

LONG-RANGE SYNCHRONIZATION AND RHYTHMIC PATTERNS CONTROL VIA COUPLING DELAY

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Abstract

The paper is devoted to study of synchronization in small neural networks with coupling delays. We show that long-range zero lag synchronization is possible even for distant neural ensembles. Weak varying of delays may provide a mechanism for control of rhythmic patterns of central pattern generators.

Key words

synchronization; phase locking; coupled oscillators; time delay

1 Introduction

Synchronization is a fundamental effect observed in all fields of physics. Particularly, there is lot of evidence that it plays crucial role in brain functioning. Neuronal synchronization is hypothesized to provide an important mechanism for the large-scale integration of distributed brain activity. Experiments show that firing of different areas of visual cortex synchronize under visual stimulation [Engel *et al.*, 1991; Courtemanche and Lamarre, 2005]. It is supposed that temporal synchronization is concerned with binding of different properties of visual object. These properties are processed in different brain areas and then are collected together to recognize the object.

Besides cognitive tasks, synchronization is important for motion control. Rhythmic motions of animals, such as walking, scratching or breathing, are controlled by rhythmic activity of the so-called central pattern generators. CPGs are neural networks that produce rhythmic outputs autonomously, i.e. without sensory or central input. From dynamical point of view CPGs are systems of coupled oscillators which undergo mutual synchronization. Phase relations between the oscillators determine output rhythmic patterns of neural activity which underlie motor patterns. Different kinds of rhythmic motions correspond to different phase relations [Yuasa and Ito, 1990; Abarbanel *et al.*, 1996; Duysens and Van der Crommert, 1998].

Dynamics of neural networks is often concerned with temporal delays in communication between the neurons. These delays come from two things – i) chemical synapses inertness and ii) finite velocity of spikes propagation along axons. Synaptic delays are usually about 1 ms, but axonal delays depend on distance between communicating neurons and may reach values of several tens on milliseconds. Large coupling delays may strongly change synchronization, that is why it is important to study how they influence dynamics of neural networks [Ermentrout and Kopell, 1998; Earl and Strogatz, 2003].

In this paper we consider small neural networks with time-delayed coupling and study their dynamics. We concentrate on features of delay-induced synchronization. Our motivation is concerned with two things. Firstly, we are interested how distant brain areas may synchronize with zero lag in spite of the fact that signal propagation between them takes large time. This problem is important for time binding and image recognition. Secondly, we examine how introduction of delays changes phase relationships between coupled oscillators. We show that delay-induced synchronization may provide a mechanism of CPG rhythmic patterns control.

2 Model

We model individual neuron as an oscillator with phase $\varphi \in [0; 1]$ growing uniformly with the velocity $d\varphi/dt = \omega$. For $\varphi = 1$, the oscillator reaches threshold, emits a pulse, and resets its phase to zero. We consider an ensemble of N non-identical neurons with frequencies ω_j interacting with a time lags. This ensemble can be described by the system

$$\frac{d\varphi_j(t)}{dt} = \omega_j + \sum_{k=1}^N G_{jk}(\varphi_j(t), \varphi_k(t - \tau_{jk})). \quad (1)$$

We use technique of phase response curves (PRCs) to describe interaction between neurons. In this

technique, coupling function is chosen in the form $G_{jk}(\varphi_j(t), \varphi_k(t-\tau_{jk})) = \mu_{jk}F(\varphi_j(t))\delta(\varphi_k(t-\tau_{jk}))$. This means that when k -th neuron emits a pulse, it is received by j -th neuron with delay τ_{jk} . When j -th neuron receives a pulse its phase instantly changes on value $\Delta\varphi = \mu_{jk}F(\varphi_j)$. This value is the so-called phase response curve. Further we use function $F(\varphi) = -\sin 2\pi\varphi$. Such form of PRC belongs to the so-called second class, which means that incoming pulses may either delay or advance neuron excitation.

Strictly speaking, (1) is the system with time lagging, so it has infinite dimension and is very difficult for studying. But because of using of PRC approach this system can be reduced to the map of finite dimension, which simplifies its investigation. The main idea of this reduction is that system dynamics consists of discrete events which occur when the neurons emit or receive pulses. During these events (the so-called H -events) the neurons phases are perturbed, and between these events they grow uniformly. We construct the map that describes how the system state changes between sequential H -events (the so-called H -map). The further study of the system dynamics is based on this map.

3 Long-range synchronization

We start from studying of mutual synchronization of two neurons with symmetric delayed coupling ($N = 2$, $\mu_{12} = \mu_{21} = \mu$, $\tau_{12} = \tau_{21} = \tau$). We say that a pair of neuron is synchronized or phase locked if they fire periodically with the same period T and constant phase shift. This dynamical mode corresponds to a stable fixed point of the H -map. We show that phase locking is possible when units frequency mismatch is limited by

$$\omega_2 - \omega_1 \leq \zeta_0 = \frac{2\mu\omega_1}{1-\mu}. \quad (2)$$

In this case synchronous modes are observed in the so-called “synchronization intervals” of coupling delay τ . These intervals are depicted in Fig. 1a for $\omega_1 = 1$, $\omega_2 = 1.1$, $\mu = 0.1$. Inside each of these intervals a pair of a stable and unstable periodic solutions of system (1) exists. Stable periodic solution corresponds to synchronization. For synchronous regimes time lag Δ between instants of neurons firing depends on τ . It weakly changes inside one synchronization interval and strongly changes from one interval to another. Notice that intervals with in-phase and antiphase synchronization alternate while τ grows.

Synchronization intervals exist for arbitrary large values of τ . Their width grows when frequency mismatch between the neurons decreases. When neurons are identical synchronization intervals cover all values of coupling delays (Fig. 1b). This means that identical neurons synchronize their activity for arbitrary value of τ . This phenomenon explains long-range synchroniza-

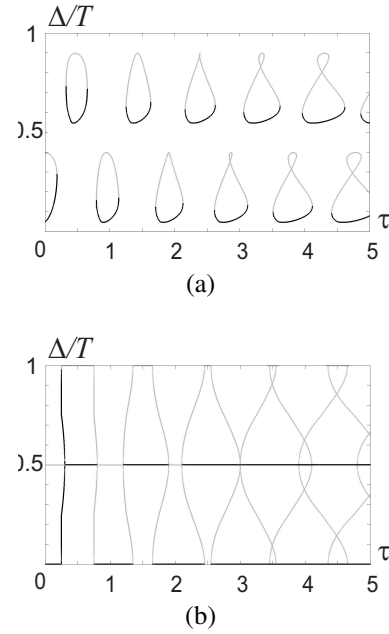


Figure 1. Periodic solutions of system (1) – phase lag Δ/T versus coupling delay τ . Black lines correspond to stable solutions, gray lines correspond to unstable ones. A pair of stable and unstable solution exists in each synchronization interval. Parameters: (a) $\omega_1 = 1, \omega_2 = 1.1, \mu = 0.1$; (b) $\omega_1 = \omega_2 = 1, \mu = 0.1$.

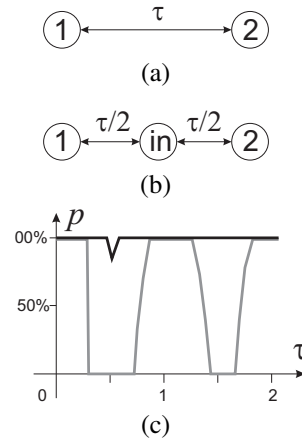


Figure 2. (a) Two directly communicating neurons with coupling delay τ . (b) Two neurons communicating via “dynamic relaying”. (c) Probabilities of zero lag synchronization versus coupling delay τ : gray line – direct coupling, black line – dynamic relaying.

tion, because synchronization occurs for all values of τ .

In a model of two identical coupled neurons synchronization may be either in-phase ($\Delta = 0$) or antiphase ($\Delta = T/2$) for different τ . But experiments provide evidence of only zero lag distant synchronization. To explain this feature we introduce the so-called “dynamical relaying” model [Vicentea *et al*, 2008]. This model assumes that two neurons do not communicate directly with each other, but both of them exchange pulses with an interneuron between them. Pulse prop-

agation between each of neurons and interneuron takes time $\tau/2$. In this case zero lag synchronization is observed for almost always for all values of τ (Fig. 2).

4 Rhythmic patterns control via delay

Let us return to the pair of two directly coupled neurons. Phase lag between the moments of their firing depend on coupling delay. This fact may be used for control of phase relationships in a small network of spiking neurons. Figure 3 illustrates this idea. In this figure phase lag Δ/T for synchronous regime is plotted for $\omega_1 = 1, \omega_2 = 1.01, \mu = 0.1$. For $\tau \in [0; 0.5]$ two overlapping synchronization intervals exist. The relative interspike lag Δ/T varies from 0 to almost 0.5 for the first interval and from 0.5 to almost 1 for the second one. Thus, varying τ one can obtain regimes with arbitrary value of interspike lag. When τ increases from 0 to 0.5, the phase shift between oscillators smoothly grows from 0 to 0.5. When τ decreases from 0.5 back to 0, the phase shift smoothly grows from 0.5 almost to 1 and then jumps to zero. Because of equality of zero and one this leap is inessential. Thus one can smoothly switch system mode between in-phase and anti-phase.

Dynamics of larger neural networks may be controlled by coupling delay as well. To illustrate this we considered a network of four globally coupled neurons depicted in Fig. 4a. The neurons are located in corners of a square, delays between neighboring neurons equal $\tau/2$, delays between diagonal neurons equal τ . Coupling strength between each pair of neurons equals $\mu = 0.1$. Frequencies ω_j have Gaussian distribution with median value $\omega = 1$ and dispersion $\sigma = 0.01$.

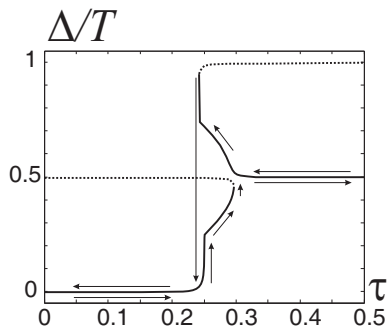


Figure 3. Phase lag control via coupling delay. When τ increases from 0 to 0.5, the relative interspike interval Δ/T almost smoothly grows from 0 to 0.5. When τ decreases, the value Δ/T smoothly grows almost to 1 and then jumps to zero.

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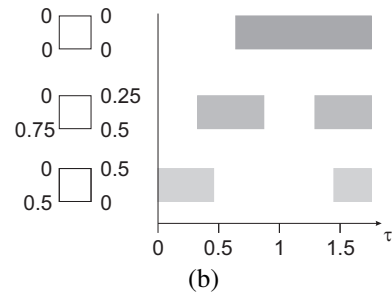
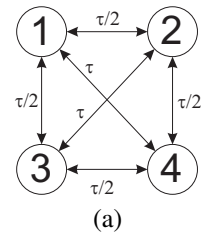


Figure 4. (a) CPG consisting of four neurons with delayed coupling. (b) Possible output rhythmic patterns and τ intervals inside which they exist.

$\mu = 0.1$. Frequencies ω_j have Gaussian distribution with median value $\omega = 1$ and dispersion $\sigma = 0.01$. This small network is an example of central pattern generator capable to demonstrate a number of rhythmic activity patterns. These patterns differ with phase relationships between neurons. Relations corresponding to three such patterns are depicted in Fig. 4b. The first pattern corresponds to global synchronization of the system, when all the phases are equal. In the second pattern all neurons fire sequentially with phase shift $\Delta\varphi = 0.25$. In the third pattern neurons divide into two pairs which fire in antiphase. These patterns exist in different intervals of τ (which may intersect) plotted in Fig. 4b. Though, one can control output rhythmic pattern of the CPG via coupling delay changing.

5 Conclusion

We have studied influence of coupling delay on dynamics of small neural networks. An interesting result is that mutual synchronization of coupled neurons is possible even for large coupling delays. This explains long-range synchronization observed in numerous experiments. Zero-lag long-range synchronization can be explained with dynamical relaying.

Another result is that coupling delay may be an instrument for output rhythmic patterns control of central pattern generators. We show that varying delays values in small network of coupled oscillatory neurons allows to change phase relationships between the units. This may be interesting for locomotion of animals or robots [Shinkichi *et al*, 2006]. Different outputs correspond to various motor patterns and various motions.

For example, when oscillators control motions of limbs, different patterns correspond to different gait types – walk, trot, gallop. One can object that it is dif-

difficult to vary all coupling delays simultaneously. But absolute values of delays do not play any crucial role, because the most important are relations between them and oscillations frequency. So varying of delays is equal to changing of network oscillation rate. Switching between patterns may occur when this frequency grows. This agrees with experimental data providing evidence that different velocities of animal motion correspond to different gaits.

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