SIMPLE ADAPTIVE CONTROL - A STABLE DIRECT MODEL REFERENCE ADAPTIVE CONTROL METHODOLOGY - BRIEF SURVEY

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Abstract: In spite of successful proofs of stability and even successful demonstrations of performance, the eventual use of Model Reference Adaptive Control (MRAC) methodologies in practical real world systems has met a rather strong resistance from practitioners and has remained very limited. Apparently, the practitioners have a hard time understanding the conditions that can guarantee stable operations of adaptive control systems under realistic operational environments. Besides, it is difficult to measure the robustness of adaptive control system stability and allow it to be compared with the common and widely used measure of phase margin and gain margin that is utilized by present, mainly LTI, controllers. Furthermore, recent counterexamples seem to show that adaptive systems may diverge even when all required conditions are fulfilled. This paper attempts to revisit the fundamental qualities of the common direct model reference adaptive control methodology based on gradient and to show that some of its basic drawbacks have been addressed and eliminated within the so-called Simple Adaptive Control methodology. The sufficient conditions that guarantee stability are clearly stated and lead to similarly clear proofs of stability, and the previous counterexamples to MRAC become just simple, successful, and stable applications of SAC.

Keywords: Control systems, stability, passivity, uncertain systems, almost strict passivity (ASP), adaptive control

1. INTRODUCTION

We will consider the square, multiple-input-multiple-output, system with the realization

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

Here, x is the n-dimensional state vector, u is the m-dimensional input vector and y is the m-dimensional output vector, and A, B, and C are matrices of corresponding dimensions. Because only a nominal model of the real-world plant is usually available for the control design and, furthermore, plant parameters may vary under various operational and environmental conditions, adaptive control methodologies seemed to be the natural solution for the problem. However, the customary stationary controllers have developed verification techniques that allow the designer to evaluate the stability robustness of the controlled system. Even if the actual gain deviates from the nominal control gains K_{nom}, the designer has a very clear idea about the admissible domain $K_{min} < K < K_{MAX}$ of possible deviation of the actual *constant* gain K that would not result in destruction of system stability. The adaptive control methodologies, as any other control techniques that use nonstationary controllers, cannot *guarantee* stability if the stationary gain K is replaced by an arbitrary, nonstationary, gain $K(t)$ even if it follows the same condition

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$K_{min} < K(t) < K_{MAX}$. Therefore, new rules and specialized methods for the proofs of stability and measure of its robustness had to be developed. Section 2 gives a short review of classical Model Reference Adaptive Control (MRAC) and its many contributions to stability analysis, along with some implied drawbacks related to the so-called "unmodeled dynamics" and to the need for "sufficient excitation" that have prevented MRAC from being used in many real-world applications in spite of its qualities. Section 3 presents the various passivity and "almost passivity" conditions that are used to proof stability with adaptive controllers. Section 4 mentions some counterexamples that diverge under MRAC, and therefore seem to demonstrate that previous assumptions may not be sufficient to guarantee stability with MRAC. Section 5 presents the Simple Adaptive Control (SAC) methodology which initially was developed as a simplified and modest alternative to MRAC, yet ultimately seems to eliminate the drawbacks related to MRAC. Section 6 shows that the almost passivity conditions are indeed sufficient to guarantee robust stability with SAC. Because the control community at large tries to avoid using adaptive controllers, the example of Section 7 illustrates the danger involved with the belief that LTI system theory and its safety gain and phase margin can be used in real-world changing environments. Section 8 then shows how basic stabilizability properties of plants and parallel feedforward can be used so SAC can be applied with systems that do not inherently satisfy the basic "almost passivity" conditions. Finally, Section 9 then shows how a simple σ-term can be used to add the necessary robustness so SAC can be applied in those situations where perfect tracking is not possible.

2. MODEL REFERENCE ADAPTIVE CONTROL

First attempts at using adaptive control techniques were developed during the sixties and were based on intuitive and even ingenious ideas (Whitaker, 1959), (Osborn, Whitaker and Kezer, 1961), yet they ended in failure, mainly because at the time there was not very much knowledge of stability analysis with nonstationary parameters. Modern methods of stability analysis that had been developed by Lyapunov at the start of the 19th century were not broadly known, much less used, in the West (Hahn, 1967). After the initial problems with adaptive control techniques of the sixties, stability analysis has become a center point in new developments related to adaptive control. Participation of some of the leading researchers in the control community at the time, such as Narendra, Landau, Åström, Kokotovič, Goodwin, Morse, Grimble and many others, added a remarkable contribution to the better modeling and to the understanding of adaptive control methodologies (Monopoli, 1974), (vanAmerongen and TenCate, 1975), (Feuer and Morse, 1978), (Morse, 1980), (Landau, 1974), (Landau, 1979), (Narendra and Valavani, 1978), (Narendra and Valavani, 1979), (Narendra, Lin and Valavani, 1980), (Narendra and Annaswami, 1989), (Goodwin, Ramadge and Caines, 1980), (Goodwin and Sin, 1984), (Astrom, 1983), (Astrom and Wittenmark, 1989), (Ioannou and Kokotovic, 1983), (Moir and Grimble, 1984), (Mareels, 1984), (Kreiselmayer and Anderson, 1986), (Ortega and Yu, 1987), (Sastri and Bodson, 1989), (Ioannou and Sun, 1996), (Bitmead, Gevers and Wertz, 1990), (Wellstead and Zarrop, 1991), (Krstic, Kanellakopoulos and Kokotovic, 1995). New tools and techniques have been developed and used and they finally led to successful proofs of stability, mainly based on the Lyapunov stability approach. The standard methodology was the Model Reference Adaptive Control approach which, as its name states, basically requires the possibly "bad" plant to follow the behavior of a "good" Model Reference.

$$\dot{x_m}(t) = A_m x_m(t) + B_m u_m(t) \tag{3}$$

$$y_m(t) = C_m x_m(t) \tag{4}$$

The control signal that feeds the plant is a linear combination of the Model state variables

$$u(t) = \sum k_i x_{mi}(t) = K x_m(t) \tag{5}$$

If the plant parameters were fully known, one could compute the corresponding controller gains that would force the plant to asymptotically follow the Model, or

$$x(t) \rightarrow x_m(t) \tag{6}$$

and correspondingly

$$y(t) \rightarrow y_m(t) \tag{7}$$

Because the entire plant state ultimately behaves exactly as the model state, MRAC is sometimes interpreted as Pole-Zero placing. However, in this report we only relate to MRAC in relation to its main aim, namely, the plant output should follow the desired behavior represented by the model output.

When the plant parameters are not (entirely) known, one is naturally lead to use adaptive control gains. The basic idea is that the plant is fed a control signal that is a linear combination of the model state through some gains. If all gains are correct, the entire plant state vector

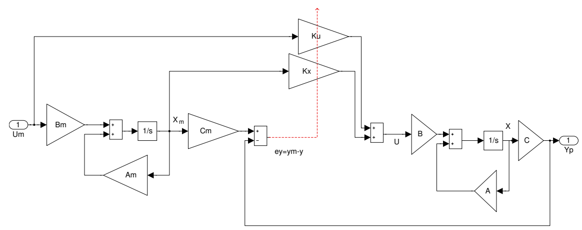

Fig. 1. Schematic representation of MRAC

would follow the model exactly. If, however, not all gains are correct, the measured plant output differs from the output of the Model Reference. The resulting "tracking error"

$$e_y(t) = y_m(t) - y(t) \tag{8}$$

can be monitored and used to generate adaptive gains. The basic idea of the adaptation is like that: assume that one component of the control signal that is fed to the plant is coming from the variable x_{mi} through the gain k_{xi}. If the gain is not perfectly correct, this component contributes to the tracking error and therefore the tracking error and the component x_{mi} are correlated. This correlation is used to generate the adaptive gain

$$\dot{k}_{xi}(t) = \gamma_i e_y(t) x_{mi}(t) \tag{9}$$

where γ_i is a parameter that affects the rate of adaptation. The adaptation should continues until the correlation diminishes and ultimately vanishes and therefore the gain derivative tends to zero and the gain itself is (hopefully) supposed to ultimately reach a constant value. In vectorial form,

$$\dot{K}_x(t) = \Sigma \gamma_i e_y(t) x_{mi}(t) = e_y(t) x_m^T(t) \Gamma_x \tag{10}$$

$$u(t) = \Sigma k_{xi} x_{mi}(t) = K_x(t) x_m(t) \tag{11}$$

As Figure 1 below shows, there are various other components that can be added to improve the performance of the MRAC system such as

$$\dot{K}_u(t) = e_y(t) u_m^T(t) \Gamma_u \tag{12}$$

so the total control signal is

$$u(t) = K_x(t) x_m(t) + K_u(t) u_m(t) \tag{13}$$

Many other elements, such as adaptive observers, etc., can be added to this basic MRAC scheme and can be found in the reference cited above, yet here we want to pursue just the basic Model Reference idea.

This approach was able to generate some rigorous proofs of stability that showed that not only the tracking error but even the entire state error

$$e_x(t) = x_m(t) - x(t) \tag{14}$$

asymptotically vanishes. This result implied that the plant behavior would asymptotically reproduce the stable model behavior and would ultimately achieve the desired performance represented by the ideal Model Reference. In particular, the Lyapunov stability technique revealed the prior conditions that had to be satisfied in order to guarantee stability and allowed getting rigorous proofs of stability of the adaptive control system. Because along with the dynamics of the state or the state error, adaptive control systems have also introduced the adaptive gains dynamics, the positive definite quadratic Lyapunov function had to contain both the errors and the adaptive gains and usually had the form

$$\begin{aligned} V(t) &= e_x^T(t) P e_x(t) \\ &+ tr\left[\left(K(t) - \widetilde{K}\right)\Gamma^{-1}\left(K(t) - \widetilde{K}\right)^T\right] \end{aligned} \tag{15}$$

Here, $\widetilde{K}$ is a set of the ideal gains that could perform perfect model following if the parameters were known, and that the adaptive control gains were supposed to asymptotically reach. Yet, in spite of successful proofs of stability, very little use has been made of adaptive control techniques in practice.

Therefore, we will first discuss some of the problems that are inherent to the classical MRAC approach and that are emphasized when one intends to use adaptive methods with such applications as large flexible space structures and similar large scale systems. First, the fact that the entire plant state vector is supposed to follow the behavior of the model state vector immediately implies that the model is basically supposed to be of the same order as the plant. If this is not the case, various problems have been shown to appear, including total instability. As real world plants are usually of very high order when compared with the nominal plant model, a so-called "unmodeled dynamics" must inherently be considered in the context of this approach. The developers of adaptive control techniques were able to show that the adaptive system still demonstrates stability robustness in spite of the "unmodeled dynamics," yet to this end they required that the "unmodeled dynamics" be "sufficiently small." Furthermore, if any state variable of the Model reference is zero, the corresponding adaptive gain is also zero. Also, if the model reaches a steady state, some of the various adaptive gains loose their independence, and this point raises the need for some "persistent excitation" or "sufficient excitation." It should be emphasized that the need for sufficiently large Models, sufficiently small "unmodeled dynamics" and "sufficient excitation" appear even if one only intends to guarantee the mere stability of the plant, before even mentioning performance. Finally, when all these basic conditions are satisfied, the stability of the adaptive control could initially be proved only if the original plant was Strictly Passive (SP), which in LTI systems implies that its input-output transfer function is Strictly Positive Real (SPR). Passivity-like conditions appear in various forms in different presentations, so they deserve a special section.

3. ON VARIOUS PASSIVITY CONDITIONS REQUIRED IN ADAPTIVE CONTROL

Positive Realness or, more precise, Passivity of systems is a useful systems property that has been first introduced in networks (Cauer, 1958), and probably introduced to dynamic systems by Kalman (Kalman, 1964) in the context of optimality. It has also shown its usefulness in the context of "absolute stability" (Popov, 1962). As we already mentioned, Positive Realness has also been shown to be useful for the proof of stability with adaptive controllers. Here, we present the state-space representation of the SPR conditions which seems to be the most useful for successful proofs of stability using Lyapunov stability theory.

Definition 1. A linear time-invariant system with a state-space realization {A, B, C}, where $A \in R^{n*n}$, $B \in R^{n*m}$, $C \in R^{m*n}$, with the m*m transfer function $T(s) = C(sI - A)^{-1}B$, is called "strictly passive (SP)" and its transfer function "strictly positive real (SPR)" if there exist two positive definite symmetric (PDS) matrices, P and Q, such that the following two relations are simultaneously satisfied:

$$PA + A^T P = -Q \tag{16}$$

$$PB = C^T \tag{17}$$

The relation between the strict passivity conditions (16)-(17) and the strict positive realness of the corresponding transfer function has been treated elsewhere (Ioannou and Tao, 1987), (Wen, 1988). Relation (16) is the common algebraic Lyapunov equation and shows that an SPR system is asymptotically stable. One can also show that conditions (16)-(17) also imply that the system is strictly minimum-phase, yet simultaneous satisfaction of both conditions (16)-(17) is far from being guaranteed even in stable and minimum-phase systems, and therefore the SPR condition seemed much too demanding. (Indeed, some colleagues in the general control community use to ask: if the system is already asymptotically stable and minimum-phase, why would one need adaptive controllers?)

For a long time, the passivity condition had been considered very restrictive (and rather obscure) and at some point the adaptive control community has been trying to drop it and to do without it. The passivity condition has been somewhat mitigated when it was shown that stability with adaptive controllers could be guaranteed even for the non-SPR system (1)-(2) if there exists a constant output feedback gain (unknown and not needed for implementation), such that a fictitious closed-loop system with the system matrix

$$A_K = A - B\widetilde{K}_e C \tag{18}$$

is SPR, namely, it satisfy the passivity conditions (16)-(17). Because in this case the original system (1)-(2) was only separated by a simple constant output feedback from strict passivity, it was called "Almost Strictly Positive Real (ASPR)" or "Almost Strictly Passive (ASP)" (Barkana and Kaufman, 1985), (Barkana, 1987). Note that such ASP systems are sometimes called (Fradkov, 2003), (FradkovHill, 1998) "feedback passive" or "passifiable." However, as we will show that any stabilizable system is also passifiable via parallel feedforward, those systems that are only at the distance of a constant feedback gain from Strict Passivity deserve a special name.

At the time, this "mitigation" of the passivity conditions did not make a great impression, because it was still not clear what systems would satisfy the new conditions. (Some even claimed that if SPR seemed to be another name for the void class of systems, the "new" class of ASPR was only adding the boundary.) Nonetheless, some ideas were available. Because a constant output gain feedback was supposed to stabilize the system, it seemed apparent that the original plant was not required to be stable. Also, because it was known that SPR systems were minimum-phase and that the product CB is Positive Definite Symmetric (PDS), it was intuitive to assume that minimum-phase systems with Positive Definite Symmetric CB were natural ASPR candidates (Barkana and Kaufman, 1985). Indeed, simple Root-locus techniques were sufficient to proof this result in SISO systems, and many examples of minimum-phase MIMO systems with CB product PDS were shown to be ASPR (Barkana and Kaufman, 1985), (Barkana, 1987). However, it was not clear how many of such MIMO system actually were ASPR. Because the ASPR property can be stated as a simple condition and because it is the main condition needed to guarantee stability with adaptive controllers, it is useful to present here the ASPR theorem for the general multi-input-multi-output systems:

Theorem 1. Any linear time-invariant system with the state-space realization {A, B, C}, where $A \in R^{n*n}$ $B \in R^{n*m}$,$C \in R^{m*n}$, with the m*m transfer function $T(s) = C(sI - A)^{-1}B$, that is minimum-phase and where the matrical product CB is PDS, is "almost strictly passive (ASP)" and its transfer function "almost strictly positive real (ASPR)."

Although the original plant is not SPR, a (fictitious) closed-loop system satisfies the SPR conditions, or in other words, there exist two positive definite symmetric (PDS) matrices, P and Q, and a positive definite gain such that the following two relations are simultaneously satisfied:

$$P(A - B\widetilde{K}_e C) + (A - B\widetilde{K}_e C)^T P = -Q \quad (19)$$

$$PB = C^T \quad (20)$$

As a matter of fact, a proof of Theorem 1 had been available in the Russian literature (Fradkov, 1976) since 1976 yet it was not known in the West. Here, many other works have later independently rediscovered, reformulated, and further developed the idea (see (Barkana, 2004) and references therein for a brief history and for a simple and direct, algebraic, proof of this important statement). Even as late as 1999, this simple ASPR condition was still presented as some algebraic condition (Huang *et al.*, 1999) that might look obscure to the control practitioner. On the other hand, (Huang *et al.*, 1999) managed to add an important contribution and emphasize the special property of ASPR systems by proving that if a system cannot be made SPR via constant output feedback, no dynamic feedback can render it SPR.

Theorem 1 has thus managed to explain the rather obscure passivity conditions with the help of new conditions that could be understood by control practitioners. It is useful to notice an important property that may makes an ASPR system to be a good candidate for stable adaptive control: if a plant is minimum-phase and its input-output matrical product CB is Positive Definite Symmetric (PDS) it is stabilizable via some static Positive Definite (PD) output feedback. Furthermore, if the output feedback gain is increased beyond some minimal value, the system remains stable even if the gain increase is nonstationary. The required positivity of the product CB could be expected, as it seemed to be a generalization of the sign of the transfer function that allows using negative feedback in SISO systems. However, although at the time it seemed to be absolutely necessary for the ASPR conditions, the required CB symmetry proved to be rather difficult to fulfill in practice, in particular in adaptive control systems where the plant parameters are not known.

After many attempts that have ended in failure, a recent publication has managed to eliminate the need for a symmetric CB. First, it was easy to observe that the Lyapunov function remains positive definite if the gain term is rewritten as follows:

$$V(t) = e_x^T(t) P e_x(t) + tr\left[S\left(K(t) - \widetilde{K}\right)\Gamma^{-1}\left(K(t) - \widetilde{K}\right)^T S^T\right] \quad (21)$$

Here, S is any nonsingular matrix. This new formulation allowed the extension of useful passivity-like properties to a new class of systems that was called W-ASPR, where $W = S^T S$, through the following definition:

Definition 2. Any linear time-invariant system with state-space realization {A, B, C}, where $A \in R^{n*n}$, $B \in R^{n*m}$, $C \in R^{m*n}$, with the m*m transfer function $T(s) = C(sI - A)^{-1}B$, is called "W-almost strictly passive (WASP)" and its transfer function "W-almost strictly positive real (WASPR)," if there exist three positive definite symmetric (PDS) matrices, P, Q, and W, and a positive definite gain $\widetilde{K}_e$ such that the following two conditions are simultaneously satisfied:

$$P(A - B\widetilde{K}_e C) + (A - B\widetilde{K}_e C)^T P = -Q \quad (22)$$

$$PB = C^T W^T \quad (23)$$

This new definition can be used with the following theorem (Barkana, Teixeira and Hsu, 2006):

Theorem 2. Any minimum-phase LTI system with a state-space realization {A, B, C}, where $A \in R^{n*n}$, $B \in R^{n*m}$, $C \in R^{m*n}$, with the m*m transfer function $T(s) = C(sI - A)^{-1}B$, where the positive definite and not necessarily symmetric matrical product CB is diagonalizable is WASP in accord with Definition 2.

Thus, (Barkana, Teixeira and Hsu, 2006) had managed to mitigate a result that has been around for more than 40 years. Nevertheless, it was very tempting to try to eliminate any restriction besides the positivity of CB. This new result was "almost" made possible by observing that, although the product of two PD matrices is not necessarily PD, the trace of the product is PD if at least one of the two matrices is PDS. Therefore, the

Lyapunov function remains positive definite if the second term in it is again rewritten as follows:

$$V(t) = e_x^T(t)Pe_x(t) + tr\left[W\left(K(t)-\widetilde{K}\right)\Gamma^{-1}\left(K(t)-\widetilde{K}\right)^T\right] \quad (24)$$

even if W is only positive definite yet not necessarily symmetric. However, we will show that in order to allow the Lyapunov derivative to be negative definite or semidefinite, the proof of stability does require the symmetry of W.

As we showed above, new developments have simplified the (sufficient?) conditions that, along with limits on the "unmodeled dynamics" and with "sufficient excitation," would be sufficient to allow rigorous and successful proofs of stability with adaptive controllers. Still, it appears that the ASPR condition was not sufficient to always guarantee the stability of MRAC. Besides, although successful proofs of stability usually ended showing that the following errors vanish asymptotically, it is rather commonly accepted that the adaptive gains may not converge to any specific limit at all, even if they are guaranteed to be bounded. Furthermore, some recent counterexamples seem to show that MRAC systems may diverge even when all previously assumed sufficient conditions are satisfied (Hsu and Costa, 1999).

4. COUNTEREXAMPLES TO MODEL REFERENCE ADAPTIVE CONTROL

In the examples of (Hsu and Costa, 1999), a 2*2 stable plant with CB positive definite is required to follow the behavior of a stable model of same order. In fact both the plant and the model have the same diagonal system matrices with negative eigenvalues, and only the input-output matrix differentiates between the two. The plant, that appears in a 2D adaptive robotic visual servoing with uncalibrated camera, is defined by the system matrices

$$A = \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix}; \quad B = \begin{bmatrix} cos\varphi & sin\varphi \\ -hsin\varphi & hcos\varphi \end{bmatrix} \quad (25)$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (26)$$

It is shown (Hsu and Costa, 1999) that standard MRAC systems become unstable even though the MRAC system was supposed to be stable because there was no "unmodeled dynamics," there was "sufficient excitation," and the assumably "sufficient" passivity conditions were also satisfied. We note that (Hsu and Costa, 1999) shows ways to avoid the problem and, using various kinds of prior knowledge, other solutions have also been proposed.

5. SIMPLE ADAPTIVE CONTROL (SAC), OR THE SIMPLIFIED APPROACH TO MODEL REFERENCE ADAPTIVE CONTROL

Various kinds of additional prior knowledge have been used and many solutions and additions have been proposed to overcome some of the various drawbacks of the basic MRAC algorithm. However, this paper sticks to the very basic idea of Model Following. Next sections will show that those basically ingenious adaptive control ideas and the systematic stability analysis they introduced had finally led to adaptive control systems that can guarantee stability robustness along with superior performance when compared with alternative, non-adaptive, methodologies. In this section we will first assume that at least one of the passivity conditions presented above holds and will deal with a particular methodology that managed to eliminate the need for the plant order and therefore can mitigate the problems related to "unmodeled dynamics" and "persistent excitation." Subsequent sections will then extend the feasibility of the methodology to those real-world systems that do not inherently satisfy the passivity conditions.

The beginning of the alternative adaptive control approach can be found in the intense activities at Rensselaer (RPI) during 1978-1983. At that time, such researchers as Kaufman, Sobel, Barkana, Balas, Wen, and others (Sobel, Kaufman and Mabus, 1982), (Kaufman *et al.*, 1981), (Barkana and Kaufman, 1982), (Barkana, Kaufman and Balas, 1983), (Barkana, 1983), (Wen and Balas, 1989) were trying to use customary adaptive control techniques with large order MIMO systems, such as planes, large flexible structures, etc. It did not take long to realize that it was impossible to even think of controllers of the same order as the plant, or even of the order of a "nominal" plant. Besides, those were inherently MIMO systems, while customary MRAC techniques at the time were only dealing with SISO systems. Because now the very reduced-order model could not be considered to be even close to the plant, one could not consider full model state following, so this aim was naturally replaced by output model following. Furthermore, as the (possibly unstable) large-order plant state could not be compared with the reduced-order model state, the model could not be thought to guarantee asymptotic stability of the plant any longer.

In order to allow stability of the reduced order adaptive control system, new adaptive control components that were not deemed to be needed by the customary MRAC had to be considered. We will show that this "small" addition had an astonishing effect towards the successful application of the modified MRAC. In brief, as it was known that stability of adaptive control systems required that the plant be stabilizable via a constant gain feedback, the natural question was why not using this direct output feedback.

Following this idea, an additional adaptive output feedback term was added to the adaptive algorithm that otherwise is very similar to the usual MRAC algorithms, namely,

$$\begin{aligned} u(t) &= K_e e_y(t) + K_x x_m(t) + K_u u_m(t) \\ &= K(t) r(t) \end{aligned} \tag{27}$$

where we denote the reference vector

$$K(t) = \left[K_e(t) \;\; K_x(t) \;\; K_u(t) \right] \tag{28}$$

Subsequently in this paper, it will be shown that the new approach uses the model as a Command Generator and therefore it is sometime called Adaptive Command Generator Tracker. Because it also uses low-order models and controllers, it was ultimately called Simple Adaptive Control (SAC). Before we discuss the differences between the new SAC approach and to adaptive control classical MRAC, it is useful to first dwell over the special role of the direct output feedback term. If the plant parameters were known, one could choose an appropriate gain $\widetilde{K}_e$ and stabilize the plant via constant output feedback control

$$u(t) = -\widetilde{K}_e y(t) \tag{29}$$

As we already mentioned above, it was known that an ASPR system (or, as we now know, a minimum-phase plant with appropriate CB product) could be stabilized by a positive definite output feedback gain. Furthermore, it was known that ASPR systems are high-gain stable, so stability of the plant is maintained if the gain value happens to go arbitrarily high beyond some minimal value. Whenever one may have sufficient prior knowledge to assume that the plant is ASPR, yet does not have sufficient knowledge to choose a good control gain, one can use the output itself to generate the adaptive gain by the rule:

$$\dot{K}_y(t) = y(t) y^T(t) \Gamma_y \tag{30}$$

and the control

$$u(t) = K_y(t) y(t) \tag{31}$$

In the more general case when the plant is required to follow the output of the model, one would use the tracking error to generate the adaptive gain

$$\dot{K}_e(t) = e_y(t) e_y^T(t) \Gamma_e \tag{32}$$

and the control

$$u(t) = K_e(t) e_y(t) \tag{33}$$

We will show how this adaptive gain addition is able to avoid some of the most difficult inherent problems related to the standard MRAC and to add robustness to its stability. Although it was developed as a natural compensation for the low-order models and was successfully applied at Rensselaer as just one element of the Simple (Model Reference) Adaptive Control methodology, it is worth mentioning that, similarly to the first proof of the ASPR property, the origins of this specific adaptive gain can again be found in an early Fradkov's work (Fradkov, 1976) in the Russian literature. Besides, later on this gain received a second birth and became very popular after 1983 in the context of adaptive control "when the sign of high-frequency gain is unknown." In this context (Nussbaum, 1983), (Morse, 1984), (Heyman, Lewis and Meyer, 1985) and after a very rigorous mathematical treatment (Byrnes and Willems, 1984), it also received a new name and it is sometimes called the Byrnes-Willems gain. Its useful properties have been thoroughly researched and some may even call this one adaptive gain Simple Adaptive Control as they were apparently able to show that it can do "almost" everything (Ilchman, Owens and Pratzel-Wolters, 1987), (Mareels and Polderman, 1996). Indeed, if an ASPR system is high-gain stable, it seems very attractive to let the adaptive gain increase to even very high values in order to achieve good performance that is represented by small tracking errors. However, although at first thought one may find that high gains are very attractive, a second thought and some more engineering experience with the real world applications make it clear that high gains may lead to saturations and may excite vibrations and other disturbances. These disturbances may not have appeared in the nominal plant model that was used for design and may not be felt in the real-world plant *unless* one uses those very high gains. Furthermore, as the motor or the plant dynamics would always require an input signal in order to keep moving and tracking the desired trajectory, it is quite clear that the tracking error cannot be zero or very small unless one uses very high gains indeed. Designers of tracking systems know that feedforward signals that come from the desired trajectory can help achieving low-error or even perfect tracking without requiring the use of dangerously high gains (and, correspondingly, exceedingly high bandwidth) in the closed loop. In the non-adaptive world, feedforward could be problematic because unlike the feedback loop, any errors in the feedforward parameters are directly and entirely transmitted to the output tracking error. Here, the adaptive control methodology can demonstrate an important advantage on the non-adaptive techniques, because the feedforward parameters are finely tuned by the very tracking er-

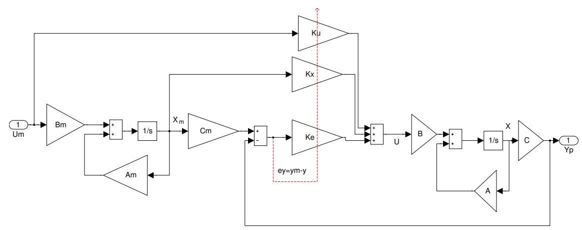

Fig. 2. Schematic representation of SAC

ror they intend to minimize. The issues discussed here and the need for feedforward again seem to show the intrinsic importance of the basic Model Following idea, and again point to the need for a model. However, the difference between the model used by SAC and the Model Reference used by the standard MRAC is that this time the so-called "Model" does not necessarily have to reproduce the plant besides incorporating the desired input-output behavior of the plant. At the extreme, it could be just a first-order pole that performs a reasonable step-response, or otherwise a higher order system, just sufficiently high to generate the desired trajectory. As it generates the command, this "model" can also be called "Command Generator" (Brousard and Berry, 1978) and the corresponding technique "Command Generator Tracker (CGT)." In summary, the adaptive control system monitors all available data, namely, the tracking error, the model states and the model input command and uses them to generate the adaptive control signal (Figure 2)

$$\dot{K}_e(t) = e_y(t) e_y^T(t) \Gamma_e \tag{34}$$

$$\dot{K}_x(t) = e_y(t) x_m^T(t) \Gamma_x \tag{35}$$

$$\dot{K}_u(t) = e_y(t) u_m^T(t) \Gamma_u \tag{36}$$

that using the concise notations (27)-(28) gives

$$\dot{K}(t) = e_y(t) r^T(t) \Gamma \tag{37}$$

and the control

$$u(t) = K(t) r(t) \tag{38}$$

It is worth noting that, initially, SAC seemed to be a very modest alternative to MRAC with apparently very modest aims and that also seemed to be very restricted by new conditions. Although at the time it probably was the only adaptive technique that could have been used in MIMO systems and with such large systems as large flexible structures, and therefore was quite immediately adopted by many researchers and practitioners, the SAC approach got a cold reception and for a long time has been largely ignored by the mainstream adaptive control. In retrospective (besides some lack of good selling) at the time this cold reception had some good reasons. Although it was called "simple" as it was quite simple to implement, the theory around SAC was not simple and many tools that were needed to support its qualities and that, slowly and certainly, revealed themselves over the year, were still missing. It subsequently not only required developing new analysis tools but also, probably even more important, better expertise at understanding their implications before they could be properly used so that they ultimately managed to highlight the very useful properties of SAC. Finally, based on developments that had spanned over more than 25 years, we will attempt to show that SAC is in fact the stable MRAC, because right from the beginning it avoids some difficulties that are inherent in the standard MRAC. First, it is useful to notice that because there is no attempt at comparison between the order or the states of the plant and the model, there is no "unmodeled dynamics." Also, because basically the stability of the system rests on the direct output feedback adaptive gain, the model is immaterial in this context and of course there is no need to mention "sufficient excitation." Besides, as we will later show and as it was observed by almost all practitioners that have tried to use it, SAC proved to be good control. While the standard MRAC may have to explain why it does not work when it is supposed to work, SAC may have to explain why it does work even in cases when the (*sufficient*) conditions are not fully satisfied. Although, similarly to any nonstationary control, in Adaptive Control it is very difficult to find the very minimal conditions that would keep the system stable, it can be shown why SAC may demonstrate some robustness even when the basic sufficient conditions are not satisfied. We note that this last point is just an observation based on experience, yet we must also note that in those cases when the basic conditions are fulfilled, they are always sufficient to guarantee the stability of the adaptive control system, with no exceptions and no counterexamples. In this respect, one can show that the MRAC "counterexamples" become just trivial, stable, and well behaving examples for SAC.

6. PROOF OF STABILITY OF SIMPLE ADAPTIVE CONTROL

One can easily prove that the WASP conditions are sufficient to prove stability using just the simple adaptive output feedback gain (32) (Barkana, Teixeira and Hsu, 2006). However, in order to avoid any misunderstandings related to the role of the unknown matrix W, here we chose to present a rigorous proof of stability for the general output model tracking case. As usual in adaptive control, one first assumes that the underlying fully deterministic output model tracking problem is solvable. A recent publication (Barkana, 2005a) shows that if the Model Reference uses a step input in order to generate the desired trajectory, the underlying tracking problem is *always* solvable. If, instead, the model input command is itself generated by an unknown system of order n_u, the model is required to be sufficiently large to accommodate this command (Barkana, 1983), (Kaufman, Barkana and Sobel, 1998), or

$$n_m + m \geq n_u \tag{39}$$

We assume that the plant to be controlled is minimum-phase and that the CB product is Positive Definite and diagonalizable though not necessarily symmetric. As we showed, the plant is WASP according to Definition 2, so it satisfies conditions (22)-(23). Under these assumptions one can use the Lyapunov function (24). Differentiating (24) and using the W-passivity relations, finally leads to the following derivative of the Lyapunov function (Appendix A)

$$\dot{V}(t) = -e_x^T(t) Q e_x(t) \tag{40}$$

One can see that $\dot{V}(t)$ in (40) is negative definite with respect to $e_x(t)$, yet only negative semidefinite with respect to the entire state-space $\{e_x(t), K(t)\}$. A direct result of Lyapunov stability theory is that all dynamic values are bounded. According to LaSalle's Invariance Principle (Kaufman, Barkana and Sobel, 1998), *all* state-variables and adaptive gains are bounded and the system ultimately ends within the domain defined by $\dot{V}(t) \equiv 0$. Because $\dot{V}(t)$ is negative definite in $e_x(t)$, the system thus ends with $e_x(t) \equiv 0$, that in turn implies $e_y(t) \equiv 0$. In other words, the adaptive control system demonstrates asymptotic convergence of the state and output error and boundedness of the adaptive gains.

6.1 On gain convergence, basic conditions for stability, optimality, robustness, etc.

Some particularly interesting questions may arise during the proof of stability. First, although the Lyapunov function was carefully selected to contain both the state error and the adaptive gains, the derivative only contains the state error. It appears as if the successful proof of stability has "managed" to eliminate any possibly negative effect of the adaptive gains. One is then entitled to ask what *positive* role the adaptive gains play (besides *not* having negative effects). This is just one more illustration of the difficulties related to the analysis of nonlinear systems. Indeed, although Lyapunov stability theory manages to prove stability, it cannot and does not provide all answers. Besides, as potential counterexamples seem to show, although the tracking error and the derivative of the adaptive gains tend to vanish, this mere result does not necessarily imply, as one might have initially thought, that the adaptive gains would reach a constant value or even a limit at all. If the adaptive gain happens to be a function such as $k(t) = sin(ln\ t)$ (suggested to us by Mark Balas), its derivative is $\dot{k}(t) = cos(ln\ t)/t$. In this example one can see that although the derivative tends to vanish in time, the gain $k(t)$ itself does not reach any limit at all. Therefore, the common opinion that seems to be accepted among experts is that the adaptive gains do not seem to converge unless the presence of some "sufficient" excitation can be guaranteed. This seem to imply that even in the most ideal, perfect following, situations, the adaptive control gains may continue wandering for ever.

However, recent results have shown that these open questions and problems are only apparent. First, even if it is not a direct result of Lyapunov analysis, one can show that the adaptive control gains always perform a steepest descent minimization of the tracking error (Barkana, 2005a). Although this "minimum" could still increase without bound in general, if the stability of the system were not guaranteed, yet this is not the case with SAC.

Second, with respect to the final gain values, when one tests an adaptive controller with a given plant, one first assumes that an underlying LTI solution for the ideal control gains exists, and then the adaptive controller is supposed to find those gains at the end of the adaptation. If the plant is known, one can first solve the deterministic tracking problem and find the ideal control gains. Then, the designer proceeds with the implementation of the adaptive controller and expects it to converge to the pre-computed ideal solution. In practice, however, one observes that, even though the tracking errors do vanish, the adaptive gains do not seem to converge. Even in those cases when the gains do demonstrate convergence, their final values are far from the ideal solution that was computed for the LTI case, and this happens even when the LTI solution is "unique."

This point may give the practitioner pause, because if there is such an uncertainty about the adaptive gains even in the ideal situations, what can be said in more difficult situations, in the presence of noise, etc.? This question has proved to be a very serious challenge and has remained open for more than 30 years, yet recently it was finally answered. It is worth mentioning that although extensions of LaSalle's Invariance Principle for non-autonomous systems have been around for quite some time (Artstein, 1977), (LaSalle, 1981), besides very few exceptions they don't seem to be widely used in Adaptive Control or in nonlinear control systems in general. This fact could be partially explained by the fact that the results are of a very general character. However, their proper interpretation and application towards the development of new basic analysis tools such as combining a Modified Invariance Principle with Gromwall-Bellman Lemma (Barkana, 1983), (Barkana, 2005a), (Kaufman, Barkana and Sobel, 1998), finally managed to provide the solution to this problem.

It was shown that if the adaptive control gains do not reach the "unique" solution that the preliminary LTI design seemed to suggest, it is not because something was wrong with the adaptive controller, but rather because the adaptive control can go beyond the LTI design. The existence of a "general" LTI solution is useful in facilitating and shorting the proof of stability, yet it is not *needed* for the convergence of the adaptive controller. While the sought after stationary controller must provide a fixed set of constant gains that would fit *any* input commands, the adaptive controller only needs that specific set of control gains that correspond to the particular input command. Even in those cases when the general LTI solution does not exist, the particular solution that the adaptive controller needs does exist (Barkana, 2005a). However, it complicates the stability analysis because it was shown that those particular solutions may allow perfect following only after a transient that adds supplementary terms to the differential equations of motion. As a consequence, the stability analysis may end with the derivative of Lyapunov function being

$$\begin{aligned}\dot{V}(t) &= W_1(t) + W_2(t) \\ &= -e_x^T(t)Qe_x(t) + F(x,t)e^{A_m t} \qquad (41)\end{aligned}$$

Although the derivative (41) still contains the negative definite term with respect to the error state, it also contains a transient term that is not negative, so the derivative is not necessarily negative definite or even semidefinite. Apparently, (41) cannot be used for any decision on stability. However, although the decision on stability is not immediate, the Modified Invariance Principle reveals that all *bounded* solutions of the adaptive system reach asymptotically the domain defined by

$$W_1(t) = -e_x^T(t)Qe_x(t) \equiv 0 \qquad (42)$$

Therefore, one must find out what those "bounded trajectories" are and it is the role of Gromwall-Bellman Lemma to actually show that, under the WASP assumption, *all* trajectories are bounded. Therefore, the previous conclusions on asymptotically perfect tracking remain valid.

Moreover, because the gains also reach that domain in space where perfect tracking is possible, this approach has also finally provided the answer to the (previously open) question on the adaptive gain convergence. Even if one assumes that the final asymptotically perfect tracking may occur while the adaptive gains continue to wander, one can show that the assumably nonstationary gains satisfy a linear differential equitation with constant coefficients and their solution is a summation of generalized exponential functions ((Barkana, 2005a) and Appendix B). This partial conclusion immediately shows that such nonlinear "counterexample" gains as that we presented above are, maybe, nice and tough mathematical challenges, yet they cannot be solutions of, and thus are actually immaterial for, the SAC tracking problem. Furthermore, because the gains are bounded, they can only be combinations of constants and converging exponentials, so they must ultimately reach constant values. Therefore, we were finally able to show (at least within the scope of SAC) that the adaptive control gains do ultimately reach a set of stabilizing constant values at the end of a steepest descent minimization of the tracking error ((Barkana, 2005a) and Appendix B).

A recent paper (Barkana, 2007) tests SAC with a few counterexamples for the standard MRAC (Hsu and Costa, 1999). The paper shows that SAC not only maintains stability in all cases that led to instability with standard MRAC, but also demonstrates very good performance.

Many practitioners that have tried it have been impressed with the ease of implementation of SAC and with its performance even in large and complex applications. Many examples seem to show that SAC maintains its stable operation even in cases when the established *sufficient* conditions do not hold. Indeed, conditions for stability of SAC have been continuously mitigated over the years, as the two successive definitions of almost passivity conditions presented in this paper may show. In order to get another qualitative estimate on SAC robustness, assume that instead of (1)-(2) the actual plant is

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x) \qquad (43)$$

$$y(t) = Cx(t) \qquad (44)$$

Assume that the nominal $\{A, B, C\}$ system is WASP, while $f(x)$ is some (linear or nonlinear) component that prevents the satisfaction of the passivity conditions. If one uses the same Lyapunov functions (24), instead of (40) one gets for the stabilization problem

$$\dot{V}(t) = -x^T(t)Qx(t) + x^T(t)Pf(x) \qquad (45)$$

and for the tracking problem

$$\dot{V}(t) = -e_x^T(t)Qe_x(t) + e_x^T(t)P[f(x) - f(x^*)] \qquad (46)$$

where x^* is the ideal trajectory, as defined in Appendix A. Note that the derivative of the Lyapunov function remains negative definite in terms of $x(t)$ or $e_x(t)$, correspondingly, if the second term in the sum is not too large, as defined (for example) by the inequality (Torres and Mehiel, 2006)

$$||f(x)|| \leq \frac{||Q||}{||P||}||x|| \qquad (47)$$

or

$$||f(x) - f(x^*)|| \leq \frac{||Q||}{||P||}||x - x^*|| \qquad (48)$$

While until very recently the main effort has been dedicated to the clarification and relaxation of the passivity conditions, similar effort is dedicated now to clarifying the limits of robustness of SAC when the basic passivity conditions are not entirely satisfied.

Besides, although much effort has been dedicated to clarification of passivity concepts in the context of Adaptive Control of stationary continuous-time systems, similar effort has been dedicated to extending these concepts to discrete-time (Barkana, 2005c) and nonstationary and nonlinear systems (Barkana, 2005d), (Bobtsov and Nagovitsina, 2007).

7. ON “SAFE” GAIN SCHEDULING VS. LACK OF STABILITY NORMS FOR ADAPTIVE CONTROL

Although the use of nonstationary gains that would adapt to the specific and momentary needs of the control systems is attractive, the somewhat uncertain conditions that would guarantee stability, and in particular the lack of verifiable norms for the stability robustness of adaptive control systems has kept their use in real-world application very low. On the other hand, the stability robustness of linear time invariant (LTI) controllers in an LTI world could always be measured in such customary practical terms as gain-margin and phase margin. Even in a changing world, when it is known that the plant parameters could change as a function of internal or external

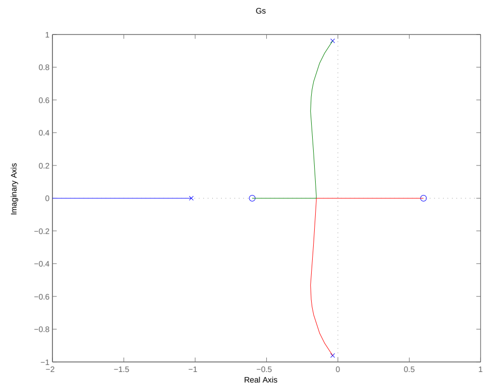

Fig. 3. Plant Root Locus

environmental and operational conditions, at the present time most designers would prefer using constant control gains and, if forced to, gain-scheduling as a function of some measure of the changing operational conditions. Although it is, or should be, quite clear to the designer that along with switching from one gain to another one looses the absolute estimator of stability given by gain margin and phase margin, experience and, probably even more, tradition, keep people preferring, using, and trusting those basically LTI design methodologies. In order to show that, along with improved performance, adaptive control can add to, not destroy, the stability robustness of systems, we here consider a simple plant, non-minimum phase yet otherwise open-loop stable system, a model that fits some missiles and other examples. The transfer function is

$$G(s) = \frac{s^2 - 0.36}{s^3 + 1.1s^2 + s + 0.95} \qquad (49)$$

Although the open loop is theoretically stable (Figure 3), one wants to use feedback gains in order to improve the stability and performance of the system. To this end, one first checks and finds out that in a given environment, the closed-loop system remains stable for any positive constant gain less than 2.64. To make sure the system is far from instability, one decides to use the constant gain k=1.3. (One knows that the environmental conditions may change, and when they do change one would switch to a different gain). Even if one knows that in the present conditions the actual control gain might vary around the nominal value of 1.3, as long the monitored conditions show that the gain stays within the “admissible” range 0 - 2.64, there seems to be no reason to worry.

Many tests using constant and even various variable gains indeed seem to show that the closed loop system maintains its stability. Even when the missile maneuvering leads to sinusoidal time-variation around the mid-range value of 1.3, namely $k(t) = 1.3 + asint$, where the parameter ’a’ is constant, the system shows stability and

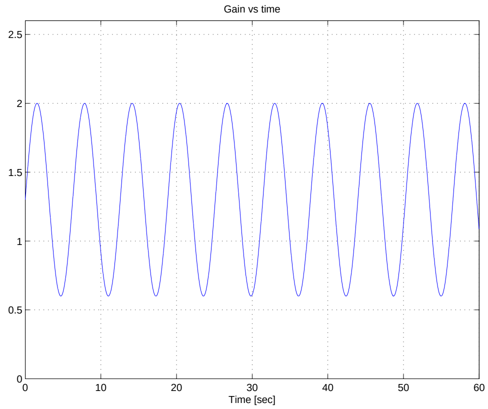

Fig. 4. Sinusoidal Gain with a=0.7

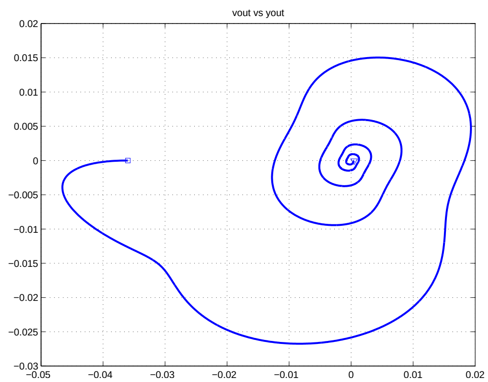

Fig. 5. Position-Velocity with a=0.7

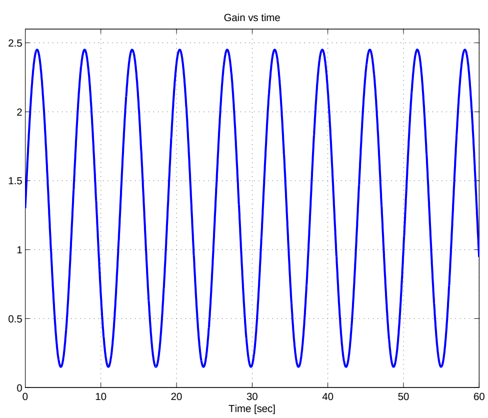

Fig. 6. Sinusoidal Gain with a=1.15

satisfactory behavior even for $a = 0.7$ (Figures 4-5).

However, when the sinusoidal variation reached $a = 1.15$ (Figures 6-7), the non-stationary gain leads to total divergence although the gain remains within the so-called "admissible" range!

In spite of the example above, even if one may not like the lack of robustness with stationary controllers in changing environments, Adaptive Control methods do not seem to be an alternative because the plant used with our example is non-minimum-phase, fact that would make Adaptive Control methodologies unusable. As we show below, a solution for plants that do not inher-

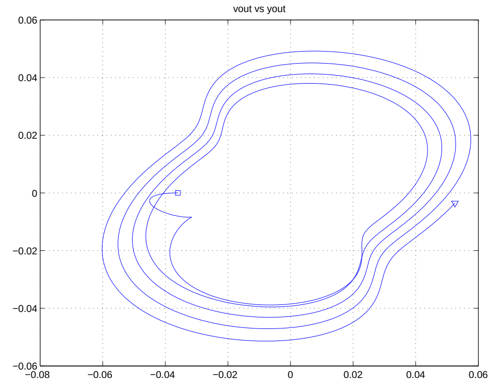

Fig. 7. Position-Velocity with a=1.15

ently satisfy the ASPR conditions, including non-minimum phase, has been available for quite a while, so we will now present the Parallel Feedforward methodology that can be used to allow applicability of Adaptive Control with almost all real-world systems.

8. PARALLEL FEEDFORWARD AND STABILITY OF SIMPLE ADAPTIVE CONTROL

Using for illustration the example of Section VIII, assume that $K_{MAX} = 2.5$ is an estimate of the highest admissible constant gain that maintains stability of the system. One would never use this value because it would not be a good control gain value. Indeed, we only use the mere *knowledge* that a (fictitious) closed-loop system using the high gain value of 2.5 would still be stable. Instead of implementing constant output feedback we use this knowledge in order to augment the system with a simple Parallel Feedforward Configuration (PFC) across the plant. If the original plant has transfer function

$$G(s) = \frac{B(s)}{A(s)} \tag{50}$$

the closed-loop system would be

$$G_{CL}(s) = \frac{B(s)}{A(s) + KB(s)} \tag{51}$$

and would be asymptotically stable. The augmented system using the inverse of the stabilizing gain $D = \frac{1}{K}$ is

$$G_a(s) = \frac{B(s)}{A(s)} + \frac{1}{K} \tag{52}$$

or

$$G_a(s) = \frac{A(s) + KB(s)}{KA(s)} \tag{53}$$

and if the closed-look system would be stable, one can see that the augmented system is minimum-phase (Figure 8). Note that although we would

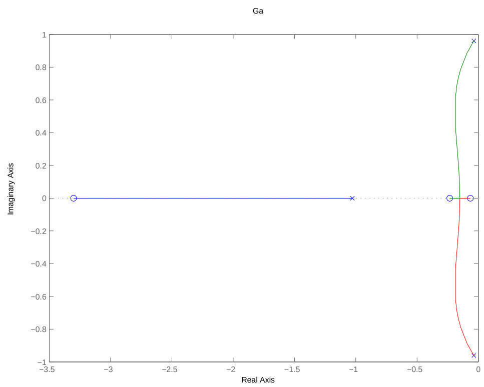

Fig. 8. Augmented Plant Root Locus

never suggest using direct input-output gains in parallel with the plant, this is a simple and useful illustration that may facilitate the understanding of the basic idea. Also, although in this paper we only dealt (and will continue to deal) with strictly causal systems, for this specific case it is useful to recall that a minimum-phase plant with relative degree 0 (zero) is also ASPR. As (53) shows, one could use the inverse of any stabilizing gain in order to get ASPR configurations. However, any such addition is added ballast to the original plant output, so using the inverse of the maximal allowed gain adds the minimal possible alteration to the plant output. The augmented system looks as follows: The augmented system has three poles and three zeros and all zeros are minimum-phase. Such a system cannot become unstable, no matter how large the *constant* gain k becomes, yet because it is ASPR one can also show that it would also stay stable no matter how large the *nonstationary* adaptive gain $k(t)$ becomes. One can easily see that the parallel feedforward has made the effective control gain that affects the plant to be:

$$k_{eff}(t) = \frac{k(t)}{1 + Dk(t)} = \frac{k(t)}{1 + \frac{k(t)}{K_{MAX}}} = \frac{1}{\frac{1}{k(t)} + \frac{1}{K_{MAX}}} \tag{54}$$

One can see that the effective gain is always below the maximal admissible constant gain (Figure 9). While this qualitative demonstration intends to provide some intuition to the designer that is used to estimate stability in terms of gain and phase margins, rigorous proofs of stability using the Lyapunov-LaSalle techniques and almost passivity conditions are also available and provide the necessary rigorous proof of stability. As we already mentioned above, the constant parallel feedforward has only been presented here for a first intuitive illustration. In practice, however, one does not want to use direct input-output across the plant that would require solving implicit loops that include the adaptive gain com-

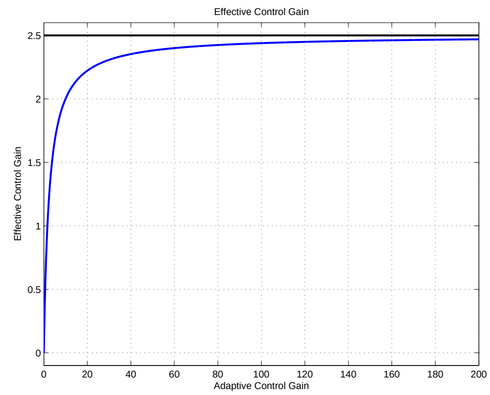

Fig. 9. Effective Gain

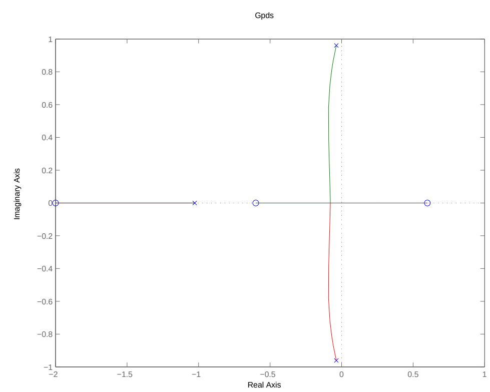

Fig. 10. Plant with PD Controller

putations. Therefore, we go to the next step that takes us to the ubiquitous PD controllers. In practice, many control systems use some form of PD controller, along with other additions that may be needed to improve performance. While the additions are needed to achieve the desired performance, in many cases the PD controller alone is sufficient to stabilize the plant. In our case, a PD controller H(s) would make the Root-locus plot to look like (Figure 10) The system is asymptotically stable for any fixed gain within the "admissible" range 0 - 2.66, so we again choose $K_{MAX} = 2.5$ as an estimate of the highest admissible constant gain that maintains stability of the system. This time however we use $D(s) = 1/H(s)$, the inverse of the PD controller, as the parallel feedforward across the plant. The Root-locus of the resulting augmented plant is shown in Figure 11. This is a strictly causal system with 4 poles and 3 strictly minimum-phase zeros and is therefore, ASPR. Although the original plant was non-minimum phase and this fact would usually forbid using adaptive controllers, here one can apply SAC and be sure that stability and asymptotically perfect tracking of the augmented system is guaranteed. The only open question is how well the actual plant output performs. In this respect, the maximal admissible gain with fictitious PD (or with any other fictitious controller) defines how small the added ballast is and how close the actual output is to the

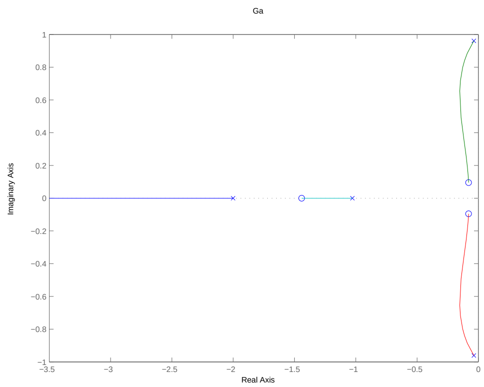

Fig. 11. Augmented Plant with strictly causal PFC

augmented output. The example here is a very bad system and was only used to illustrate the problems one may encounter using constant gain in changing environments and cannot be expected to result in good behavior without performing much more study and basic control design. The examples above have been used to present a simple principle: if the system can be stabilized by the controller H(s), then the augmented system $G_a(s) = G(s) + H^{-1}(s)$ is minimum-phase. Proper selection of the relative degree of $H^{-1}(s)$ will thus render the augmented system ASPR (Barkana, 1987). This last statement implies that "passivability" of systems is actually dual to stabilizability. If a stabilizing controller is known, its inverse in parallel with the plant can make the augmented system ASPR. When sufficient prior knowledge is available to design a stabilizing controller, some researchers prefer to use this knowledge and directly design the corresponding parallel feedforward (Iwai and Mizumoto, 1992) or "shunt" (Fradkov, 1994). When the "plant" is a differential equation, it is easy to assume that the order or the relative degree is available and then a stabilizing controller or the parallel feedforward can be implemented. However, in real world, where the "plant" could be a plane, a flexible structure or a ship, the available knowledge is the result of some wind-tunnel or other experimental tests that may result in some approximate frequency response or approximate modeling, sufficient to allow some control design, yet in general do not provide reliable knowledge on the order or relative degree of the real plant. On the other hand (although it may very much want some adaptive control to help improving performance if it only could be trusted), the control community actually continues to control real-world systems with fixed controllers. Therefore, in our opinion the question "How can you find a stabilizing controller?" should not be given any excessive emphasis. In any case, if there is sufficient prior knowledge to directly design the feedforward there is definitely sufficient information to design a stabilizing configuration, and vice versa. Note that the example of this section is a bad system that was on purpose selected to provide a counterexample for the stability with assumably "constant" gains. Although the stability of the augmented system with adaptive control is guaranteed, the plant output may not behave very well, even with the added parallel feedforward. In any case, even in those cases when the parallel feedforward is too large to allow good performance as monitored at the actual plant output, the behavior of the, possibly both unstable and non-minimum phase, plant within the augmented system is stable and it was shown to allow stable identification schemes (Johansson and Robertsson, 2002) and thus, lead to better understanding of the plant towards better, adaptive or non-adaptive, control design. Still, as recently shown with a non-minimumphase UAV example (Barkana, 2005b) and with many other realistic examples (Kaufman, Barkana and Sobel, 1998), prior knowledge usually available for design allows using basic preliminary design and then very small additions to the plant that not only result in robust stability of the adaptive control system even with originally non-minimum phase plants, but that also lead to performance that is ultimately superior to other control methodologies. A recent publication uses the parallel feedforward compensator for safe tuning of MIMO Adaptive PID Controllers (Iwai, Mizumoto, Nakashima and Kumon, 2007) and another (Ben Yamin, Yaesh and Shaked, 2007) shows how to implement Simple Adaptive Controllers with guaranteed H_∞ performance.

9. ROBUSTNESS OF SIMPLE ADAPTIVE CONTROL WITH DISTURBANCES

The presentation so far showed that a simple adaptive controller can guarantee stability of any system that is minimum-phase if the CB product is Positive Definite and diagonalizable if not symmetric. In case these conditions do not inherently hold, basic knowledge on the stabilizability properties of the plant, usually known, can be used to fulfill them via Parallel Feedforward Configurations. Therefore, the proposed methodology seems to fit almost any case where asymptotically perfect output tracking is possible. However, after we presented the eulogy of the adaptive output feedback gain (32), it is about time to also present what could become its demise, if not properly treated. When persistent disturbances such as random noise or very high frequency vibrations are present, perfect tracking is not possible. Even when the disturbance is known and various variations of the Internal Model Principle can be devised (Fradkov and Andrievsky, 2007) to filter them out, some residual tracking error may always be present. While tracking with small final errors could be acceptable, it is clear that the adaptive gain term (32) would, slowly but certainly, increase without limit. Indeed, theoretically, ASPR systems maintain stability with arbitrarily high gains and in some cases (in case of missiles, for example) the adaptive system mission could end even before problems are even observed. However, allowing the build-up of high gains that do not come in response to any actual requirement is not acceptable, because in practice they may lead to numerical problems and saturation effects. However, very early we observed how the robustness of SAC with disturbances can be guaranteed by adding Ioannou's σ-term (Ioannou and Kokotovic, 1983) with the error adaptive gain that would now be

$$\dot{K}_e(t) = e_y(t)e_y^T(t)\Gamma_e - \sigma K_e(t) \tag{55}$$

Finally, this new addition is literally making SAC an *adaptive* controller (see (Barkana, 2005a) and (Kaufman, Barkana and Sobel, 1998) and references therein): while the control gains always perform a steepest descent minimization of the tracking error, the error gain defined in (55) goes up-and-down fitting the right gain to the right situation in accord with the changing operational needs.

10. CONCLUSIONS AND FUTURE WORKS

This report presented the various components and main properties of a special version of Model Reference Adaptive Control, called Simple Adaptive Control. Initially, because it was attempting to use low-order models with large order plants and also because of some initial lack of mathematical tools, SAC scope and performance seemed to be very modest when compared with the customary MRAC. However, with the development of special mathematical analysis tools along with the gradually deeper understanding of its special qualities, SAC appears to have ultimately become the stable MRAC.

Appendix A. PROOF OF STABILITY

The underlying deterministic tracking problem assumes that there exists an "ideal control"

$$u^*(t) = \widetilde{K}_x x_m(t) + \widetilde{K}_u u_m(t) \tag{A.1}$$

that could keep the plant along an "ideal trajectory" that performs perfect tracking. In other words, the ideal plant

$$\dot{x}^*(t) = Ax^*(t) + Bu^*(t) \tag{A.2}$$

$$y^*(t) = Cx^*(t) \tag{A.3}$$

moves along "ideal trajectories" such that

$$y^*(t) = y_m(t) \tag{A.4}$$

We assume that the underlying LTI problem is solvable and thus, that some ideal gains $\widetilde{K}_x$ and $\widetilde{K}_u$ exist (Barkana, 2005a). Because the plant and the model can have different dimensions, the "following error" is defined to be the difference between the ideal and the actual plant state

$$e_x(t) = x^*(t) - x(t) \tag{A.5}$$

and correspondingly

$$\begin{aligned} e_y(t) &= y_m(t) - y(t) = y^*(t) - y(t) \\ &= Cx^*(t) - Cx(t) = Ce_x(t) \end{aligned} \tag{A.6}$$

Differentiating $e_x(t)$ gives:

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}^*(t) - \dot{x}(t) \qquad \text{(A.7)} \\ &= Ax^*(t) + Bu^*(t) - Ax(t) - Bu(t) \\ \dot{e}_x(t) &= Ae_x(t) - Bu(t) + Bu^*(t) \qquad \text{(A.8)} \end{aligned}$$

$$\begin{aligned} \dot{e}_x(t) &= Ae_x(t) - B(K_e e_y(t) + K_x x_m(t) + K_u u_m(t)) \\ &+ B(\widetilde{K}_x x_m(t) + \widetilde{K}_u u_m(t)) \end{aligned} \tag{A.9}$$

Adding and subtracting $B\widetilde{K}_e e_y(t) = B\widetilde{K}_e C e_x(t)$ above gives

$$\dot{e}_x(t) = \left(A - B\widetilde{K}_e C\right) e_x(t) - B\left(K(t) - \widetilde{K}\right) r(t) \tag{A.10}$$

where for convenience we denoted

$$\widetilde{K} = \left[\widetilde{K}_e \ \widetilde{K}_x \ \widetilde{K}\right] \tag{A.11}$$

The derivative of the Lyapunov function (24) is

$$\begin{aligned} \dot{V}(t) &= e_x^T(t)P\dot{e}_x(t) + \dot{e}_x^T(t)Pe_x(t) \\ &+tr\left[W\dot{K}(t)\Gamma^{-1}\left(K(t) - \widetilde{K}\right)\right] \\ &+tr\left[W\left(K(t) - \widetilde{K}\right)\Gamma^{-1}\dot{K}(t)^T\right] \end{aligned} \tag{A.12}$$

Using relations (22)-(23) gives

$$\begin{aligned} &\dot{V}(t) = x^T(t)\left[PA_K + A_K^T P\right]x(t) \\ &-x^T C^T W\left(K - \widetilde{K}\right)Cx - x^T C^T\left(K - \widetilde{K}\right)WCx \\ &+x^T C^T W\left(K - \widetilde{K}\right)Cx + x^T C^T\left(K - \widetilde{K}\right)WCx \end{aligned} \tag{A.13}$$

The last two terms in (A.13), originating in the derivative of the adaptive gain terms in V(t), cancel the previous, possibly troubling, non-positive, terms and thus lead to the Lyapunov derivative

$$\dot{V}(t) = -e_x^T(t)Qe_x(t) \tag{A.14}$$

Appendix B. GAIN CONVERGENCE

Let the linear time-invariant plant (1)-(2) track the output of the model (3)-(4). In general, in the past we have assumed that the model uses step inputs to generate the desired command (Barkana, 2005a). For the more general command following case, we assume that the command itself is generated by an unknown input generator

$$\dot{x}_u(t) = A_u x_u(t) \tag{B.1}$$

$$u(t) = C_u x_u(t) \tag{B.2}$$

We want to check what the ultimate adaptive control gains that perform perfect tracking could be. When the error is zero, the input control to the plant is a linear combination of available measures.

$$u(t) = K_x x_m(t) + K_u u_m(t) \tag{B.3}$$

Assume that the plant moves along such "ideal trajectories" and the nonstationary gains are such that the plant output $y^*(t) = Cx^*(t)$ perfectly tracks the model output, namely, $e_y(t) = 0$, or

$$Cx^*(t) = C_m x_m(t) \tag{B.4}$$

Differentiating (B.4) gives

$$C\dot{x}^*(t) = C_m \dot{x}_m(t) \tag{B.5}$$

or

$$CAx^*(t) + CBu(t) = C_m A_m x_m(t) + C_m B_m u_m(t) \tag{B.6}$$

$$\begin{aligned} &CAx^*(t) + CBK_x x_m(t) + CBK_u u_m(t) \\ &\quad = C_m A_m x_m(t) + C_m B_m u_m(t) \end{aligned} \tag{B.7}$$

$$\begin{aligned} CAx^*(t) = &\left[C_m A_m - CBK_x\right] x_m(t) \\ &+ \left[C_m B_m - CBK_u\right] u_m(t) \end{aligned} \tag{B.8}$$

We assume that CA is maximal rank and get

$$\begin{aligned} &x^*(t) = \\ &(CA)^T \left(CA\,(CA)^T\right)^{-1} \left[C_m A_m - CBK_x(t)\right] x_m(t) \\ &+ (CA)^T \left(CA\,(CA)^T\right)^{-1} \left[C_m B_m - CBK_u(t)\right] u_m(t) \\ &+ x_0^*(t) \end{aligned} \tag{B.9}$$

or

$$x^*(t) = S_x(t) x_m(t) + S_u(t) u_m(t) + x_0^*(t) \tag{B.10}$$

Here,

$$S_x(t) = (CA)^T \left(CA\,(CA)^T\right)^{-1} \left[C_m A_m - CBK_x(t)\right] \tag{B.11}$$

$$S_u(t) = (CA)^T \left(CA\,(CA)^T\right)^{-1} \left[C_m B_m - CBK_u(t)\right] \tag{B.12}$$

and $x_0^*(t)$ represents those functions that satisfy

$$CAx_0^*(t) \equiv 0 \tag{B.13}$$

and are solutions of the plant differential equation

$$\dot{x}_0^*(t) = Ax_0^*(t) \tag{B.14}$$

that result in

$$\dot{y}_0^*(t) = C\dot{x}_0^*(t) = CAx_0^*(t) \equiv 0. \tag{B.15}$$

Note that the differential equation (B.14) of the supplementary term $x_0^*(t)$ does not contain control terms because those would be included in the other terms in (B.10). Because CB is nonsingular one gets from (B.4)

$$u(t) = (CB)^{-1} \left[C_m A_m x_m(t) + C_m B_m u_m(t) - CAx^*(t)\right] \tag{B.16}$$

or

$$u(t) = K_x(t) x_m(t) + K_u(t) u_m(t) \tag{B.17}$$

Here

$$K_x(t) = (CB)^{-1} \left(C_m A_m - CAS_x(t)\right) \tag{B.18}$$

$$K_u(t) = (CB)^{-1} \left(C_m B_m - CAS_u(t)\right) \tag{B.19}$$

$$\begin{aligned} \dot{x}^*(t) = &S_x(t)\dot{x}_m(t) + \dot{S}_x(t) x_m(t) \\ &+ S_u(t)\dot{u}_m(t) + \dot{S}_u(t) u_m(t) + \dot{x}_0^*(t) \end{aligned} \tag{B.20}$$

$$\begin{aligned} \dot{x}^*(t) = &S_x(t) A_m x_m(t) + S_x(t) B_m u_m(t) \\ &+ \dot{S}_x(t) x_m(t) + S_u(t) C_u A_u x_u(t) \\ &+ \dot{S}_u(t) C_u x_u(t) + \dot{x}_0^*(t) \end{aligned} \tag{B.21}$$

$$\begin{aligned} \dot{x}^*(t) = &Ax^*(t) + Bu(t) = AS_x(t) x_m(t) \\ &+ AS_u(t) u_m(t) + Ax_0^*(t) \\ &+ BK_x(t) x_m(t) + BK_u(t) u_m(t) \end{aligned} \tag{B.22}$$

$$\begin{aligned} &AS_x(t) x_m(t) + AS_u(t) u_m(t) + Ax_0^*(t) \\ &+ BK_x(t) x_m(t) + BK_u(t) u_m(t) \\ &= S_x(t) A_m x_m(t) + S_x(t) B_m C_u x_u(t) \\ &+ \dot{S}_x(t) x_m(t) + S_u(t) C_u A_u x_u(t) \\ &+ \dot{S}_u(t) C_u x_u(t) + \dot{x}_0^*(t) \end{aligned} \tag{B.23}$$

The terms in $x_0^*(t)$ and $\dot{x}_0^*(t)$ cancel each other and we get

$$\begin{aligned} &AS_x(t) x_m(t) + AS_u(t) C_u A_u x_u(t) \\ &+ B\,(CB)^{-1} \left(C_m A_m - CAS_x(t)\right) x_m(t) \\ &+ B\,(CB)^{-1} \left(C_m B_m - CAS_u(t)\right) C_u A_u x_u(t) \\ &= S_x(t) A_m x_m(t) + S_x(t) B_m C_u x_u(t) \\ &+ \dot{S}_x(t) x_m(t) + S_u(t) C_u A_u x_u(t) \\ &+ \dot{S}_u(t) C_u x_u(t) \end{aligned} \tag{B.24}$$

and finally

$$M_x x_m(t) + M_u x_u(t) = 0 \tag{B.25}$$

Here,

$$M_x = \dot{S}_x(t) - AS_x(t) + S_x(t)A_m \\ -B\,(CB)^{-1}\,(C_m A_m - CAS_x(t)) \quad \text{(B.26)}$$

$$M_u = \dot{S}_u(t)C_u + S_x(t)B_m C_u + S_u(t)C_u A_u \\ -AS_u(t)C_u A_u \\ -B\,(CB)^{-1}\,(C_m B_m - CAS_u(t))\,C_u A_u \quad \text{(B.27)}$$

We first consider the case when the signals $x_m(t)$ and $x_m(t)$ are "sufficiently rich" so the equations can be separated and the differential equations of $S_x(t)$ and $S_u(t)$ are

$$\dot{S}_x(t) - \left[I - B\,(CB)^{-1}\,C\right] AS_x(t) + S_x(t)A_m \\ = B\,(CB)^{-1}\,C_m A_m \quad \text{(B.28)}$$

$$\dot{S}_u(t)C_u + \left[I - \left(I - B\,(CB)^{-1}\,C\right) A\right] S_u(t)C_u A_u \\ = -S_x(t)B_m C_u + B\,(CB)^{-1}\,C_m B_m C_u A_u \quad \text{(B.29)}$$

One can see that (B.28) is a linear differential equation with constant coefficients, excited by a constant forcing function. Therefore, the solution of (B.28) is a combination of generalized exponentials of the form

$$S_{xij}(t) = \sum ct^m e^{bt} \sin(\beta t + \varphi) \quad \text{(B.30)}$$

Equation (B.29) is then a linear differential equation with constant coefficients excited by exponential and constant forcing functions, and its solution is therefore also a combination of generalized exponentials. Now, from (B.18) and (B.19) one can see that the components of the control gains are also combinations of constants and generalized exponentials of the form

$$k_{ij}(t) = a_{ij} + \sum ct^m e^{bt} \sin(\beta t + \varphi) \quad \text{(B.31)}$$

On the other hand, because one knows that the adaptive gains that perform asymptotically perfect tracking are bounded, any divergent exponentials are excluded. What is left is a combination of converging exponentials, constants and eventually stable sinusoidal functions. However, because we know that the derivative of the adaptive gain vanishes in time, steady sinusoidal functions are also excluded. Therefore, those adaptive gains that can perform perfect tracking can only be combinations of constants and converging exponentials that ultimately tend to reach a constant limit as time goes to infinity.

Now we must consider the more general case when the signals cannot be considered "sufficiently rich." The solution for the model (3)-(4) supplied with the command (B.1)-(B.2) has the form

$$x_m(t) = Ex_u(t) + e^{A_m t} d_0 \quad \text{(B.32)}$$

At $t = 0$ one gets

$$x_m(0) = Ex_u(0) + d_0 \quad \text{(B.33)}$$

$$d_0 = x_m(0) - Ex_u(0) \quad \text{(B.34)}$$

Substituting (B.32) in (3) gives

$$E\dot{x}_u(t) = EA_u x_u(t) = A_m Ex_u(t) + A_m e^{A_m t} d_0 \\ +B_m C_u x_u(t) \quad \text{(B.35)}$$

$$(A_m E - EA_u + B_m C_u)\,x_u(t) + A_m e^{A_m t} d_0 = 0 \quad \text{(B.36)}$$

Therefore, at steady state the model is

$$x_m(t) = Ex_u(t) \quad \text{(B.37)}$$

where E satisfies the equation

$$A_m E - EA_u + B_m C_u = 0 \quad \text{(B.38)}$$

Substituting (B.38) in (B.28) gives

$$M_{x_1} Ex_u(t) + M_{x_2} x_u(t) = 0 \quad \text{(B.39)}$$

Here

$$M_{x_1} = \dot{S}_x(t) - AS_x(t) + S_x(t)A_m \\ - B\,(CB)^{-1}\,(C_m A_m - CAS_x(t)) \quad \text{(B.40)}$$

$$M_{x_2} = \dot{S}_u(t)C_u + S_x(t)B_m C_u + S_u(t)C_u A_u \\ - AS_u(t)C_u A_u \\ - B\,(CB)^{-1}\,(C_m B_m - CAS_u(t))\,C_u \quad \text{(B.41)}$$

Equation (B.41) becomes

$$M(t)x_u(t) = 0 \quad \text{(B.42)}$$

Here

$$M(t) = \dot{S}_x(t)E + S_x(t)A_m E + S_x(t)B_m C_u \\ -AS_x(t)E + B\,(CB)^{-1}\,CAS_x(t)E \\ +\dot{S}_u(t)C_u + S_u(t)C_u A_u + AS_u(t)C_u A_u \\ +B\,(CB)^{-1}\,CAS_u(t)C_u \\ -B\,(CB)^{-1}\,C_m\,(A_m E + B_m C_u) \quad \text{(B.43)}$$

and after some algebra

$$M(t) = \dot{S}_x(t)E + \dot{S}_u(t)C_u \\ + (S_x(t)E + S_u(t)C_u)\,A_u \\ - \left(I + B\,(CB)^{-1}\,C\right) A\,(S_x(t)E - S_u(t)C_u) \\ -B\,(CB)^{-1}\,C_m EA_u \quad \text{(B.44)}$$

We first use relation (B.44) to again show that the perfect following problem has many solutions. If we first choose those solutions that satisfy

$$S_x(t)E - S_u(t)C_u = 0 \quad \text{(B.45)}$$

$$S_u(t)C_u = S_x(t)E = S_1(t) \quad \text{(B.46)}$$

we get

$$M(t) = \dot{S}_1(t) + S_1(t) - B\,(CB)^{-1}\,C_m EA_u \quad \text{(B.47)}$$

The equation

$$\dot{S}_1(t) + S_1(t) - B\,(CB)^{-1}\,C_m E A_u = 0 \quad \text{(B.48)}$$

is a stable linear differential equation with constant coefficients. Therefore, is given by a combination of exponential function and ultimately reaches a constant limit, S_f. However, it now implies that only a linear combination of the ultimate gains satisfies a relation of the form

$$S_x(t)E = S_u(t)C_u = S_f \quad \text{(B.49)}$$

Similarly only a linear combination of the ultimate adaptive control gains satisfies a relation of the form

$$K_x(t)F + K_u(t)G = K_f \quad \text{(B.50)}$$

While any set of *constant* gains that satisfy (B.50) would perform perfect tracking, *nonstationary* gains could also do. Moreover, in order to simplify the equation and show that it has solutions, we only considered those particular solutions that satisfy (B.49), yet the selection is almost arbitrary, and the equation

$$M(t)x_u(t) = 0 \quad \text{(B.51)}$$

has many more solutions than (B.48), in general. Therefore, any effort of proving ultimate convergence of the adaptive gains actually seems to end in failure. There is no doubt that, in principle, perfect tracking can occur while the bounded time-varying gains keep wandering across some hyper-surface described, for example, by (B.50) or by any corresponding equation. However, although such solutions for the perfect tracking exist, one may still ask whether those nonstationary gains can be the ultimate values of the adaptation process. Can the steepest descent minimization end with some ever wandering gains? As we conclude below, most certainly, not. First, although it is hard to translate engineering intuition into rigorous mathematics, it is "felt" that the lack of "richness" that the perfect following equation shows does not express the "richness" of signals that exists during the entire process of adaptation up to and until "just before" perfect tracking. Yet, somewhat more rigorously, the same argument that seems to fail the Lyapunov-LaSalle approach can now be used to redeem it. Along with equation (B.50) of the ultimate hyper-surface that contains them, the ultimate adaptive gains are also located on the hyper-ellipsoid that corresponds to the final value of the Lyapunov function. If the initial value of the Lyapunov function is V(t=0) = V0, its final value is given by

$$\lim_{t\to\infty} V\,(t) = V_0 - \int_0^\infty e_x^T\,(t)Qe_x\,(t)\,dt = V_f \quad \text{(B.52)}$$

or, in case there are transient terms in the Lyapunov derivative

$$\lim_{t\to\infty} V\,(t) = V_0 - \left[\int_0^\infty e_x^T\,(t)Qe_x\,(t) + transient\right] dt = V_f \quad \text{(B.53)}$$

As the errors ultimately vanish and the monotonically increasing output gain $K_e(t)$ reaches an ideal stabilizing gain value, the adaptive control gains are located on the hyper-ellipsoid defined by

$$\begin{aligned} trace[(K_x(t) - \widetilde{K}_x)^T\Gamma_x^{-1}(K_x(t) - \widetilde{K}_x) \\ + (K_u(t) - \widetilde{K}_u)^T\Gamma_u^{-1}(K_u(t) - \widetilde{K}_u)] = V_f \end{aligned} \quad \text{(B.54)}$$

with the set $\left\{\widetilde{K}_x, \widetilde{K}_u\right\}$ at its center. Because any set of constant gains that satisfy the perfect tracking equation can play the role of ideal gains set that is used in the Lyapunov function, choosing the set $\left\{\widetilde{K}_{x_1}, \widetilde{K}_{u_1}\right\}$ finds the final gain on hyper-ellipsoid with the center in $\left\{\widetilde{K}_{x_1}, \widetilde{K}_{u_1}\right\}$, namely,

$$\begin{aligned} & trace[(K_x(t) - \widetilde{K}_{x1})^T\Gamma_x^{-1}(K_x(t) - \widetilde{K}_{x1}) \\ & \quad + (K_u(t) - \widetilde{K}u1)^T\Gamma_u^{-1}(K_u(t) - \widetilde{K}_{u1})] \\ & = V_{f1}(K_u(t) - \widetilde{K}_u)] = V_f \end{aligned} \quad \text{(B.55)}$$

However, assuming the fictitious set $\left\{\widetilde{K}_{x_2}, \widetilde{K}_{u_2}\right\}$ finds the final gain on a *different* hyper-ellipsoid with the center in $\left\{\widetilde{K}_{x_2}, \widetilde{K}_{u_2}\right\}$. Therefore, for the same adaptation process, that starts and ends with the same values, this thinking experiment finds the final gains located at the intersection of infinitely many *distinct* hyper-ellipsoids, so their common intersection is a point or a "line" of Lebesque measure zero. Although this argument may requires more polishing, it points to the fact that, ultimately, the adaptive gains do converge to a limit. In some cases, the rate of convergence may be slow and simulations may show the gain varying for a long-long time. Hence, it is important to know the gains do not vary at random and that, even if sometimes slowly, they certainly tend to reach their final bounded constant limit.

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