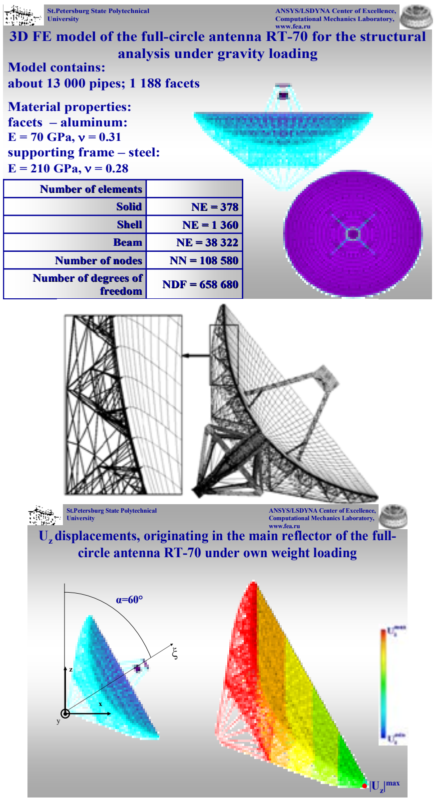



Figure 1. The finite-element model of the radio-telescope metallic structure.

The radio-telescope metallic structure as a mechanical system can be described with various levels of detail. The initial data of such models are working drawings of separate elements and units, properties of materials of these elements, conditions of fixing, installation and adjustment. Using these data and special computer programs such as Ansys, Nastran, etc., which are a finite-element environment for solving of a wide variety of engineering problems, including analysis of designs, the computer full-scale finite-element model is worked out. Such model allows to define geometrical, weight, inertial and flexibility parameters of separate elements, units, and mechanical object as a whole.

In the full-scale finite-element model (FEM) the equations of motion are formulated in the master coordinates:

$$\begin{aligned}
&\ddot{\eta} = \delta\Omega\dot{\eta} + \Omega\eta + \Pi u,\\
&\xi = H\eta,\ \Omega = H^{-1}\mathrm{P}^{-1}\Xi H,\\
&\Pi = H^{-1}\mathrm{P}^{-1}\Gamma,\\
&\mathrm{P}\ddot{\xi} = \delta\Xi\dot{\xi} + \Xi\xi + \Gamma u,
\end{aligned} \tag{1}$$

where $\eta$ is the vector of master coordinates; $\xi$ is the vector of physical coordinates; $u$ is the vector of external loadings; $\Omega$ is the diagonal matrix of squares of the eigen frequencies ordered on increase; $H$ is the basis transformation matrix from basis of master coordinates to basis of physical coordinates; $\mathrm{P}$ is the symmetric matrix of inertia; $\Xi$ is the symmetric stiffness matrix; $\Pi$ is the matrix of external loadings in basis of physical coordinates; $\Gamma$ is the matrix of external loadings in basis of master coordinates; $\delta$ is the dissipation factor.

As an example of the finite-element analysis the modeling of metallic structure of the fully rotating millimetre-wave radio-telescope which main dish diameter is 70 meters (RT-70, Suffa) can be considered. These research is carrying out at the St. Petersburg State Polytechnical University [Borovkov A. I., Shevchenko D. V., Gimmelman V. G., 2003].

Using the Ansys environment for RT-70 MS as a result of the finite-element analysis the matrixes $\Omega$ and $H$ were obtained. These matrixes are non-stationary and depend on initial state in the general case. The vector $\eta$ cans have thousand of variables. If diagonal elements of matrix $\Omega$ are sorted ascending then the influence of variables of the $\eta$ with greater serial numbers on the physical variables $\xi$ will be the less, than the serial number is more.

It gives grounds for reduction of the system (1) by rejection of a large part of variables with big numbers. The matrix $H$ becomes rectangular (extended on columns), therefore the transformation of the reduced system in basis of master coordinates to the reduced system in basis of physical coordinates is ambiguous and thus the symmetry of matrix of inertia and stiffness matrix can be broken. In this case there is a good reason to consider the model as a system of firm bodies connected by elastic elements. Thus, the number of firm bodies should correspond to the number of the eigen frequencies accounted for in model. We have called coordinates of such system as generalized coordinates, and velocities as generalized velocities.

We have presented such model in a symbol form. It allows to investigate structural and topological properties of RT MS, gyroscopic effects, influence of limped nonlinearities as dry friction, backlash, etc., and also influence of separate parameters on dynamics of the system as a whole by rather simple computing means.

As an example the reduced mathematical symbol lumped parameters model of MS will be considered. This model consists of the seven firm bodies (Fig. 2).

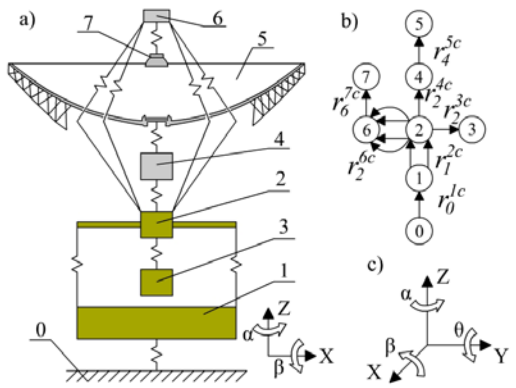
## 3 The nonlinear analytical model of the radio-telescope metallic structure

We have presented the metallic structure as a system of seven firm bodies (Fig.2). Each body has six degrees of freedom and its position in space is defined by six generalized coordinates. The angular and linear displacements of firm bodies from each other are chosen as coordinates. Firm bodies are connected by elastic elements which deformations obey a generalized Hooke's law. *The base coordinate system*

$E^0$ is connected with the Earth (Fig.2, b) and which is set in space by three unit length vectors (orts): $e_x^0$, $e_y^0$, $e_z^0$:

$$E^0 = \begin{pmatrix} e_x^0 & e_y^0 & e_z^0 \end{pmatrix},$$

$$e_x^0 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^{\mathrm{T}},\ e_y^0 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^{\mathrm{T}},\ e_z^0 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}.$$




a) The calculated scheme of RT-70 MS: 0 – the Earth; 1 – the Platform; 2 – the Base; 3 – the Counterbalance; 4 – the Main Dish Base; 5 – the Main Dish; 6 – the Girder; 7 – the Counterreflector; b) The MS calculated scheme graph; c) The coordinate axes and the elementary rotation angles.

Figure 2. The representation of RT-70 MS as a calculated scheme with limp parameters

We shall define elementary «units» from which equations of the system in the form of matrix algorithms will be constructed. For this purpose we will consider the kinematic pair, consisting of the *platform* (body $i$ ) and the *base* (body $j$ ). Bodies have the coordinate systems *(the body axes)* $E^i$ and $E^j$ accordingly which origins are situated in the firm bodies weight centers. At the initial moment of time with the body $j$ *the constructional coordinate system* $E^{jc}$, received by parallel displacement of $E^i$ on a vector $r_i^{jc} = \begin{pmatrix} x_i^{jc} & y_i^{jc} & z_i^{jc} \end{pmatrix}^{\mathrm{T}}$, is connected. Because of external influences and elasticity of design the angular and linear displacements of the body $j$ relatively $E^{jc}$ take place.

The position of $E^j$ in $E^{jc}$ is defined *by the elementary rotation angles* $\beta_{jc}^j$ $\theta_{jc}^j$, $\alpha_{jc}^j$ relatively orts $e_x^{jc}$ $e_y^{jc}$, $e_z^{jc}$ and *the variable vector of parallel displacement* $r_{jc}^j = \begin{pmatrix} x_{jc}^j & y_{jc}^j & z_{jc}^j \end{pmatrix}^{\mathrm{T}}$. Two bases $E^{jc}$ and $E^j$ transform each other *by the rotation matrix* $C_j^{jc}$:

$$E^j = E^{jc} C_j^{jc}.$$

The matrix $C_j^i$ is a product of *elementary rotation matrixes*:

$$C_j^{jc} = C_j^{jc,1}(\beta_{jc}^j) C_j^{jc,2}(\theta_{jc}^j) C_j^{jc,3}(\alpha_{jc}^j),$$

$$C_j^{jc,1}(\beta_{jc}^j) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta_{jc}^j) & -\sin(\beta_{jc}^j) \\ 0 & \sin(\beta_{jc}^j) & \cos(\beta_{jc}^j) \end{bmatrix},$$

$$C_j^{jc,2}(\theta_i^j) = \begin{bmatrix} \cos(\theta_{jc}^j) & 0 & \sin(\theta_{jc}^j) \\ 0 & 1 & 0 \\ -\sin(\theta_{jc}^j) & 0 & \cos(\theta_{jc}^j) \end{bmatrix},$$

$$C_j^{jc,3}(\alpha_{jc}^j) = \begin{bmatrix} \cos(\alpha_{jc}^j) & -\sin(\alpha_{jc}^j) & 0 \\ \sin(\alpha_{jc}^j) & \cos(\alpha_{jc}^j) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The transition from one coordinate system to another, caused by some vector of parallel displacement (constant or variable), is carried out using *matrix of parallel displacement*:

$$T_j^{jc}(r_{jc}^j) = \begin{bmatrix} E & 0 \\ \langle r_{jc}^j \rangle & E \end{bmatrix},\ E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\langle r_{jc}^j \rangle = \begin{bmatrix} 0 & -z_{jc}^j & y_{jc}^j \\ z_{jc}^j & 0 & -x_{jc}^j \\ -y_{jc}^j & x_{jc}^j & 0 \end{bmatrix},$$

where $\langle r_{jc}^j \rangle$ is a matrix formed by elements of the vector of parallel displacement (in this case - $r_{jc}^j$ ).

For our kinematic pair the motion $E^j$ in $E^{jc}$ is carried out using vector of parallel displacement $r_{jc}^j$ and rotation matrix $C_j^{jc}$, *the general matrix of coordinate system transformation* has a form:

$$L_j^{jc} = T_j^{jc} \cdot [C_j^{jc}],\ [C_j^{jc}] = \begin{bmatrix} C_j^{jc} & 0 \\ 0 & C_j^{jc} \end{bmatrix}.$$

The derivative of the matrix $L_m^n$ is defined by equation $\dot{L}_m^n = L_m^n \cdot \Phi_m^{n,m}$, where $\Phi_m^{n,m} = \begin{bmatrix} \langle \omega_m^{n,m} \rangle & 0 \\ \langle v_m^{n,m} \rangle & \langle \omega_m^{n,m} \rangle \end{bmatrix}$, $\langle \omega_m^{n,m} \rangle$ and $\langle v_m^{n,m} \rangle$ are the matrixes formed by a vector of angular and linear velocities $V_m^{n,m} = \left[ v_m^{n,m} \mid \omega_m^{n,m} \right]$ of the body $m$ relatively the body $n$, in body axes of the body $m$. Coordinate columns of vectors of linear $v_m^{n,m}$ and angular $\omega_m^{n,m}$ velocities $m$ - body in $E_m$ are called the *quasi-velocities.*

For this kinematic pair firm bodies with indexes $i$ and $j$ with six degrees of freedom, three angles $\beta_{jc}^j$, $\theta_{jc}^j$, $\alpha_{jc}^j$ and components of the vector of parallel displacement $r_{jc}^j$ are chosen as pair *generalized coordinates* $q_{jc}^j$ (2):

$$q_{jc}^j = \begin{bmatrix} x_{jc}^{j,x} & y_{jc}^j & z_{jc}^j & \beta_{jc}^j & \theta_{jc}^j & \alpha_{jc}^j \end{bmatrix}^{\mathrm{T}} \qquad (2)$$

*The generalized velocities* $\dot{q}_m^n$ of the given kinematic pair are defined in (3):

$$\dot{q}_{jc}^{j} = \begin{bmatrix} \dot{x}_{jc}^{j} & \dot{y}_{jc}^{j} & \dot{z}_{jc}^{j} & \dot{\beta}_{jc}^{j} & \dot{\theta}_{jc}^{j} & \dot{\alpha}_{jc}^{j} \end{bmatrix}^{\mathrm{T}} \quad (3)$$

The kinematic equation of kinematic pair $i, j$ with six degrees of freedom has a form:

$$V_j^{jc,j} = M_j^{jc}\dot{q}_{jc}^{j},\ M_j^{jc} = \begin{bmatrix} C_j^{jc,\mathrm{T}} & 0 \\ 0 & \varepsilon_j^{jc} \end{bmatrix},$$
$$\varepsilon_{jc}^{j} = \left[ C_j^{jc,\mathrm{T}}(\alpha_{jc}^{j})C_j^{jc,\mathrm{T}}(\theta_{jc}^{j})e_x^{jc} \mid C_j^{jc,\mathrm{T}}(\alpha_{jc}^{j})e_y^{jc} \mid e_z^{jc} \right], \quad (4)$$

where $\varepsilon_j^{jc}$ is the *Euler's matrix.*

During system research there is a necessity to impose holonomic constraints on its motion using joints. Then displacements on some generalized coordinates are absent and, hence, the corresponding generalized velocities are equal to zero. We will enter the *vectors of axis mobility* $f_k^j$ of a pair $i, j$, where $k$ is a number of generalized coordinate on which holonomic constraint is imposed.

$$f_k^j = \begin{pmatrix} 0 & 0 & \overset{k}{1} & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}},\ \left\| f^j \right\| = \begin{bmatrix} \ldots & \overset{k}{\left| f_k^j \right|} & \ldots \end{bmatrix}.$$

Then equation (4) becomes $V_j^{jc,j} = M_j^{jc}\left\| f^j \right\| \dot{q}_{jc}^{j}$. As all firm bodies of MS calculated scheme have six degrees of freedom the matrix $\left\| f^j \right\|$ is identity, and will not be taken into consideration.

By similar way the generalized coordinates for each kinematic pair are certain according to the graph in the figure 2, where constructional displacements taking place in system are also specified.

In the compact form the vector of the generalized coordinates $q_C$ and the vector of generalized velocities $\dot{q}_C$ we have a form (5):

$$q_C = \left[ q_{1c}^1 \mid q_{2c}^2 \mid q_{3c}^3 \mid q_{4c}^4 \mid q_{5c}^5 \mid q_{6c}^6 \mid q_{7c}^7 \right],$$
$$\dot{q}_C = \left[ \dot{q}_{1c}^1 \mid \dot{q}_{2c}^2 \mid \dot{q}_{3c}^3 \mid \dot{q}_{4c}^4 \mid \dot{q}_{5c}^5 \mid \dot{q}_{6c}^6 \mid \dot{q}_{7c}^7 \right]. \quad (5)$$

The kinematic equation of all system becomes:

$$V_C = M_C \dot{q}_C,$$
$$V_C = \left[ V_1^{1c,1} \mid V_2^{2c,2} \mid V_3^{3c,3} \mid V_4^{4c,4} \mid V_5^{5c,5} \mid V_6^{6c,6} \mid V_7^{7c,7} \right],$$
$$M_C = diag\left[ M_1^{1c}, M_2^{2c}, M_3^{3c}, M_4^{4c}, M_5^{5c}, M_6^{6c}, M_7^{7c} \right].$$

It is essential to transform $V_C$ to quasi-velocities relatively the base coordinate system $V_C^0$, the matrix defining this transformation is called a *configuration matrix of system* $L_C$ (6). The kinematic equation in $V_C^0$ has a form:

$$V_C^0 = L_C^{\mathrm{T}} V_C,\ V_C^0 = L_C^{\mathrm{T}} M_C \dot{q}_C,$$
$$V_C^0 = \left[ V_1^{0,1} \mid V_2^{0,2} \mid V_3^{0,3} \mid V_4^{0,4} \mid V_5^{0,5} \mid V_6^{0,6} \mid V_7^{0,7} \right], \quad (6)$$
$$L_C = \begin{bmatrix} E & L_2^1 & L_3^1 & L_4^1 & L_5^1 & L_6^1 & L_7^1 \\ 0 & E & L_3^2 & L_4^2 & L_5^2 & L_6^2 & L_7^2 \\ 0 & 0 & E & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E & L_5^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E & L_7^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & E \end{bmatrix},$$

$$L_2^1 = T_{2c}^1 L_2^{2c},\ L_3^2 = T_{3c}^2 L_3^{3c},\ L_4^2 = T_{4c}^2 L_4^{4c},\ L_5^4 = T_{5c}^4 L_5^{5c},$$
$$L_6^2 = T_{6c}^2 L_6^{6c},\ L_7^6 = T_{7c}^6 L_7^{7c},\ L_3^1 = L_2^1 L_3^2,\ L_4^1 = L_2^1 L_4^2,$$
$$L_5^2 = L_4^2 L_5^4,\ L_5^1 = L_2^1 L_5^2,\ L_6^1 = L_2^1 L_6^2\ L_7^2 = L_6^2 L_7^6,$$
$$L_7^1 = L_2^1 L_7^2.$$

For the further constructions we enter a new matrix:

$$S = M_C^T L,\ V_C = S^T \dot{q}_C. \quad (7)$$

The matrix $S$ is called a *system structural matrix* and contains the information about the system structure.

The MS dynamic equations has a form (8):

$$\Theta_k^k \dot{V}_k^{0,k} + \Phi_k^{0,k}\Theta_k^k V_k^{0,k} = R_k^k + F_k^k,\ k = 1,2,\ldots 7, \quad (8)$$

where $R_k^k = \left[ R_k^{k,x} \mid R_k^{k,y} \mid R_k^{k,z} \mid Mr_k^{k,x} \mid Mr_k^{k,y} \mid Mr_k^{k,z} \right]$ are internal forces ($R_k^{k,x}$) and torques ($Mr_k^{k,x}$) acting on the $k$ –body represented in the body axes; $F_k^k = \left[ F_k^{k,x} \mid F_k^{k,y} \mid F_k^{k,z} \mid M_k^{k,x} \mid M_k^{k,y} \mid M_k^{k,z} \right]$ are external forces ($F_k^{k,x}$) and torques ($M_k^{k,x}$) acting on the $k$ –body represented in the body axes; $\Theta_k^k$ is a matrix of inertia of the firm body, this matrix is defined in the body axes. In our case internal forces and the moments are forces of viscous and dry friction $N_k^k$, force of elastic interaction $El_k^k$, constructional damping $D_k^k$, external forces are control $U_k^k$, gravity $G_k^k$ and wind loading $W_k^k$.

The equations (8) becomes (9):

$$\Theta_C \dot{V}_C^0 + \Phi_C^0 \Theta_C V_C^0 =$$
$$= Rn_C + N_C + El_C + U_C + G_C + W_C, \quad (9)$$

where

$$\Phi_C^0 = diag\left[ \Phi_1^{0,1}, \Phi_2^{0,2}, \Phi_3^{0,3}, \Phi_4^{0,4}, \Phi_5^{0,5}, \Phi_6^{0,6}, \Phi_7^{0,7} \right],$$
$$\Theta_C = diag\left[ \Theta_1^1, \Theta_2^2, \Theta_3^3, \Theta_4^4, \Theta_5^5, \Theta_6^6, \Theta_7^7 \right],$$
$$N_C = \left[ N_1^1 \mid N_2^2 \mid 0 \mid 0 \mid 0 \mid 0 \mid 0 \right],$$
$$El_C = \left[ El_1^1 \mid El_2^2 \mid El_3^3 \mid El_4^4 \mid El_5^5 \mid El_6^6 \mid El_7^7 \right],$$
$$D_C = \left[ D_1^1 \mid D_2^2 \mid D_3^3 \mid D_4^4 \mid D_5^5 \mid D_6^6 \mid D_7^7 \right],$$
$$G_C = \left[ G_1^1 \mid G_2^2 \mid G_3^3 \mid G_4^4 \mid G_5^5 \mid G_6^6 \mid G_7^7 \right],$$
$$W_C = \left[ 0 \mid W_2^2 \mid W_3^3 \mid 0 W_5^5 \mid W_6^6 \mid W_7^7 \right].$$

Having multiplied the equation (9) at the left on the structural matrix of system $S$, we lead system to the generalized forces, having considered (7) we receive:

$$S\Theta_C S^{\mathrm{T}} \ddot{q}_C + (S\Theta_C \dot{S}^{\mathrm{T}} + S\Phi_C^0 \Theta_C S^{\mathrm{T}})\dot{q}_C =$$
$$= Q_n + Q_c + Q_u + Q_g + Q_w + Q_d,$$
$$A(q_C) = S\Theta_C S^{\mathrm{T}},\ B(q_C, \dot{q}_C) = S\Theta_C \dot{S}^{\mathrm{T}} + S\Phi_C^0 \Theta_C S^{\mathrm{T}},$$
$$A(q_C)\ddot{q}_C + B(q_C, \dot{q}_C)\dot{q}_C = Q_n + Q_c + Q_u + Q_g + Q_w,$$
$$\dot{S} = \dot{M}_C^{\mathrm{T}} L_C + M_C^{\mathrm{T}} \dot{L}_C,\ \dot{L}_j^i = L_j^i \Phi_j^{i,j},$$
$$\dot{M}_j^i = \begin{bmatrix} \dot{C}_j^{i,\mathrm{T}} & 0 \\ 0 & \dot{\varepsilon}_j^i \end{bmatrix},\ \dot{C}_j^{i,T} = \left\langle \omega_j^{i,j} \right\rangle^{\mathrm{T}} C_j^{i,\mathrm{T}},$$
$$\dot{\varepsilon}_j^i = [\left\langle e_{13} \right\rangle^{\mathrm{T}} C_j^{i,3}(\alpha_j^i)^{\mathrm{T}} C_j^{i,2}(\theta_j^i)^{\mathrm{T}} \dot{\alpha}_j^i +$$
$$+ C_j^{i,3}(\alpha_j^i)^{\mathrm{T}} C_j^{i,2}(\theta_j^i)^{\mathrm{T}} \left\langle e_{12} \right\rangle^{\mathrm{T}} \dot{\theta}_j^i \mid \left\langle e_{13} \right\rangle^{\mathrm{T}} C_j^{i,3}(\alpha_j^i)^{\mathrm{T}} \dot{\alpha}_j^i e_{12} \mid 0 ].$$

It is essential to define the generalized forces of elastic interaction $Q_C$ and constructional damping $Q_d$. For each elastic link we enter a matrix of elasticity $C_i$ and a matrix of damping $D_i$:

$$Q_c = -Cq_c,\ \ Q_d = -D\dot{q}_C,$$

$$C = diag\left[C_1, C_2, C_3, C_4, C_5, C_6, C_7\right],$$

$$D = diag\left[D_1, D_2, D_3, D_4, D_5, D_6, D_7\right].$$

A vector of acceleration of free falling is set in the base coordinate system $g_0$, hence, it is necessary to pass from $E_0$ in the body axes $E_i$. Gravity forces of the system $Q_g$ are defined by the equation:

$$Q_g = SG_C.$$

The control $U_C$ acting on system has a form:

$$U_1^1 = \begin{bmatrix}0 & 0 & 0 & M_{Dv1} & 0 & 0\end{bmatrix}^{\mathrm{T}},$$

$$U_3^3 = \begin{bmatrix}0 & F_{Dv2} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}},$$

$$U_C = \left[U_1^1 \,|\, 0 \,|\, U_3^3 \,|\, 0 \,|\, 0 \,|\, 0\right],$$

where $U_1^1$ is the control on an azimuthal platform; $U_3^3$ is the control on swinging part; $M_{Dv1}$ is the moment created by a drive of an azimuthal platform; $F_{Dv2}$ is the force created by a drive of the swinging part.

Expression for the generalized control will be:

$$Q_u = SU_C\,.$$

The numerical example shown bellow is a result of calculations of RT-70 MS finite-element model. These data were used in described above RT MS symbol model.

As a vector of physical coordinates are chosen the angles of rotations relatively of an azimuthal axis of platform, base, base of main dish, main dish, girder, counter-reflector and counterbalance:

$$\alpha = [\alpha_{pl}, \alpha_{bas}, \alpha_{bmd}, \alpha_{md}, \alpha_{gir}, \alpha_{cr}, \alpha_{cb}]^{\mathrm{T}} \text{ (rad)}.$$

Following a principle of superposition, we consider, that each chosen physical coordinate corresponds the line of a matrix $H$. What spectrum of eigen frequencies from a matrix $\Omega$ corresponds to chosen physical vector $\alpha$ we shall define by comparison among themselves numerical values of elements a vector-line of a matrix $H$.

As a result we have reduced matrixes $\Omega_r$, $H_r$:

$\Omega_r$ = -1000diag(0, 0.062, 0.362, 0.511, 0.562, 0.695, 4.147] (1/c$^2$),

$$H_r = \begin{bmatrix}
-0.378 & -0.3394 & 0.2688 & -0.0243 & 0.0009 & 0.0049 & 0.0628\\
-0.378 & -0.2571 & -0.1131 & 0.0244 & -0.0011 & 0.0085 & -0.9598\\
-0.378 & -0.1291 & -0.2194 & 0.1078 & -0.0086 & -0.0687 & 0.1452\\
-0.378 & 0.7304 & 0.0372 & -0.0123 & 0.0009 & 0.0056 & -0.0019\\
-0.378 & -0.2816 & -0.2322 & 0.0947 & 0.0071 & 0.222 & 0.1992\\
-0.378 & -0.3161 & -0.6441 & 0.9769 & 0.9999 & -0.9725 & -0.0315\\
-0.378 & -0.2989 & -0.6297 & -0.1543 & 0.0041 & -0.0148 & 0.1143
\end{bmatrix}$$

Ansys easily allows to calculate the moments of inertia of the chosen firm bodies:

$\mathrm{P}_r$ = 10$^7$ [2.1 0.4 0.6472 1.6 0.32 0.0011 0.4] (kg*s$^2$)/rad).

The stiffness matrix calculated from the equation

$$\Xi_r = \mathrm{P}_r H_r \Omega_r H_r^{-1}.$$

$\Xi_r$ = 1.0e+010

$$\begin{bmatrix}
-0.5350 & 0.5350 & 0 & 0 & 0 & 0 & 0\\
0.5350 & -1.4754 & 0.5050 & 0 & 0.2280 & 0 & 0.2074\\
0 & 0.5050 & -0.5890 & 0.0840 & 0 & 0 & 0\\
0 & 0 & 0.0840 & -0.0840 & 0 & 0 & 0\\
0 & 0.2280 & 0 & 0 & -0.2286 & 0.0006 & 0\\
0 & 0 & 0 & 0 & 0.0006 & -0.0006 & 0\\
0 & 0.2074 & 0 & 0 & 0 & 0 & -0.2074
\end{bmatrix}$$

(kgм/rad)

The natural spectrum, which components are calculated under the formula

$$f = sort(sqrt(diag(-\Omega_r))/(2*pi)).$$

$f$=[0 1.2512 3.0281 3.5985 3.7730 4.1964 10.2493]. (Hz)

## 4 Conclusion

The definition of parameters of mathematical models of the radio-telescope metallic structure, which are required for estimation of inaccessible to direct measurement components of state vector, is connected with a minimization problem of dimension of state vector.

In this article the solving of this problem, when requirements to accuracy of the estimations increase, is shown to be possible only with using methods of mathematical programming and finite-element analysis.

The obtained models are quasistationary, and can be used as estimators of state vectors. Because of it their parameters are "freezing" on specified time interval and provides recalculation when one stationary point change to another.

The given approach to modeling of RT MS has allowed to prove an opportunity to create the precision control system for millimetre-wave RT which main dish diameter is 70 meters. The modeling accuracy of pointing of this system is 4 seconds of angle.

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