Nonlinear Gyromoment Spacecraft Attitude Control with Precise Pointing the Flexible Antennas

Ye.I. Somov, S.A. Butyrin, S.Ye. Somov

Abstract—New approach for modelling a physical hysteresis damping the flexible spacecraft structure oscillations, is developed. New results on spacecraft attitude guidance and digital gyromoment nonlinear control with precise pointing the large-scale flexible antennas, are presented.

I. INTRODUCTION

A correct mathematical description of physical hysteresis is a basic problem for an internal friction theory (N.N. Davidsonov, 1938; A.Yu. Ishlinskii, 1944; W. Prager, 1956; J.F. Besseling, 1958; Ye.S. Sorokin, 1960; Ya.G. Panovko [1]; G.S. Pisarenko [2]; V.A. Palmov [3]; L.F. Kochneva [4] et al.) with regard to the well-known flexible-plastic micro-deformations of materials. The rigorous mathematical aspects for qualitative analysis of general hysteresis models are represented in a number of research works [5]. Recently, new deformations of materials. The rigorous mathematical aspects for qualitative analysis of general hysteresis models are represented in a number of research works [5]. Recently, new approach was developed for description of physical hysteresis [6], [7], which is based on set-valued differential equation represented in a number of research works [5].

II. MODEL OF PHYSICAL HYSTERESIS

Let $x(t)$ is a real piecewise-differentiated function for $t \in T_{t_0} = [t_0, +\infty)$. Let there be the values $\tilde{x}_\nu = x(t_\nu)$ of the function in the time moments $t_\nu, \nu \in \mathbb{N}_0 \equiv \{0, 1, 2, \ldots \}$, when the last changing a sign of a speed $\tilde{x}(t)$ was happened, e.g.

$$\tilde{x}_\nu \equiv x(t_\nu); t_\nu : \text{Sign}\tilde{x}(t_\nu + 0) \neq \text{Sign}\tilde{x}(t_\nu - 0).$$

A local function $\tilde{x}_\nu(t)$ on each a local time semi-interval $T_\nu \equiv [t_\nu, t_{\nu+1})$ is introduced as

$$\tilde{x}_\nu(t) = x(t) - \tilde{x}_\nu \forall t \in T_\nu,$$

and the functional $k_\nu(x(t)) \equiv k_\nu(k, \tilde{p}, \tilde{x}_\nu)$ of the hysteresis function shape is defined as

$$k_\nu(x(t)) = \alpha \left(1 - (1 - p) \exp(-\tilde{p}|\tilde{x}_\nu|)\right), \quad t \in T_\nu,$$

where $k, p, \tilde{p}$ are constant positive parameters. For a constant parameter $\alpha > 0$ and $x_0 \equiv x(t_0)$ a normed hysteresis function $r(t) = \text{Hst}(\cdot, x(t))$ with memory

$$r(t) = \text{Hst}(a_h, \alpha_h, k_\nu(x(t)), r_\nu, x(t)),$$

and restriction on its module by parameter $a_h > 0$, is defined as a right-sided solution of the equations

$$D^+ r = \begin{cases} k_\nu |r - a_h \text{Sign}\tilde{x}(t)|^{\alpha_h} \tilde{x}(t) & |r| < a_h, \\ 0 & |r| \geq a_h; \end{cases}$$

$$r(t_0 + 0) = r_0.$$

Differential equation in (5) has a discontinuous right side and ambiguously depends on forcing function $x(t)$ and its speed $\dot{x}(t)$, e.g. it depends on all own pre-history which is expressed by the functional $k_\nu(x)$ (3). At initial condition $y_0 \equiv y_0 = y(t_0)$ for $x = x_0$ the hysteresis function $y(t)$ is defined as follows

$$y(t) \equiv m \text{Hst}(a_h, \alpha_h, k_\nu, r_\nu, x(t)); r_\nu \equiv y_0/m, \quad y(t) \geq y_0/m.$$

with the constant positive scale coefficient $m > 0$. In developed model (1) – (6) a parameter $\tilde{p}$ determines on the whole a degree of convergence for a trajectory $y(t) = F_h(\cdot, x(t))$ in the plane $xOy$ on symmetric limiting static loop under a harmonic forcing function $x(t) = A\sin \omega t$ with fixed values $A, \omega$ and any initial condition $y_0 = y_0$ with $|y_0|/m < a_h$.

For this model all requirements are realized, including the famous requirements on a model vibro-correctness by [5],

Fig. 1. Results of a hysteresis model testing.
and also on a frequency independence and a fine return on a main symmetric limiting hysteresis loop after a short-term passage on a displaced local hysteresis loop [3], [4]. Last properties are verified in prearranged scale by Fig. 1 for the hysteresis model with parameters: \( m = 1, \alpha_h = 1.5, \alpha_a = 200, k = 5.125 \times 10^{-4}, p = 2, \tilde{p} = 0.75 \times 10^{-3} \). Moreover the forcing function have the form:

\[
x(t) = \begin{cases}
  A \sin \omega t & (0 \leq t \leq \tau_1) \cap (\tau_2 \leq t \leq \tau_3);
  B(1+\sin \omega t) & \tau_1 \leq t \leq \tau_2,
\end{cases}
\]

\[
A = 200; \: B = 40; \: \omega_1 = 1; \: \omega_2 = 5; \: \tau_3 = 40; \: \tau_1 \equiv 5\pi - \tau^*; \: \tau_2 = 7\pi - \tau^*; \: \tau^* \approx 0.03415\pi.
\]

### III. MATHEMATICAL MODELS

Let us introduce the inertial reference frame (IRF) \( \mathbb{I}_0 \), the geodesic Greenwich reference frame (GRF) \( \mathbb{E}_c \), and the geodesic horizon reference frame (HRF) \( \mathbb{E}_h^b \). There are also standard defined the SC body reference frame (BRF) \( \mathbb{B} \) (\( \mathbb{O}_{xyz} \)), the orbit reference frame (ORF) \( \mathbb{O} (\mathbb{O}_{xyz}^\phi \mathbb{O}_{xv}^\beta \mathbb{O}_{xv}^\gamma \mathbb{O}_{xv}^\alpha) \) and the antenna (sensor) reference frame (SFR) \( \mathbb{S} (\mathbb{O}_{xyz}^g \mathbb{O}_{xv}^\phi \mathbb{O}_{xv}^\beta \mathbb{O}_{xv}^\gamma) \) with an origin \( S \). The BRF attitude with respect to the IRF \( \mathbb{I}_0 \) is defined by quaternion \( \Lambda = (\lambda_0, \lambda) \), \( \lambda = (\lambda_1, \lambda_2, \lambda_3) \), and with respect to the ORF — by the column \( \phi = \{ \phi_i, i = 1, 2, 3 \equiv 1: 3 \} \) of Euler-Krylov elementary angles \( \phi_i \) in the sequence 1’2’3’.

Let us vectors \( \omega(t) \) and \( v(t) \) are standard denotations of the SC body vector angular rate and its mass center velocity with respect to the IRF, respectively.

Applied further symbols \((\cdot, \cdot), \{ \cdot \}, [\cdot] \) for vectors and \([a \times \cdot], (\cdot)^\dagger \) for matrixes are conventional denotations. For a fixed position of flexible structures on the SC body with some simplifying assumptions and \( t \in T_{0a} = [0, +\infty) \) a SC spatial motion model is appeared as follows:

\[
\dot{\Lambda} = \Lambda \dot{\omega}/2; \quad \dot{\Lambda} = \{ \dot{\Lambda}, \dot{v}_\delta, \dot{\omega}, \dot{q} \} = \{ \dot{F}^\gamma, \dot{F}^\delta, \dot{F}^\rho \}; \quad \dot{\beta} = \dot{u}^\delta; \quad (7)
\]

\[
F^\delta = -m(\omega \times v_\delta) + \omega \times (L \times \omega - 2J) + R^c; \quad L = M_q q;
\]

\[
F^\omega = -L \times (\omega \times v_\delta) + M^g - \omega \times G + M^g; \quad M^g = -A h \dot{\beta};
\]

\[
F^\rho = -\left( -\Omega^j \right)^2 m_j r_j(t); \quad G = G^0 + D_k^p q; \quad q = \{ q_j \};
\]

\[
r_j(t) = H_s t; \quad m_j \equiv m \{ \alpha^i h^j, k^l, x_{o_j}, y_{o_j}, z_{o_j} \}; \quad x_{o_j} = q_j(t)/m_j;
\]

\[
A = \left[ \begin{array}{c|c|c}
  m_l & -[L^x] & M_q^y \\
  [L^x] & J & D_q \\
  M_q^y & D_q & I
\end{array} \right]; \quad A_h = \left[ \begin{array}{c}
  [\partial \mathcal{H} / \partial \beta] \nend{array} \right]; \quad G^0 = J \omega + \mathcal{H}(\beta); \quad \mathcal{H}(\beta) = h_y \Sigma h_y(\beta_p),
\]

where \( v_\delta \) is a velocity deflection with respect to its nominal value by the gyroline (GD), \( h_y \) is a constant own angular momentum (AM) of each gyroline (GD), \( s_{o_j} = q_j(t_0)/m_j \) with a concordance of initial conditions (6). Parameters \( m_j, \alpha^i h^j, \alpha^g, \) and \( k_j, p_j, n_j \) by functionals \( \text{Hst}(\cdot) \) and \( \kappa_k(x_j) \) for tones of the SC structure oscillations are defined by an identification procedure starting from analysis of experimental hysteresis loop for normed deformation \( \varepsilon \) and strength \( \sigma \) of the structure material, see Fig. 2. At standard linear modelling one can have

\[
F^g = \left\{ -((\delta_j^g / \pi)) \left( \Omega_j^g q_j + (\Omega_j^g)^2 q_j \right) \right\}, \quad (8)
\]

where \( \delta_j^g \in [10^{-2}, 2 \times 10^{-4}] \) is decrement by \( j \)-tone of the SC structure flexible oscillations. The antenna’s flexibility results in additional angular deflection column \( \delta \phi \equiv \{ \delta_i, i = 1: 3 \} \) of the SRF \( \mathbb{S} \) with respect its nominal position in the BRF, including the antenna’s line-of-sight \( \mathbb{S}^z \):

\[
\delta \phi = q_d, \quad (9)
\]

where matrix \( q_d \) is calculated by its shape modes. The gyro moment cluster (GMC) is applied at scheme 2-SPE, based on four GDs [8], fig. 3.

### IV. THE PROBLEM STATEMENT

In precession theory of the control moment gyros the GMC control torque vector \( M^g \) is presented as follows:

\[
M^g = -H = -A_h(\beta) \dot{u}^\delta(t); \quad \beta = \dot{u}^\delta = \{ u_j^g(t) \}. \quad (10)
\]

Here \( u_j^g(t) = a_j^g Z \text{Sat}(\text{Qntg}(u^g_{pk}, B_u), T_u) \) with a constant \( a^g \) and a control period \( T_u = t_{k+1} - t_k \), \( k \in \mathbb{N}_0 \); discrete functions \( u_{pk}^g = u_{jk}^g(t_k) \) are outputs of nonlinear control law (NCL), and functions \( \text{Sat}(x, a) \) and \( \text{Qntg}(x, a) \) are general-usage ones, while the holder model with the period \( T_u \) is such: \( y(t) = Z h(x_k, T_u) = x_k \forall t \in [t_k, t_{k+1}) \).

Applied onboard measuring subsystem is based on initial gyro unit corrected by the fine fixed-head star trackers. This subsystem is intended for precise determination of the SC BRF angular position with respect to the IRF \( \mathbb{I}_0 \). Applied contemporary filtering & alignment calibration algorithms and a discrete astatic observer give finally a fine discrete estimating the SC angular motion coordinates by the quaternion \( \Lambda^m = \Lambda^h \Lambda^n \), \( s \in \mathbb{N}_0 \), where \( \Lambda^h = \Lambda(t_a), \Lambda^n \) is a “noise-drift” digital quaternion and a measurement period \( T_a = t_{a+1} - t_a \leq T_u \) is multiply with respect to a control period \( T_u \). When the SC is moving at a distant part of the high-eccentricity orbit (HEO) by Moltinuya type (with apogee 46370 km and perigee 7370 km, fig. 4) there are fulfilled sequence of its angular modes:

1° the SC antenna pointing to a given point at the Earth surface and then the target tracking during given time interval \( T_a \equiv [t^0_a, t^1_a] \);

2° the SC antenna guidance from any Earth point to next the same point during time \( t \in T_p \equiv [t^0_p, t^1_p], T_p \equiv t^1_p + T_p \) where \( T_p \) is given, see fig. 4.
V. SYNTHESIS OF FEEDBACK CONTROL

At the SC lifetime up to 15 years its structure inertial and flexible characteristics are slowly changed in wide boundaries, and the solar array panels (SAPs) are slowly rotated on the angle $\gamma(t) \in [0, 2\pi]$ with respect to the SC body for their tracking the Sun direction. Therefore at inertial matrix $A$, and partial frequencies $Q^\circ(t)$ of the SC structure are not complete certain. Let us $\Lambda$ continuous of the $X$, where $\dot{X} = F(X, U)$ by tasks $1^\circ, 2^\circ$ and the GMC control $U = \{u_{nk}\}$ on the quaternion values $A^{\circ}$ when the SC structure characteristics are uncertain and its damping is very weak, $\delta^3 \approx 10^{-3}$ in (8).

For the VLF $\nu : H \rightarrow \mathbb{R}^k$ with components $\nu^s(x) \geq 0$, $\nu^s(0) = 0$, $s = 1 : k$ and the norm $\|\nu(x)\| = \max\{\nu^s(x), s = 1 : k\}$, defined are the scalar function $\bar{\nu}(x) = \max\{\nu^s(x), s = 1 : k, 1 \leq k \leq k\}$ and a lower right derivative with respect to (11):

$$\bar{\nu}'(x) \equiv \lim_{\delta t \to 0^+} \{\nu(x + \delta t \bar{X}(t, x)) - \nu(x)/\delta t\}.$$

**Theorem.** Let there exist the VLF $\nu$, so that:

1. $(\exists a \in \mathbb{R}^k) (\forall x \in H) \rho(x) \leq a \cdot \bar{\nu}(x)$;  
2. $(\exists b \in \mathbb{R}^k) (\forall x_0 \in H_0) \|\nu(x_0)\| \leq (b, \rho^0(x_0))$;  
3. $\exists \gamma \in \mathbb{R}^*$ and a function $\varphi_{\gamma}(\cdot)$ exists so that $\gamma \leq \varphi_{\gamma}(a, \gamma)$;  
4. $\forall (t, x) \in (T_0 \times H)$ the conditions are satisfied:
   a) $\nu_0'(x) \leq \bar{\nu}(t, x) \equiv \nu_0(x) + \bar{\nu}(t, x)$;  
   b) Hurwitz condition for positive matrix $P$;  
   c) Wazewski condition on quasi-monotonicity for the function $\bar{\nu}(t, y)$;  
   d) Caratheodory condition for the function $\bar{\nu}(t, y)$, bounded in each domain $\Omega^\circ_r = (T_0 \times S^e_r)$, where $r > 0$ and $S^e_r = \{y \in \mathbb{R}^k : \|y\| < r\}$;  
   e) $\bar{\nu}(t, y)/\|y\| \to 0$ for $y \to 0$ uniformly with respect to time $t \in T_0$, where $\nu_0 = \nu - \gamma$. Then solution $x(t) = 0$ of the system (11) is $\rho^0$-exponential invariant and the matrix $B$ has the form $B = c \cdot ab^\dagger$ with $c \in \mathbb{R}^4$.

**Proof.** The basis of inequality for vector norm $\rho(x(t))$ is attained by the comparison principle, using the maximum right-sided solution $\bar{\nu}(t) \equiv \bar{\nu}(t_0, x_0; t)$ of a comparison system $\dot{x}_c(t) = P x_c(t) + \bar{\nu}(t, x_c(t))$.

There is such an important problem: by what approach is it possible to create constructive techniques for constructing the VLF $\nu(x)$ and simultaneous synthesis of a nonlinear control law $u = U(x)$ for the close-loop system (11) with given vector norms $\rho(x)$ and $\rho^0(x_0)$? Recently, a pithy technique on constructing VLF at such synthesis has been elaborated. This method is based on a nonlinear transformation of the NCS model and solving the problem in two stages.

In stage 1, the right side $F(\cdot)$ in (11) is transformed as $\tilde{F}(\cdot) = f(x) + G(x)u + \tilde{F}(t, x, u)$, some principal variables in a state vector $x \in H \subseteq \mathbb{R}^n$ with $n \leq n$, $x_0 \in H_0 \subseteq H$ are selected and a simplified nonlinear model of the object (11) is presented in the form of an affine quite smooth nonlinear control system

$$\dot{x} = F(x, u) \equiv f(x) + G(x)u \equiv f(x) + \sum g_i(x) u_j,$$

which is structurally synthesized by the EFL technique. In this aspect, based on the structural analysis of given vector norms $\rho(x)$ and $\rho^0(x)$, and also vector-functions $f(x)$ and $g_i(x)$, the output vector-function $h(x) = \{h_i(x)\}$ is carefully selected. Furthermore, the nonlinear invertible (one-to-one) coordinate transformation $z = \Phi(x) \forall x \in S_h \subseteq H$ with $\Phi(0) = 0$ is analytically obtained with simultaneous constructing the VLF. Finally, bilateral component-wise inequalities for the vectors $z(x, y, \nu(x), \rho(x), \rho^0(x_0))$ are derived, it is most desirable to obtain the explicit form for the
nonlinear transformation \( x = \Psi(z) \), inverse with respect to \( z = \Phi(x) \), and the VLF aggregation procedure is carried out with analysis of proximity for a singular directions in the Jacobian \( [\partial \Phi(x, U(x))] / \partial x \).

In stage 2, the problem of nonlinear CL synthesis for the complete model of the NCS (11), taking rejected coordinates, nonlinearities and restrictions on control, into account is solved by the VLF-method. If a forming control is digital, a measurement the model’s state is discrete and incomplete, then a simplified nonlinear discrete object’s model is obtained by Taylor-Lie series, a nonlinear digital CL is formed and its parametric synthesis is carried out with a simultaneously construct a discrete sub-vector VLF.

VI. THE ANTENNA GUIDANCE LAWS

The analytic matching solution have been obtained for problem of the SC angular guidance at its antenna pointing to the Earth target and the same target tracking at time \( t \in T_n \) with \( t^0 = t_0 + T_n \). Solution is based on a vector composition of all elementary motions in the GRF \( \mathbf{E}_0 \), the SRF \( \mathbf{S} \) and orthogonal matrix \( \mathbf{C}_n^t \equiv \mathbf{C} = \parallel \mathbf{c}_i \parallel \) which defines the SRF \( \mathbf{S} \) orientation with respect to the HRF \( \mathbf{E}_0 \). Normed to the communication oblique range \( D \) vector \( \mathbf{v} \) and the SC body programmed angular rate vector \( \boldsymbol{\omega}^p \) with respect the GRF \( \mathbf{E}_0 \), are presented in the SRF \( \mathbf{S} \) as \( \mathbf{v}_s^e = \{ \mathbf{v}_s^{e_i} ; i = 1 : 3 \} \) and \( \boldsymbol{\omega}^p_s = \{ \mathbf{\omega}^{sp}_s ; i = 1 : 3 \} \).

Calculation of vector \( \mathbf{v}_e^p \) is carried out by explicit analytical relations

\[
\mathbf{\omega}^{sp}_{e1} = -\frac{\mathbf{v}_e^p\mathbf{c}_2\mathbf{c}_1^{T} + \mathbf{v}_s^p\mathbf{c}_3\mathbf{c}_1^{T}}{2\mathbf{c}_{11}}; \quad \mathbf{\omega}^{sp}_{e2} = -\frac{\mathbf{v}_e^p\mathbf{c}_3\mathbf{c}_1^{T} + \mathbf{v}_s^p\mathbf{c}_2\mathbf{c}_1^{T}}{2\mathbf{c}_{11}}; \quad \mathbf{\omega}^{sp}_{e3} = \mathbf{v}_e^p. \tag{12}
\]

By numerical solution of the quaternion differential equation \( \mathbf{\Lambda}^p = \mathbf{\Lambda}^e \mathbf{\omega}^{sp} / 2 \) one can obtain values of vectors \( \mathbf{\Lambda}^p(t_0) \) for the time moments \( t_s \in T_n \) with period \( T_q = t_{s+1} - t_s, \ s = 0 : n_q, \ n_q = T_n / T_q \) when initial value \( \mathbf{\Lambda}^p(t_0) \) is given. Further solution is based on extrapolation of the vectors' \( \mathbf{\Lambda}_k^p \equiv \mathbf{\Lambda}_k^p(t_k) \) values which are defined for \( t_k \in T_n \) with step \( T_n = t_{k+1} - t_k, \ k = 0 : n_k, \ n_k \equiv T_n / T_q \). Extrapolation is carried out by a set of \( n_k \) 3-degree vector splines with analytical obtaining a high-precision approximation the SRF \( \mathbf{S} \) guidance motion with respect to the GRF \( \mathbf{E}_0 \), on vectors of both angular rate and acceleration. Required functions \( \mathbf{\Lambda}^p(t), \mathbf{\omega}^p(t) \) and \( \mathbf{\epsilon}^p(t) \) for this guidance mode are calculated by explicit formulas.

Fast onboard algorithms for the SC antenna guidance by its rotation maneuver into given time interval \( t \in T_p \) with restrictions to \( \mathbf{\omega}^p(t) \) and \( \mathbf{\epsilon}^p(t) \) corresponding restrictions on \( \mathbf{h}(\beta(t)) \) and \( \mathbf{\beta}(t) \) in a class of the SC angular motions, were elaborated. Here the boundary conditions on left \((t = t_0)\) and right \((t = t_p)\) trajectory ends are given as follows:

\[
\mathbf{\Lambda}^p(t_0) = \mathbf{\Lambda}^p_0; \quad \mathbf{\omega}^p(t_0) = \mathbf{\omega}^p_0; \quad \mathbf{\epsilon}^p(t_0) = \mathbf{\epsilon}^p_0; \tag{13}
\]

\[
\mathbf{\Lambda}^p(t_p) = \mathbf{\Lambda}^p_f; \quad \mathbf{\omega}^p(t_p) = \mathbf{\omega}^p_f; \quad \mathbf{\epsilon}^p(t_p) = \mathbf{\epsilon}^p_f. \tag{14}
\]

Developed approach to the problem is based on necessary and sufficient condition for solvability of Darboux problem. Solution is presented as composition of three \((k = 1 : 3)\) simultaneously derived elementary rotations of embedded bases \( \mathbf{E}_k \) about units \( \mathbf{e}_k \) of Euler axes, which position is defined from the boundary conditions (13) and (14) for initial spatial problem. For all 3 elementary rotations with respect to units \( \mathbf{e}_k \) the boundary conditions are analytically assigned. Into the HRF \( \mathbf{I}_B \) the quaternion \( \mathbf{\Lambda}^p(t) \) is defined by the production

\[
\mathbf{\Lambda}^p(t) = \mathbf{\Lambda}^p_f \mathbf{\Lambda}^p_1(t) \mathbf{\Lambda}^p_2(t) \mathbf{\Lambda}^p_3(t), \tag{15}
\]

where \( \mathbf{\Lambda}^p_k(t) = (\cos(\varphi^p_k(t)/2), \sin(\varphi^p_k(t)/2) \mathbf{e}_k) \), \( \mathbf{e}_k \) is unit of Euler axis by \( k \)'s rotation, and functions \( \varphi^p_k(t) \) present the elementary rotation angles in analytical form. These functions were selected in class of splines by 5 degree. Explicit time functions \( \mathbf{\Lambda}^p(t), \mathbf{\omega}^p(t) \) and \( \mathbf{\epsilon}^p(t) \) are applied on onboard computer for the time moments \( t_s \) by the SC antenna guidance at its both pointing \((t_s \in T_n)\) and rotation maneuver \((t_s \in T_p)\).

VII. THE SC STRUCTURE OSCILLATIONS

Presented in [9] and applied here the distribution law \( f_\mu(\beta) = 0 \) of the GMC normed AM \( \mathbf{h}(\beta) = \Sigma \mathbf{h}_n(\beta) \) between GD’s pairs ensures its singular state only at separate time moments (with Lebesgue zero measure) and bijectively connects the vector \( \mathbf{M}^p(t) \) with vectors \( \beta(t) \) and \( \mathbf{\beta}(t) = \mathbf{u}^p(t) \). Therefore for preliminary study it is rational to consider the column \( \mathbf{M}^p = \{ \mathbf{M}^i_p, i = 1 : 3 \} \) as control vector. Applying the state vector \( \mathbf{x} = \{ \mathbf{\phi}, \mathbf{v}_s, \mathbf{\omega}, \mathbf{q}, \mathbf{\dot{q}} \} \) and denotations \( u(t) = \mathbf{M}^p(t), y(t) = \mathbf{\phi}(t) \) for a linearizing procedure of the SC model (7) at neighbourhood of the SC equilibrium in the ORF \( \mathbf{O} \) one can obtain continuous models

\[
\mathbf{\dot{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}; \quad \mathbf{y} = \mathbf{C} \mathbf{x} = \mathbf{C} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B},
\]

for which research the SC angular motion it is necessary to take into account only asymmetric modal shapes of the structure oscillations, see fig. 5. Comparison of linear (8) and hysteresis (7) modelling the SC structure weak-damped oscillations was developed by numerical methods. The SC natural frequency characteristics on roll channel are presented in Fig. 6 at linear modelling with decrement \( \delta^3 = 2 \times 10^{-3} \) for all tones. These characteristics were computed on the
basis of a transfer function $W_p(s) = \phi_2(s)/M_2^b(s)$. At hysteresis modeling the same “frequency characteristics” were also computed by numerical simulation for a set of input amplitudes $M_2^b$. Obtained results are close, but resonance “peaks” have very narrow form for hysteresis modelling.

VIII. FILTERING AND CONTROL

Actual error quaternion is $E = (e_0, e) = \hat{A}^p(t) \circ \hat{A}$, the Euler parameters’ vector $E = \{e_0, e\}$, and the attitude error’s matrix is $C_t = C(E) = I_3 - 2[e \times]Q_t^e$, $Q_t^e = Q(E) = I_3 e_0 + [e \times]$ with $\det(Q_t^e) = e_0$. As for contemporary communication SC, their large-scale SAPs and antennas have first minimal natural frequency $\approx 1\text{ Hz}$ by the structure flexible oscillations. At given digital control period $T_u$ discrete frequency characteristics are computed via absolute pseudo-frequency $\lambda = 2t g(\omega T_u/2)/T_u$. For period’s multiple $n_q$ and a filtering period $T_q = T_u/n_q$ applied filter have the discrete transfer function

$$W_t(z_q) = (1 + b_1)/(1 + b_1 z_q^{-1});$$

$$b_1 \equiv -\exp(-T_q/T_t); \quad z_q \equiv \exp(sT_q)$$

and discrete frequency characteristics $\tilde{W}_t(j\lambda)$ via own absolute pseudo-frequency $\lambda = (2/T_q)tg(\omega T_q/2) = n_q(2/T_u)tg(\lambda T_u/2)/n_q$:

$$\tilde{W}_t(j\lambda) \equiv \tilde{W}_t(j\lambda) = K_{\lambda}\tilde{\lambda}^\lambda (\tilde{\lambda}^\lambda - q_0^\lambda)/(\tilde{\lambda}^\lambda - p_0^\lambda),$$

where $\tilde{\lambda} = j\lambda; K_{\lambda}^\lambda = (1 + b_1)/(1 - b_1); q_0^\lambda = -(2/T_q)$ and $p_0^\lambda = -K_{\lambda}^\lambda(2/T_q)$. Therefore logarithmic amplitude characteristics $\text{Lm}(\lambda)$ (LAC) and logarithmic phase characteristics $\phi(\lambda)$ (LPC) of discrete filter (16) are appeared as:

$$\text{Lm}(\lambda) = 20[\text{lg} K_{\lambda}^\lambda + \text{lg}[tg(\text{arctg}(\lambda T_u/2)/n_q)]$$

$$- \text{lg}[tg(\text{arctg}(\lambda T_u/2)/n_q) + K_{\lambda}^\lambda]];$$

$$\phi(\lambda) = \text{arctg}(\lambda T_u/2)/n_q$$

$$- \text{arctg}[tg(\text{arctg}(\lambda T_u/2)/n_q)/K_{\lambda}^\lambda].$$

As an example fig. 7 presents frequency characteristics of discrete filter (16) for control period $T_u = 4\text{ s}, S_1 = 2\text{ s}$ and the period’s multiple $n_q = 4$. Applied digital control law is as follows:

$$e_k = -2e_k^i; \quad g_{k+1} = B g_k + C e_k;$$

$M_2^b = K_2^b(g_k + P e_k),$$

matrices $B, C, P$ and $K_2^b$ have diagonal form with

$$a_i = [(2/T_u) - 1]/[(2/T_u) + 1];$$

$$b_i = [(2/T_u) - 1]/[(2/T_u) + 1];$$

$$p_i = (1 - b_i)/(1 - a_i);$$

$$c_i = p_i(b_i - a_i),$$

where $T_1, T_2$ and $k_2^b$ are constant parameters which are thoroughly selected for the robust properties of gynmoment control system. Here digital information on only the SC attitude filtered error vector $e_k^i$ is applied for forming the control vector (19), moreover measured error quaternion is $E = (e_0, e_2) = \hat{A}^p(t) \circ \hat{A}_m^p$, the measured Euler parameters’ vector $E = \{e_0, e_2\}$, then filtering is executed by the relations

$$\dot{x}_{s+1} = A\dot{x}_s + B e_s;$$

$$e_s = \hat{C} x_s + \hat{D} e_s,$$
X. Conclusions

New approach for modelling a physical hysteresis was developed and its application for digital gyromoment attitude control of a flexible spacecraft structure was considered. In progress of [10] – [11] new results on the SC antenna guidance, a digital gyromoment spacecraft attitude control and a flexible antenna fine pointing, were presented.

Fig. 8. The SC open-loop pseudo-frequency characteristics on roll channel: — without discrete filter, —— with discrete filter.

Fig. 9. Transient processes by attitude angles with respect the ORF.

Fig. 10. Transient processes by angular rates.

Fig. 11. The SC antenna oscillations at sequence of the guidance modes.

Fig. 12. The SC antenna’s weak-damped oscillations at a longtime tracking.

References


