Nonlinear Gyromoment Spacecraft Attitude Control with Precise Pointing the Flexible Antennas

Ye.I. Somov, S.A. Butyrin, S.Ye. Somov

***Abstract*— New approach for modelling a physical hysteresis damping the flexible spacecraft structure oscillations, is developed. New results on spacecraft attitude guidance and digital gyromoment nonlinear control with precise pointing the large-scale flexible antennas, are presented.**

I. Introduction

A correct mathematical description of physical hysteresis is a basic problem for an internal friction theory (N.N. Davidenkov, 1938; A.Yu. Ishlinskii, 1944; W. Prager, 1956; J.F. Besseling, 1958; Ye.S. Sorokin, 1960; Ya.G. Panovko [1]; G.S. Pisarenko [2]; V.A. Palmov [3]; L.F. Kochneva [4] et al.) with regard to the well-known flexible-plastic micro-deformations of materials. The rigorous mathematical aspects for qualitative analysis of general hysteresis models are represented in a number of research works [5]. Recently, new approach was developed for description of physical hysteresis [6], [7], which is based on set-valued differential equation with discontinuous right-side. The paper briefly presents new results on modelling a hysteresis damping and describes in detail their application to a communication spacecraft (SC) attitude guidance and robust control with precise pointing the large-scale flexible weak-damping antennas.

II. Model of Physical Hysteresis

Let $x(t)$ is a real piecewise-differetiated function for $t \in \mathrm{T}_{t_0} \equiv [t_0, +\infty)$. Let there be the values $\check{x}_\nu = x(t_\nu)$ of the function in the time moments $t_\nu,\ \nu \in \mathbb{N}_0 \equiv [0, 1, 2, \cdots)$, when the *last* changing a *sign* of a speed $\dot{x}(t)$ was happened, e.g.

$$\check{x}_\nu \equiv x(t_\nu)|t_\nu : \mathrm{Sign}\dot{x}(t_\nu + 0) \neq \mathrm{Sign}\dot{x}(t_\nu - 0). \quad (1)$$

A *local* function $\tilde{x}_\nu(t)$ on each a local time semi-interval $\mathrm{T}_\nu \equiv [t_\nu, t_{\nu+1})$ is introduced as

$$\tilde{x}_\nu(t) = x(t) - \check{x}_\nu \quad \forall t \in \mathrm{T}_\nu, \quad (2)$$

and the functional $k_\nu(x(t)) \equiv k_\nu(k, p, \tilde{p}, \tilde{x}_\nu)$ of the hysteresis function *shape* is defined as

$$k_\nu(x(t)) = k(1 - (1-p)\ \exp(-\tilde{p}|\tilde{x}_\nu|)),\ t \in \mathrm{T}_\nu, \quad (3)$$

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where $k, p, \tilde{p}$ are constant positive parameters. For a constant parameter $\alpha_h > 0$ and $x_0 \equiv x(t_0)$ a *normed* hysteresis function $r(t) = \mathrm{Hst}(\cdot, x(t))$ with memory

$$\begin{aligned} r(t) &= \mathrm{Hst}(a_h, \alpha_h, k_\nu(x(t)), r_o, x(t)); \\ r(t_0) &\equiv r_o = \mathrm{Hst}(a_h, \alpha_h, k_\nu(x_0), r_o, x_0) \end{aligned} \quad (4)$$

and restriction on its module by parameter $a_h > 0$, is defined as a right-sided solution of the equations

$$\mathrm{D}^+ r = \begin{cases} k_\nu\ |r - a_h \mathrm{Sign}\,\dot{x}(t)|^{\alpha_h}\ \dot{x}(t) & |r| < a_h \\ 0 & |r| \geq a_h \end{cases}; \qquad r(t_0 + 0) = r_o. \quad (5)$$

Differential equation in (5) has a discontinuous right side and ambiquitely depends on forcing function $x(t)$ and its speed $\dot{x}(t)$, e.g. it depends on all own pre-history which is expressed by the functional $k_\nu(x)$ (3). At initial condition $y_o \equiv y_0 = y(t_0)$ for $x = x_0$ the hysteresis function $y(t)$ is defined as follows

$$y(t) \equiv m\ \mathrm{Hst}(a_h, \alpha_h, k_\nu, r_o, x(t)); r_o \equiv y_o/m, \quad (6)$$

with the constant positive scale coefficient $m > 0$. In developed model (1) – (6) a parameter $\tilde{p}$ determines on the whole a degree of convergence for a trajectory $y(t) = F_h(\cdot, x(t))$ in the plane $x\mathrm{O}y$ on symmetric limiting static loop under a harmonic forcing function $x(t) = A \sin \omega t$ with fixed values A, ω and any initial condition $y_o = y_0$ with $|y_o|/m < a_h$. For this model all requirements are realized, including the famous requirements on a model vibro-correctness by [5],

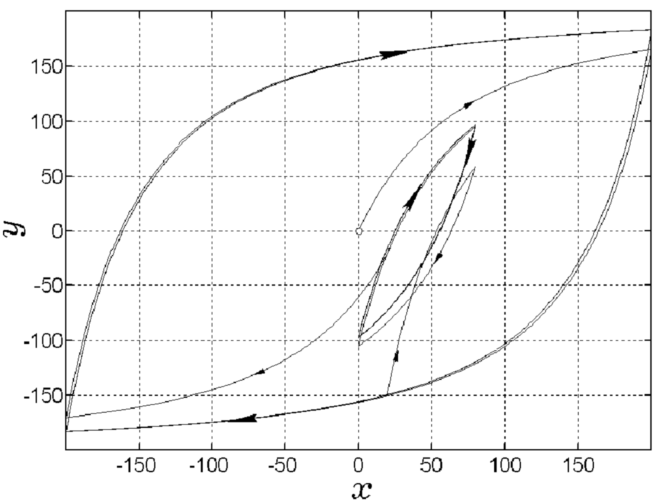

Fig. 1. Results of a hysteresis model testing

and also on a frequency independence and a fine return on a main symmetric limiting hysteresis loop after a short-term passage on a displaced local hysteresis loop [3], [4]. Last properties are verified in prearranged scale by Fig. 1 for the hysteresis model with parameters: $m = 1,\ \alpha_h = 1.5, a_h = 200, k = 5.125\,10^{-4}, p = 2,\ \tilde{p} = 0.75\,10^{-3}$. Moreover the forcing function have the form:

$$x(t) = \begin{cases} A\sin\omega_1 t & (0 \le t < \tau_1)\ \&\ (\tau_2 \le t \le \tau_3); \\ B(1+\sin\omega_2 t) & \tau_1 \le t < \tau_2, \end{cases}$$

$$A = 200;\ B = 40;\ \omega_1 = 1;\ \omega_2 = 5;\ \tau_3 = 40;$$
$$\tau_1 \equiv 5\pi - \tau^*;\ \tau_2 = 7\pi - \tau^*;\ \tau^* \approx 0.03415\pi.$$

III. Mathematical models

Let us introduce the inertial reference frame (IRF) $\mathbf{I}_\oplus$, the geodesic Greenwich reference frame (GRF) $\mathbf{E}_\mathrm{e}$ and the geodesic horizon reference frame (HRF) $\mathbf{E}^\mathrm{h}_\mathrm{e}$. There are also standard defined the SC body reference frame (BRF) $\mathbf{B}$ ($\mathrm{O}xyz$), the orbit reference frame (ORF) $\mathbf{O}$ ($\mathrm{O}x^\mathrm{o}y^\mathrm{o}z^\mathrm{o}$) and the antenna (sensor) reference frame (SRF) $\boldsymbol{\mathcal{S}}$ ($\mathrm{S}x^\mathrm{s}y^\mathrm{s}z^\mathrm{s}$) with an origin S. The BRF attitude with respect to the IRF $\mathbf{I}_\oplus$ is defined by quaternion $\boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda})$, $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$, and with respect to the ORF — by the column $\boldsymbol{\phi} = \{\phi_i, i = 1,2,3 \equiv 1:3\}$ of *Euler-Krylov* elementary angles ϕ_i in the sequence $13'2''$. Let us vectors $\boldsymbol{\omega}(t)$ and $\mathbf{v}(t)$ are standard denotations of the SC body vector angular rate and its mass center velocity with respect to the IRF, respectively. Applied further symbols $\langle\cdot,\cdot\rangle, \times, \{\,\cdot\,\}, [\,\cdot\,]$ for vectors and $[\mathbf{a}\times], (\cdot)^\mathrm{t}$ for matrixes are conventional denotations. For a fixed position of flexible structures on the SC body with some simplifying assumptions and $t \in \mathrm{T}_{t_0} = [t_0, +\infty)$ a SC spatial motion model is appeared as follows:

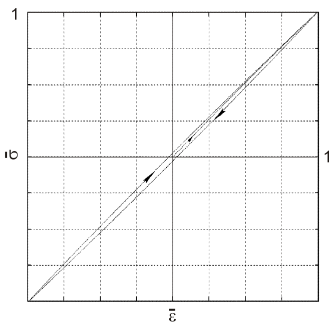

Fig. 2. The normed hysteresis

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda}\circ\boldsymbol{\omega}/2;\ \ \mathbf{A}^\mathrm{o}\{\overset{*}{\mathbf{v}}_\delta, \dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}\} = \{\mathbf{F}^\mathrm{v}_\delta, \mathbf{F}^\omega, \mathbf{F}^q\};\ \ \dot{\boldsymbol{\beta}} = \mathbf{u}^\mathrm{g}; \quad (7)$$

$$\mathbf{F}^\mathrm{v}_\delta = -\mathrm{m}(\boldsymbol{\omega}\times\mathbf{v}_\delta) + \boldsymbol{\omega}\times(\mathbf{L}\times\boldsymbol{\omega} - 2\dot{\mathbf{L}}) + \mathbf{R}^\mathrm{c};\ \mathbf{L} = \mathbf{M}_q\,\mathbf{q};$$

$$\mathbf{F}^\omega = -\mathbf{L}\times(\boldsymbol{\omega}\times\mathbf{v}_\delta) + \mathbf{M}^\mathbf{g} - \boldsymbol{\omega}\times\mathbf{G} + \mathbf{M}^\mathrm{o};\mathbf{M}^\mathbf{g} = -\mathbf{A}_\mathrm{h}\dot{\boldsymbol{\beta}};$$

$$\mathbf{F}^q = \{-(\Omega^q_j)^2 m_j r_j(t)\};\quad \mathbf{G} = \mathbf{G}^o + \mathbf{D}_q\dot{\mathbf{q}};\quad \mathbf{q} = \{q_j\};$$

$$r_j(t) = \mathrm{Hst}(a^j_h, \alpha^j_h, k^j_\nu, x_{oj}, x_j(t));\quad x_j(t) = q_j(t)/m_j;$$

$$\mathbf{A}^\mathrm{o} = \begin{bmatrix} \mathrm{m}\mathbf{I}_3 & [-\mathbf{L}\times] & \mathbf{M}_q \\ [\mathbf{L}\times] & \mathbf{J} & \mathbf{D}_q \\ \mathbf{M}^\mathrm{t}_q & \mathbf{D}^\mathrm{t}_q & \mathbf{I} \end{bmatrix};\quad \begin{array}{l} \mathbf{A}_\mathrm{h} = [\,\partial\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})\,/\,\partial\boldsymbol{\beta}\,]; \\ \mathbf{G}^o = \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}); \\ \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = \mathrm{h}_g\ \Sigma\ \mathbf{h}_p(\beta_p), \end{array}$$

where $\mathbf{v}_\delta$ is a velocity deflection with respect to its nominal value by the gravity forces, h_g is a constant own angular momentum (AM) of each gyrodine (GD), vectors $\boldsymbol{\beta} = \{\beta_p\}$; $\boldsymbol{\omega} = \{\omega_i\}$; $\mathbf{M}^\mathbf{g} = \{\mathrm{M}^\mathrm{g}_i\}$ and $x_{oj} = q_j(t_0)/m_j$ with a concordance of initial conditions (6). Parameters m_j, a^j_h, α^j_h and $k_j, p_j, \tilde{p}_j$ by functionals $\mathrm{Hst}(\cdot)$ and $k^j_\nu(x_j)$ for tones of the SC structure oscillations are defined by an identification procedure starting from analysis of experimental hysteresis loop for normed deformation $\bar{\varepsilon}$ and strength $\bar{\sigma}$ of the structure material, see Fig. 2. At standard linear modelling one can have

$$\mathbf{F}^q = \{-((\delta^q_j/\pi)\,\Omega^q_j\,\dot{q}_j + (\Omega^q_j)^2\ q_j)\}, \quad (8)$$

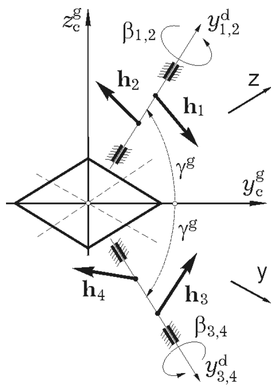

Fig. 3. The *2-SPE* scheme

where $\delta^q_j \in [10^{-2}, 2\,10^{-4}]$ is decrement by j-tone of the SC structure flexible oscillations. The antenna's flexibility results in additional angular deflection column $\delta\boldsymbol{\phi} \equiv \{\delta\phi_i, i = 1:3\}$ of the SRF $\boldsymbol{\mathcal{S}}$ with respect its nominal position in the BRF, including the antenna's line-of-sight $\mathrm{S}x^\mathrm{s}$:

$$\delta\boldsymbol{\phi} = \mathbf{Q}_q\,\mathbf{q}, \quad (9)$$

where matrix $\mathbf{Q}_q$ is calculated by its shape modes. The gyro moment cluster (GMC) is applied at scheme *2-SPE*, based on four GDs [8], fig. 3.

IV. The problem statement

In precession theory of the control moment gyros the GMC control torque vector $\mathbf{M}^\mathbf{g}$ is presented as follows:

$$\mathbf{M}^\mathbf{g} = -\overset{*}{\boldsymbol{\mathcal{H}}} = -\mathbf{A}_\mathrm{h}(\boldsymbol{\beta})\mathbf{u}^\mathrm{g}(t);\ \dot{\boldsymbol{\beta}} = \mathbf{u}^\mathrm{g}(t) \equiv \{\mathrm{u}^\mathrm{g}_p(t)\}. \quad (10)$$

Here $\mathrm{u}^\mathrm{g}_p(t) = a^\mathrm{g}\,\mathrm{Zh}[\mathrm{Sat}(\mathrm{Qntr}(u^g_{pk}, b_u), B_u), T_u]$ with a constant a^g and a control period $T_u = t_{k+1} - t_k$, $k \in \mathbb{N}_0$; discrete functions $u^g_{pk} \equiv u^g_p(t_k)$ are outputs of nonlinear control law (NCL), and functions $\mathrm{Sat}(x, a)$ and $\mathrm{Qntr}(x, a)$ are general-usage ones, while the holder model with the period T_u is such: $y(t) = \mathrm{Zh}[x_k, T_u] = x_k\ \forall t \in [t_k, t_{k+1})$.

Applied onboard measuring subsystem is based on initial gyro unit corrected by the fine fixed-head star trackers. This subsystem is intended for precise determination of the SC BRF $\mathbf{B}$ angular position with respect to the IRF $\mathbf{I}_\oplus$. Applied contemporary filtering & alignment calibration algorithms and a discrete astatic observer give finally a fine discrete estimating the SC angular motion coordinates by the quaternion $\boldsymbol{\Lambda}^\mathrm{m}_s = \boldsymbol{\Lambda}_s\circ\boldsymbol{\Lambda}^\mathrm{n}_s$, $s \in \mathbb{N}_0$, where $\boldsymbol{\Lambda}_s \equiv \boldsymbol{\Lambda}(t_s)$, $\boldsymbol{\Lambda}^\mathrm{n}_s$ is a "noise-drift" digital quaternion and a measurement period $T_q = t_{s+1} - t_s \le T_u$ is multiply with respect to a control period T_u. When the SC is moving at a distant part of the high-elliptical orbit (HEO) by *Molniya* type (with apogee 46370 km and perigee 7370 km, fig. 4) there are fulfilled sequence of its angular modes:

1° the SC antenna pointing to a given point at the Earth surface and then the target tracking during given time interval $\mathrm{T}_n \equiv [t^n_0, t^n_f]$.
2° the SC antenna guidance from any Earth point to next the same point during time $t \in \mathrm{T}_p \equiv [t^p_0, t^p_f]$, $t^p_f \equiv t^p_0 + T_p$ where T_p is given, see fig. 4.

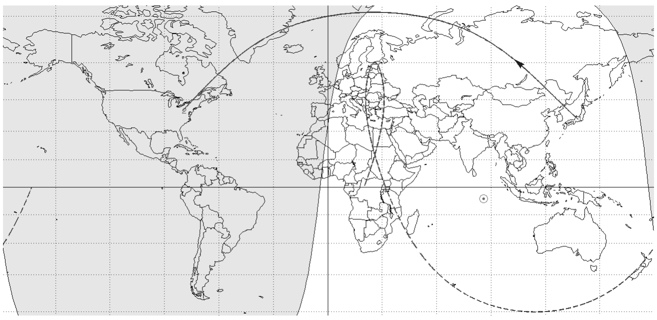

Fig. 4. Antenna pointing by the HEO spacecraft guidance

At the SC lifetime up to 15 years its structure inertial and flexible characteristics are slowly changed in wide boundaries, and the solar array panels (SAPs) are slowly rotated on the angle $\gamma(t) \in [0,\, 2\pi]$ with respect to the SC body for their tracking the Sun direction. Therefore at inertial matrix $\mathbf{A}^{\mathrm{o}}$ and partial frequencies Ω_j^q of the SC structure are not complete certain. Let us $\mathbf{\Lambda}^p(t)$ is a quaternion, $\boldsymbol{\omega}^p(t)$ and $\dot{\boldsymbol{\omega}}^p(t) = \boldsymbol{\varepsilon}^p(t)$ are angular rate and acceleration vectors of the SC body programmed motion in the IRF $\mathbf{I}_{\oplus}$. Problems consist in synthesis of the SC antenna guidance laws for calculating $\mathbf{\Lambda}^p(t)$, $\boldsymbol{\omega}^p(t)$, $\boldsymbol{\varepsilon}^p(t)$ by tasks 1°, 2° and the GMC control $\mathbf{u}_k^g = \{u_{pk}^g\}$ on the quaternion values $\mathbf{\Lambda}_s^{\mathrm{m}}$ when the SC structure characteristics are uncertain and its damping is very weak, $\delta_j^q \approx 10^{-3}$ in (8).

V. Synthesis of Feedback Control

Applied general approach to synthesis of *nonlinear* control system (NCS) with a partial measurement of its state is presented, moreover the method of *vector Lyapunov functions* (VLF), which has a strong mathematical basis for analysis of stability and other dynamical properties of various nonlinear interconnected systems with the *discontinuous right-hand side*, is used in cooperation with the *exact feedback linearization* (EFL) technique. Let there be given a nonlinear controlled object

$$D^{+}\mathrm{x}(t) = \mathcal{F}(\mathrm{x}(t), \mathrm{u}); \quad \mathrm{x}(t_0) = \mathrm{x}_0; \; t \in \mathrm{T}_{t_0},$$

where $\mathrm{x}(t) \in \mathcal{H} \subset \mathbb{R}^n$ is a state vector with an initial condition $\mathrm{x}_0 \in \mathcal{H}_0 \subseteq \mathcal{H}$; $\mathrm{u} = \{u_j\} \in \mathrm{U} \subset \mathbb{R}^r$ is a control vector. Let some *vector norms* $\rho(\mathrm{x}) \in \overline{\mathbb{R}}_+^l$ and $\rho^0(\mathrm{x}_0) \in \overline{\mathbb{R}}_+^{l_0}$ also be *given*. For any control law (CL) $\mathrm{u} = \mathcal{U}(\mathrm{x})$ the closed-loop system has the form

$$D^{+}\mathrm{x}(t) = \mathcal{X}(t, \mathrm{x}); \quad \mathrm{x}(t_0) = \mathrm{x}_0, \tag{11}$$

where $\mathcal{X}(t,\mathrm{x}) = \mathcal{F}(\mathrm{x}, \mathcal{U}(\mathrm{x})), \mathcal{X} : \mathrm{T}_{t_0} \times \mathcal{H} \to \mathcal{H}$ is a *discontinuous* operator. Assuming the existence and the non-local continuability of the *right-sided* solution $x(t) \equiv \mathrm{x}(t_0, \mathrm{x}_0; t)$ of the system (11) for its *extended definition* in the aspect of physics, the most important dynamic property is obtained, that is $\rho\rho^0$*-exponential invariance* of the solution $x(t) = 0$ under the *desired* $\gamma \in \overline{\mathbb{R}}_+^l$:

$$(\exists \alpha \in \mathbb{R}_+)\, (\exists \mathcal{B} \in \overline{\mathrm{B}}_+^{l \times l_0})\, (\exists \delta \in \mathbb{R}_+^{l_0})\, (\forall \rho^0(\mathrm{x}_0) < \delta)$$
$$\rho(x(t)) \le \gamma + \mathcal{B}\; \rho^0(\mathrm{x}_0)\; \exp(-\alpha(t - t_0)) \quad \forall t \in \mathrm{T}_{t_0}.$$

For the VLF $\upsilon : \mathcal{H} \to \overline{\mathbb{R}}_+^k$ with components $\upsilon^s(\mathrm{x}) \ge 0$, $\upsilon^s(0) = 0, s = 1 : k$ and the norm $\|\upsilon(\mathrm{x})\| = \max\{\upsilon^s(\mathrm{x}), s = 1 : k\}$, defined are the scalar function $\overline{\upsilon}(\mathrm{x}) = \max\{\upsilon^s(\mathrm{x}), s = 1 : l_k, 1 \le l_k \le k\}$ and a *lower right derivative* with respect to (11):

$$\underline{\upsilon}'(\mathrm{x}) \equiv \lim_{\delta t \to 0+} \{(\upsilon(\mathrm{x} + \delta t\, \mathcal{X}(t, \mathrm{x})) - \upsilon(\mathrm{x}))/\delta t\}.$$

Theorem. *Let there exist the VLF* υ, *so that*:

1) $(\exists \mathrm{a} \in \mathbb{R}_+^l)\; (\forall \mathrm{x} \in \mathcal{H}) \qquad \rho(\mathrm{x}) \le \mathrm{a} \cdot \overline{\upsilon}(\mathrm{x})$;
2) $(\exists \mathrm{b} \in \mathbb{R}_+^{l_0})\; (\forall \mathrm{x}_0 \in \mathcal{H}_0)\; \|\upsilon(\mathrm{x}_0)\| \le \langle \mathrm{b}, \rho^0(\mathrm{x}_0) \rangle$;
3) $\exists \gamma_c \in \overline{\mathbb{R}}_+^k$ *and a function* $\varphi_\gamma(\cdot)$ *exists so that* $\gamma_c \le \varphi_\gamma(\mathrm{a}, \gamma)$;
4) $\forall\; (t, \mathrm{x}) \in (\mathrm{T}_{t_0} \times \mathcal{H})$ *the conditions are satisfied*:
   a) $\underline{\upsilon}'_\gamma(\mathrm{x}) \dot{\le} \mathrm{f}_c(t, \upsilon_\gamma(\mathrm{x})) \equiv \mathrm{P}\upsilon_\gamma(\mathrm{x}) + \tilde{\mathrm{f}}_c(t, \upsilon_\gamma(\mathrm{x}))$;
   b) *Hurwitz condition for positive matrix* P;
   c) *Ważewski condition on quasi-monotonicity for the function* $\tilde{\mathrm{f}}_c(t, \mathrm{y})$;
   d) *Carateodory condition for the function* $\tilde{\mathrm{f}}_c(t, \mathrm{y})$, *bounded in each domain* $\Omega_c^r = (\mathrm{T}_{t_0} \times \mathcal{S}_c^r)$, *where* $r > 0$ *and* $\mathcal{S}_c^r = \{\mathrm{y} \in \mathbb{R}^k : \|\mathrm{y}\|_{\mathrm{E}} < r\}$;
   e) $(\tilde{\mathrm{f}}_c(t, \mathrm{y})/\|\mathrm{y}\|) \overset{t \in \mathrm{T}_{t_0}}{\Longrightarrow} 0$ *for* $\mathrm{y} \to 0$ *uniformly with respect to time* $t \in \mathrm{T}_{t_0}$,

where $\upsilon_\gamma = \upsilon - \gamma_c$. *Then solution* $x(t) = 0$ *of the system* (11) *is* $\rho\rho^0$*-exponential invariant and the matrix* $\mathcal{B}$ *has the form* $\mathcal{B} = c \cdot \mathrm{ab}^{\mathrm{t}}$ *with* $c \in \mathbb{R}_+$.

Proof. The basis of inequality for vector norm $\rho\,(x(t))$ is attained by the *comparison principle*, using the *maximum right-sided* solution $\overline{x}_c(t) \equiv \overline{\mathrm{x}}_c(t_0, \mathrm{x}_{c0}; t)$ of a *comparison system* $\dot{\mathrm{x}}_c(t) = \mathrm{P}\,\mathrm{x}_c(t) + \tilde{\mathrm{f}}_c(t, \mathrm{x}_c(t))$.

There is such an important problem: by what approach is it possible to create *constructive* techniques for constructing the VLF $\upsilon(\mathrm{x})$ and *simultaneous* synthesis of a nonlinear control law $\mathrm{u} = \mathcal{U}(\mathrm{x})$ for the close-loop system (11) with given vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x}_0)$? Recently, a pithy technique on constructing VLF at such synthesis has been elaborated. This method is based on a *nonlinear transformation* of the NCS model and solving the problem in two stages.

In stage 1, the right side $\mathcal{F}(\cdot)$ in (11) is transformed as $\mathcal{F}(\cdot) = \mathrm{f}(\mathrm{x}) + \mathrm{G}(\mathrm{x})\,\mathrm{u} + \tilde{\mathcal{F}}(t, \mathrm{x}(t), \mathrm{u})$, some *principal variables* in a state vector $\mathrm{x} \in \tilde{\mathcal{H}} \subset \mathbb{R}^{\tilde{n}} \subseteq \mathbb{R}^n$ with $\tilde{n} \le n$, $\mathrm{x}_0 \in \tilde{\mathcal{H}}_0 \subseteq \tilde{\mathcal{H}}$ are selected and a *simplified nonlinear model* of the object (11) is presented in the form of an affine *quite smooth* nonlinear control system

$\dot{\mathrm{x}} = \mathrm{F}(\mathrm{x}, \mathrm{u}) \equiv \mathrm{f}(\mathrm{x}) + \mathrm{G}(\mathrm{x})\mathrm{u} \equiv \mathrm{f}(\mathrm{x}) + \sum \mathrm{g}_j(\mathrm{x}) u_j$, which is structurally synthesized by the EFL technique. In this aspect, based on the structural analysis of *given* vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x})$, and also vector-functions $\mathrm{f}(\mathrm{x})$ and $\mathrm{g}_j(\mathrm{x})$, the *output vector-function* $\mathrm{h}(\mathrm{x}) = \{\mathrm{h}_i(\mathrm{x})\}$ is carefully selected. Furthermore, the nonlinear invertible (one-to-one) coordinate transformation $\mathrm{z} = \Phi(\mathrm{x})\; \forall \mathrm{x} \in \mathcal{S}_{\mathrm{h}} \subseteq \tilde{\mathcal{H}}$ with $\Phi(0) = 0$ is analytically obtained with *simultaneous* constructing the VLF. Finally, bilateral component-wise inequalities for the vectors $\mathrm{x}, \mathrm{z}, \upsilon(\mathrm{x}), \rho(\mathrm{x}), \rho^0(\mathrm{x}_0)$ are derived, it is most desirable to obtain the *explicit* form for the nonlinear transformation $\mathrm{x}=\Psi(\mathrm{z})$, inverse with respect to $\mathrm{z}=\Phi(\mathrm{x})$, and the VLF aggregation procedure is carried out with analysis of proximity for a singular directions in the *Jacobian* $[\partial \mathrm{F}(\mathrm{x}, \mathcal{U}(\mathrm{x}))/\partial \mathrm{x}]$.

In stage 2, the problem of nonlinear CL synthesis for the *complete model* of the NCS (11), taking rejected coordinates, nonlinearities and restrictions on control, into account is solved by the VLF-method. If a forming control is digital, a measurement the model's state is discrete and incomplete, then a simplified nonlinear discrete object's model is obtained by *Teylor-Lie* series, a *nonlinear* digital CL is formed and its parametric synthesis is carried out with a simultaneously construct a *discrete sub-vector* VLF.

VI. The antenna guidance laws

The analytic matching solution have been obtained for problem of the SC angular guidance at its antenna pointing to the Earth target and the same target tracking at time $t\in \mathrm{T}_n$ with $t_f^n\equiv t_0^n+T_n$. Solution is based on a vector composition of all elemental motions in the GRF $\mathbf{E}_{\mathrm{e}}$ using the HRF $\mathbf{E}_{\mathrm{e}}^{\mathrm{h}}$, the SRF $\boldsymbol{\mathcal{S}}$ and orthogonal matrix $\mathbf{C}_{\mathrm{h}}^{\mathrm{s}} \equiv \tilde{\mathbf{C}} = \| \tilde{c}_{ij} \|$ which defines the SRF $\boldsymbol{\mathcal{S}}$ orientation with respect to the HRF $\mathbf{E}_{\mathrm{e}}^{\mathrm{h}}$. Normed to the communication oblique range D vector $\mathbf{v}$ and the SC body programmed angular rate vector $\boldsymbol{\omega}^p$ with respect the GRF $\mathbf{E}_{\mathrm{e}}$ are presented in the SRF $\boldsymbol{\mathcal{S}}$ as $\mathbf{v}_{\mathrm{e}}^{\mathrm{s}} = \{\tilde{\mathrm{v}}_{\mathrm{e}i}^{\mathrm{s}}, i=1:3\}$ and $\boldsymbol{\omega}_{\mathrm{e}}^{sp} = \{\omega_{\mathrm{e}i}^{sp}, i=1:3\}$. Calculation of vector $\boldsymbol{\omega}_{\mathrm{e}}^{sp}$ is carried out by *explicit* analytical relations

$$\omega_{\mathrm{e}1}^{sp} = -\frac{\tilde{\mathrm{v}}_{\mathrm{e}3}^{\mathrm{s}}\tilde{c}_{21} + \tilde{\mathrm{v}}_{\mathrm{e}2}^{\mathrm{s}}\tilde{c}_{31}}{2\tilde{c}_{11}};\ \omega_{\mathrm{e}2}^{sp} = -\tilde{\mathrm{v}}_{\mathrm{e}3}^{\mathrm{s}};\ \omega_{\mathrm{e}3}^{sp} = \tilde{\mathrm{v}}_{\mathrm{e}2}^{\mathrm{s}}. \tag{12}$$

By numerical solution of the quaternion differential equation $\dot{\boldsymbol{\Lambda}}_{\mathrm{e}}^{sp} = \boldsymbol{\Lambda}_{\mathrm{e}}^{sp}\circ\boldsymbol{\omega}_{\mathrm{e}}^{sp}/2$ one can obtain values of vectors $\boldsymbol{\lambda}_{es}^{sp} \equiv \boldsymbol{\lambda}_{\mathrm{e}}^{sp}(t_s)$ for the time moments $t_s \in \mathrm{T}_n$ with period $T_q = t_{s+1} - t_s$, $s = 0 : n_q$, $n_q = T_n/T_q$ when initial value $\boldsymbol{\Lambda}_{\mathrm{e}}^{sp}(t_0^n)$ is given. Further solution is based on *extrapolation* of the vectors' $\boldsymbol{\lambda}_{\mathrm{e}k}^{\mathrm{s}} \equiv \boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{s}}(t_k)$ values which are defined for $t_k \in \mathrm{T}_n$ with step $T_a = t_{k+1} - t_k$, $k = 0 : n_a$, $n_a \equiv T_n/T_a$. Extrapolation is carried out by a set of n_a 3-degree *vector* splines with analytical obtaining a high-precise approximation the SRF $\boldsymbol{\mathcal{S}}$ guidance motion with respect to the GRF $\mathbf{E}_{\mathrm{e}}$ on vectors of both angular rate and acceleration. Required functions $\boldsymbol{\Lambda}^p(t)$, $\boldsymbol{\omega}^p(t)$ and $\boldsymbol{\varepsilon}^p(t)$ for this guidance mode are calculated by *explicit* formulas.

Fast onboard algorithms for the SC antenna guidance by its rotation maneuver into given time interval $t \in \mathrm{T}_p$ with restrictions to $\boldsymbol{\omega}^p(t)$ and $\boldsymbol{\varepsilon}^p(t)$ corresponding restrictions on $\mathbf{h}(\boldsymbol{\beta}(t))$ and $\dot{\boldsymbol{\beta}}(t)$ in a class of the SC angular motions, were elaborated. Here the boundary conditions on left ($t=t_0^p$) and right ($t=t_f^p$) trajectory ends are given as follows:

$$\boldsymbol{\Lambda}^p(t_0^p)=\boldsymbol{\Lambda}_0^p;\ \boldsymbol{\omega}^p(t_0^p)=\boldsymbol{\omega}_0^p;\ \boldsymbol{\varepsilon}^p(t_0^p)=\boldsymbol{\varepsilon}_0^p; \tag{13}$$

$$\boldsymbol{\Lambda}^p(t_f^p)=\boldsymbol{\Lambda}_f^p;\ \boldsymbol{\omega}^p(t_f^p)\equiv\boldsymbol{\omega}_f^p;\ \boldsymbol{\varepsilon}^p(t_f^p)=\boldsymbol{\varepsilon}_f^p. \tag{14}$$

Developed approach to the problem is based on necessary and sufficient condition for solvability of *Darboux* problem. Solution is presented as composition of three ($k=1:3$) *simultaneously* derived elementary rotations of embedded

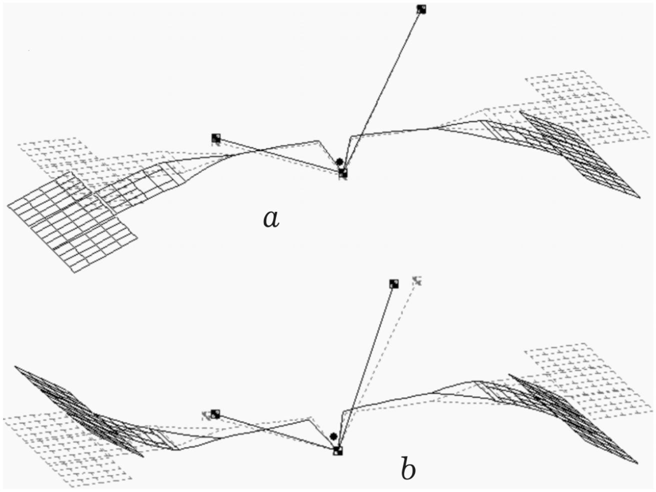

Fig. 5. Symmetric (a) and asymmetric (b) modal shapes of the SAPs

bases $\mathbf{E}_k$ about units $\mathbf{e}_k$ of *Euler* axes, which position is defined from the boundary conditions (13) and (14) for initial spatial problem. For all 3 elementary rotations with respect to units $\mathbf{e}_k$ the boundary conditions are analytically assigned. Into the IRF $\mathbf{I}_{\oplus}$ the quaternion $\boldsymbol{\Lambda}^p(t)$ is defined by the production

$$\boldsymbol{\Lambda}^p(t) = \boldsymbol{\Lambda}_0^p\circ\boldsymbol{\Lambda}_1^p(t)\circ\boldsymbol{\Lambda}_2^p(t)\circ\boldsymbol{\Lambda}_3^p(t), \tag{15}$$

where $\boldsymbol{\Lambda}_k^p(t) = (\cos(\varphi_k^p(t)/2), \sin(\varphi_k^p(t)/2)\ \mathbf{e}_k)$, $\mathbf{e}_k$ is unit of *Euler* axis by k's rotation, and functions $\varphi_k^p(t)$ present the elementary rotation angles in analytical form. These functions were selected in class of splines by 5 degree. Explicit time functions $\boldsymbol{\Lambda}^p(t)$, $\boldsymbol{\omega}^p(t)$ and $\boldsymbol{\varepsilon}^p(t)$ are applied at onboard computer for the time moments t_s by the SC antenna guidance at its both pointing ($t_s\in\mathrm{T}_n$) and rotation maneuver ($t_s\in\mathrm{T}_p$).

VII. The SC Structure Oscillations

Presented in [9] and applied here the distribution law $f_\rho(\boldsymbol{\beta}) = 0$ of the GMC normed AM $\mathbf{h}(\boldsymbol{\beta}) = \Sigma\, \mathbf{h}_p(\beta_p)$ between GD's pairs ensures its singular state only at separate time moments (with *Lebesgue* zero measure) and bijectively connects the vector $\mathbf{M}^{\mathbf{g}}(t)$ with vectors $\boldsymbol{\beta}(t)$ and $\dot{\boldsymbol{\beta}}(t) = \mathbf{u}^{\mathrm{g}}(t)$. Therefore for preliminary study it is rational to considerate the column $\mathbf{M}^{\mathbf{g}}(t)=\{\mathrm{M}_i^{\mathrm{g}}, i=1:3\}$ as control vector. Applying the state vector $\mathbf{x} = \{\boldsymbol{\phi}, \mathbf{v}_\delta, \boldsymbol{\omega}, \dot{\mathbf{q}}, \mathbf{q}\}$ and denotations $\mathbf{u}(t) = \mathbf{M}^{\mathbf{g}}(t)$, $\mathbf{y}(t) = \boldsymbol{\phi}(t)$ for a linearizing procedure of the SC model (7) at neighbourhood of the SC equilibrium in the ORF $\mathbf{O}$ one can obtain continuous models

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u},\ \mathbf{y} = \mathbf{C}\mathbf{x};\ \mathbf{W}(\mathrm{s}) = \mathbf{C}(\mathbf{I}\mathrm{s} - \mathbf{A})^{-1}\mathbf{B},$$

for which research the SC angular motion it is necessary to take into account only asymmetric modal shapes of the structure oscillations, see fig. 5. Comparison of linear (8) and hysteresis (7) modelling the SC structure weak-damped oscillations was developed by numerical methods. The SC natural frequency characteristics on roll channel are presented in Fig. 6 at linear modelling with decrement $\delta_j^q = 2\,10^{-3}$ for all tones. These characteristics were computed on the

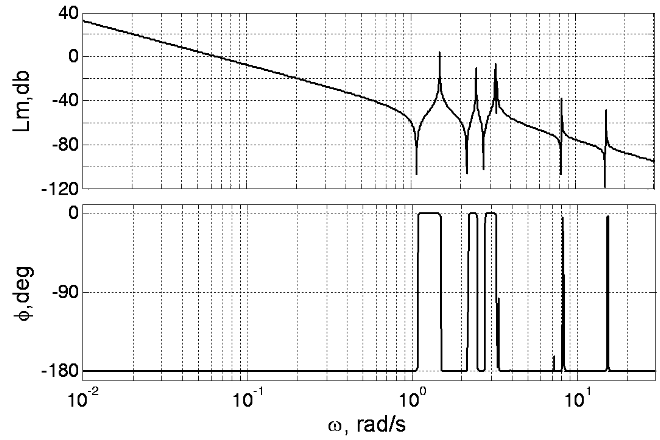

Fig. 6. The spacecraft natural frequency characteristics on roll channel

basis of a transfer function $\mathrm{W}_y(s) = \phi_2(s)/\mathrm{M}_2^{\mathrm{g}}(s)$. At hysteresis modeling the same "frequency characteristics" were also computed by numerical simulation for a set of input amplitudes $\mathrm{M}_2^{\mathrm{g}}$. Obtained results are close, but resonance "peaks" have very narrow form for hysteresis modelling.

VIII. Filtering and Control

Actual error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \tilde{\mathbf{\Lambda}}^p(t) \circ \mathbf{\Lambda}$, the *Euler* parameters' vector $\boldsymbol{\mathcal{E}} = \{e_0, \mathbf{e}\}$, and the attitude error's matrix is $\mathbf{C}_e \equiv \mathbf{C}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 - 2[\mathbf{e}\times]\mathbf{Q}_e^{\mathrm{t}}$, $\mathbf{Q}_e \equiv \mathbf{Q}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 e_0 + [\mathbf{e}\times]$ with $\det(\mathbf{Q}_e) = e_0$. As for contemporary communication SC, their large-scale SAPs and antennas have first minimal natural frequency ≈ 0.1 Hz by the structure flexible oscillations. At given digital control period T_u discrete frequency characteristics are computed via absolute pseudo-frequency $\lambda = 2\mathrm{tg}(\omega T_u/2)/T_u$. For period's multiple n_{q} and a filtering period $T_q = T_u/\mathrm{n_q}$ applied filter have the discrete transfer function

$$\begin{gathered} \mathrm{W_f(z_q)} = (1 + \mathrm{b}_1)/(1 + \mathrm{b}_1 \mathrm{z_q^{-1}}); \\ \mathrm{b}_1 \equiv -\exp(-T_q/T_{\mathrm{f}});\ \mathrm{z_q} \equiv \exp(\mathrm{s}T_q) \end{gathered} \tag{16}$$

and discrete frequency characteristics $\tilde{\mathrm{W}}_{\mathrm{f}}(j\lambda_{\mathrm{q}})$ via own absolute pseudo-frequency $\lambda_{\mathrm{q}} = (2/T_q)\mathrm{tg}(\omega T_q/2) = n_{\mathrm{q}}(2/T_u)\mathrm{tg}(\mathrm{arctg}(\lambda T_u/2)/n_{\mathrm{q}})$:

$$\tilde{\mathrm{W}}_{\mathrm{f}}(j\lambda_{\mathrm{q}}) \equiv \tilde{\mathrm{W}}_{\mathrm{f}}(\tilde{\mathrm{s}}_{\mathrm{q}}) = \mathrm{K}_{\mathrm{f}}^{\lambda}\ (\tilde{\mathrm{s}}_{\mathrm{q}} - q_{\mathrm{q}}^{\lambda})/(\tilde{\mathrm{s}}_{\mathrm{q}} - p_{\mathrm{q}}^{\lambda}), \tag{17}$$

where $\tilde{\mathrm{s}}_{\mathrm{q}} = j\lambda_{\mathrm{q}}; \mathrm{K}_{\mathrm{f}}^{\lambda} = (1 + \mathrm{b}_1)/(1 - \mathrm{b}_1) < 1$, $q_{\mathrm{q}}^{\lambda} = -(2/T_q)$ and $p_{\mathrm{q}}^{\lambda} = -\mathrm{K}_{\mathrm{f}}^{\lambda}(2/T_q)$. Therefore logarithmic amplitude characteristics $\mathrm{Lm}(\lambda)$ (LAC) and logarithmic phase characteristics $\phi(\lambda)$ (LPC) of discrete filter (16) are appeared as:

$$\begin{aligned} \mathrm{Lm}(\lambda) = 20[&\lg \mathrm{K}_{\mathrm{f}}^{\lambda} + \lg|j\mathrm{tg}(\mathrm{arctg}(\lambda T_u/2)/n_{\mathrm{q}})| \\ &- \lg|j\,\mathrm{tg}\,(\mathrm{arctg}(\lambda T_u/2)/n_{\mathrm{q}}) + \mathrm{K}_{\mathrm{f}}^{\lambda}|]; \\ \phi\,(\lambda) = \ &[\mathrm{arctg}\,(\lambda T_u/2)/n_{\mathrm{q}}] \\ &-\mathrm{arctg}[\mathrm{tg}\,(\mathrm{arctg}(\lambda T_u/2)/n_{\mathrm{q}})/\mathrm{K}_{\mathrm{f}}^{\lambda}]. \end{aligned} \tag{18}$$

As an example fig. 7 presents frequency characteristics of discrete filter (16) for control period $T_u = 4$ s, $T_{\mathrm{f}} = 2$ s and the period's multiple $n_{\mathrm{q}} = 4$. Applied digital control law is

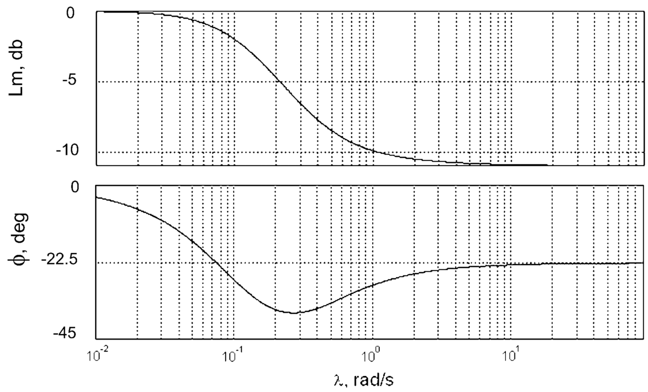

Fig. 7. The LAC and LPC of discrete filter

as follows:

$$\begin{gathered} \boldsymbol{\epsilon}_k = -2\mathbf{e}_k^{\mathrm{f}};\ \mathbf{g}_{k+1} = \mathbf{B}\,\mathbf{g}_k + \mathbf{C}\,\boldsymbol{\epsilon}_k; \\ \mathbf{M}_k^{\mathrm{g}} = \mathbf{K}^{\mathrm{g}}(\mathbf{g}_k + \mathbf{P}\boldsymbol{\epsilon}_k), \end{gathered} \tag{19}$$

matrices $\mathbf{B}, \mathbf{C}, \mathbf{P}$ and $\mathbf{K}^{\mathrm{g}}$ have diagonal form with

$$\begin{aligned} a_i &= [(2/\mathrm{T}_u)\tau_{1i} - 1]/[(2/\mathrm{T}_u)\tau_{1i} + 1]; \\ b_i &= [(2/\mathrm{T}_u)\tau_{2i} - 1]/[(2/\mathrm{T}_u)\tau_{2i} + 1]; \\ p_i &= (1 - b_i)/(1 - a_i);\ c_i = p_i(b_i - a_i), \end{aligned}$$

where τ_{1i}, τ_{2i} and k_i^{g} are constant parameters which are thoroughly selected for the robust properties of gymomoment control system. Here digital information on only the SC attitude *filtered* error vector $\mathbf{e}_k^{\mathrm{f}}$ is applied for forming the control vector (19), moreover measured error quaternion is $\mathbf{E}_s = (e_{0s}, \mathbf{e}_s) = \tilde{\mathbf{\Lambda}}^p(t_s) \circ \mathbf{\Lambda}_s^{\mathrm{m}}$, the measured *Euler* parameters' vector $\boldsymbol{\mathcal{E}}_s = \{e_{0s}, \mathbf{e}_s\}$, then filtering is executed by the relations

$$\tilde{\mathbf{x}}_{s+1} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{B}}\mathbf{e}_s; \mathbf{e}_s^{\mathrm{f}} = \tilde{\mathbf{C}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{D}}\mathbf{e}_s, \tag{20}$$

where matrices $\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ have diagonal form with $\tilde{a}_i = -\mathrm{b}_1^{\mathrm{f}}$; $\tilde{b}_i = \mathrm{b}_1^{\mathrm{f}}$; $\tilde{c}_i = -(1 + \mathrm{b}_1^{\mathrm{f}})$ and $\tilde{d}_i = 1 + \mathrm{b}_1^{\mathrm{f}}$. The discrete filtering efficiency is demonstrated by fig. 8 for the same SC roll channel. There is not stability on this channel without the discrete filter (16) and the same ones (20). In stage 2 a *nonlinear* digital CL is formed, its parametric synthesis is carried out with a simultaneously construct a *discrete* VLF.

IX. Computer simulation

The *MatLab* system was applied for simulation and analysis of the flexible SC antenna pointing model (7) and (8) with discrete filtering (20), discrete CL (19) and forming the GMC digital control (10). For considered variants of digital gyromoment control some results on the SC stabilization in the ORF $\mathbf{O}$ are presented in fig. 9 and fig. 10 when for $t = t_0 = 0$ initial conditions on all variables are zero except $\phi_i(0) = 0.5$ deg for all $i = 1 : 3$. Additional angular deflection by antenna's flexibility $\delta\boldsymbol{\phi}$ (9) are presented in fig. 11 (sequence of the SC antenna guidance modes: 1° — pointing and target tracking, 2° — guidance by a rotation maneuver to next target and again 1° — the same next target tracking, see fig. 4) and in fig. 12 (the antenna's weak-damped oscillations at a longtime tracking).

X. CONCLUSIONS

New approach for modelling a physical hysteresis was developed and it application for digital gyromoment attitude control of a flexible spacecraft structure was considered. In progress of [10] – [11] new results on the SC antenna guidance, a digital gyromoment spacecraft attitude control and a flexible antenna fine pointing, were presented.

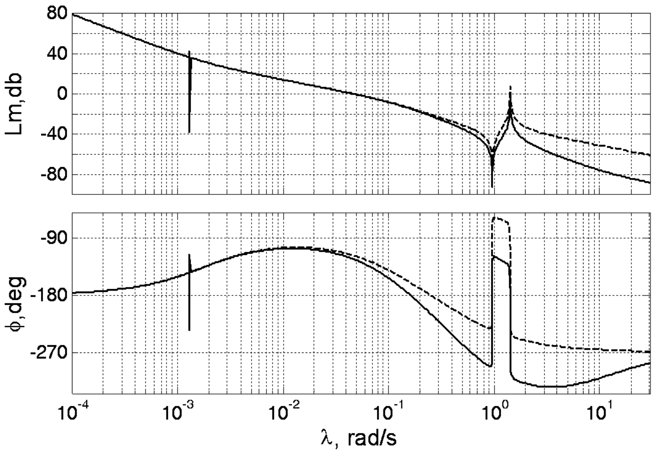

Fig. 8. The SC open-loop pseudo-frequency characteristics on roll channel: - - - without discrete filter, — with discrete filter.

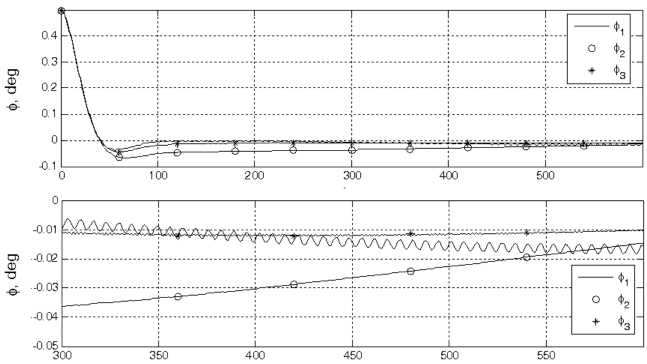

Fig. 9. Transient processes by attitude angles with respect the ORF

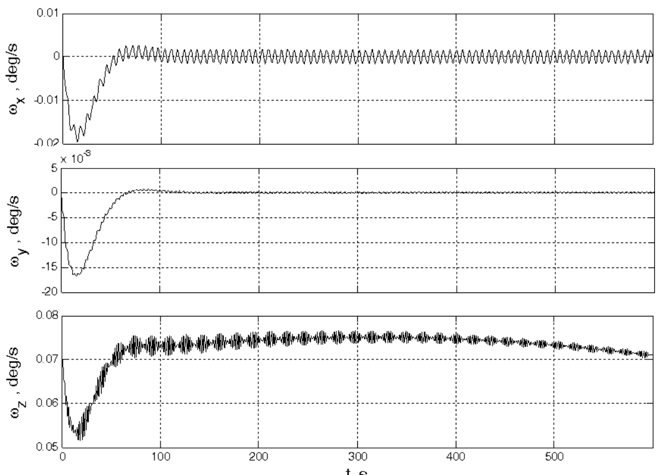

Fig. 10. Transient processes by angular rates

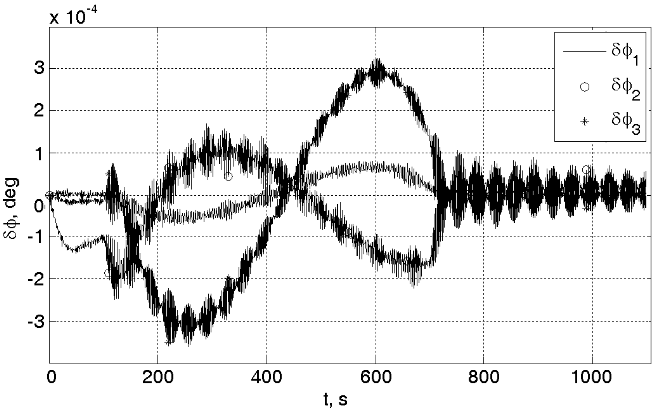

Fig. 11. The SC antenna oscillations at sequence of the guidance modes

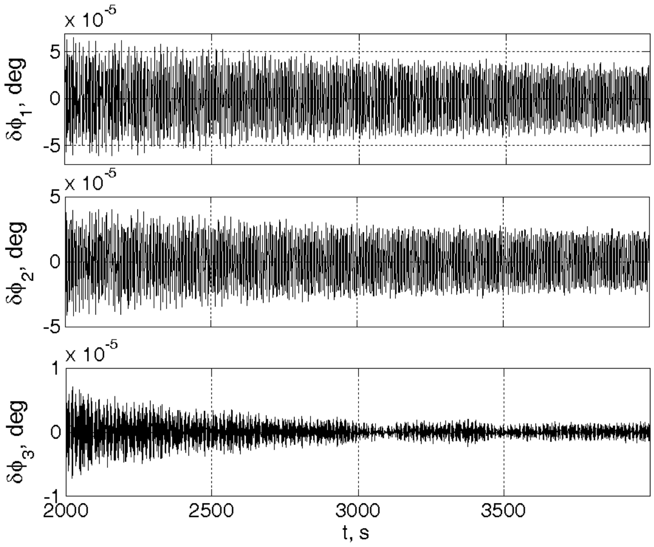

Fig. 12. The SC antenna's weak-damped oscillations at a longtime tracking

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