# Controlling chaos in spatially extended beam-plasma system with the help of continuous delayed feedback

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The problem of controlling oscillations in spatially extended systems is nowadays actual and important. For microwave electronics devices this task is connected with controlling characteristics of generated signals. We study the problem of controlling complex dynamics in a beam-plasma system (fluid model of Pierce diode) based on the concepts of controlling chaos in the systems with few degrees of freedom. The presented method is connected with stabilization of unstable homogeneous equilibrium state and the unstable spatio-temporal periodical states analogous to unstable periodic orbits. Stabilization is realized with the help of continuous delayed feedback, by the method based on Pyragas [1] works. In the Pyragas scheme the system is synchronized with its own state taken one orbit period earlier, by continuous change of control parameter. The most attractive feature of the proposed method is that the continuous control signal is given to one of the boundaries of the system what makes this method convenient to use in practice, for example, for microwave beam-plasma systems.

Pierce diode [2] is a model of beam-plasma system consisting of two plane parallel infinite grids pierced by electron beam. The space between the grids is filled with the neutralizing ions. The dynamics of this system is defined by the so-called Pierce parameter $\alpha = \omega_p L/v_0$, where $\omega_p$ is the plasma frequency. With $\alpha > \pi$ the instability leads to the appearance of the virtual cathode in the diode space. At the same time, when $\alpha \sim 3\pi$, the instability is limited by non-linearity and the electron beam passes through the diode space without appearance of a virtual cathode. In this case the system can demonstrate different non-linear dynamics, from periodic to chaotic.

The stationary homogeneous equilibrium state of the electron beam in Pierce diode is characterized by the following distribution of the space charge potential, density and the velocity of the electron beam: $\bar{v}(x) = 1.0, \quad \bar{\rho}(x) = 1.0, \quad \bar{\varphi}(x) = 0.0.$

The continuous delayed feedback is realized by changing the potential of the right boundary of the system by the signal taken from the fixed point of the diode space. Practically, this scheme of delaying feedback can be realized, for example, with the help of delay lines on magnetostatic waves or acoustic waves [3]. Such method allows to select

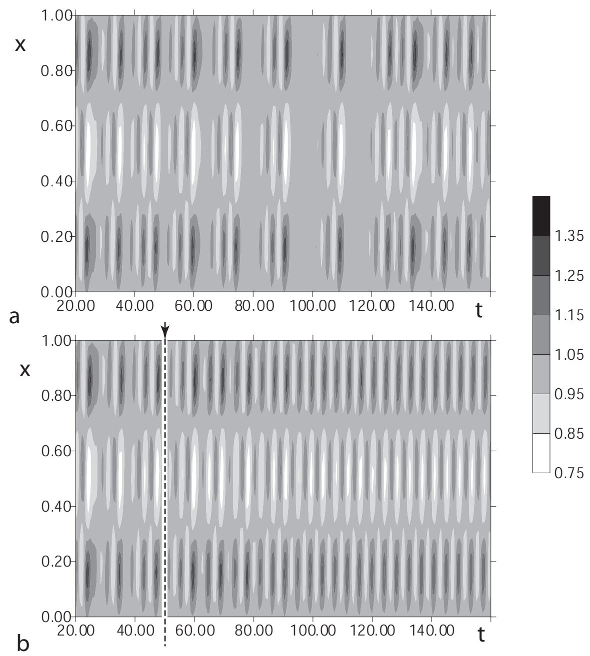


Рис. 1: Spatio–temporal dynamics of Pierce diode in chaotic regime (*a*) and in the regime of stabilization of the periodic orbit $T = 4.173$ (*b*).

the required delay time.

After switching on the continuous feedback the amplitude of autonomous chaotic oscillations rapidly decrease and stabilization of the unstable state is observed. The controlling signal in the feedback line becomes rather small in comparison with the signal before stabilization (near 0.01%) and thus, the regime of chaos control is realized in the system. As a quantitative characteristics of the stability of the discussed equilibrium state, the maximum Lyapunov exponent $\Lambda$ is calculated and its dependance on the feedback parameters in different regimes is analysed. Another method of controlling which we study is the stabilization of unstable spatio-temporal states of the system dynamics. They are analogous to well-known unstable periodic orbits of the chaotic attractor of few-dimensional systems. The procedure of picking them out includes reconstruction the system attractor from the time series of the space charge density oscillations. For stabilization of unstable periodical orbits we take the scheme where the feedback signal is formed as:

$$\varphi(x = 1.0, t) = f_{\text{fb}}^{T}(t) = K(\rho(x_{\text{fix}}, t) - \rho(x_{\text{fix}}, t - T_k)) = K\xi(t). \tag{1}$$

Here $T_k$ is the delay time equal to the period of the $k$th unstable orbit. Fig. 1 shows the spatio–temporal dynamics of the system in cases of autonomous oscillations and in the regime of stabilization (the value of the space charge density $\rho(x,t)$ is shown by colour nuances scaling). The arrow and dotted line in $b$ shows the moment of switching on the delayed feedback. This scheme is effective for the stabilization of the unstable orbits with low periods when the values of the maximum Lyapunov exponent $\lambda$ and the orbit period $\tau$ fulfil the condition $\lambda\tau \le C$, where $C$ is constant depending on the system. We also study the case of stabilization of the orbits with higher period $T_k$ for which this condition does not fulfil. The scheme which we consider is a modification of the method described in [4], when the feedback signal depends not only from the system state at the moment $(t - T_k)$, as it was in the previous scheme, but also from the states at the moments of time $(t - mT_k)$, with some weight coefficients. Following this work [4], we show that the described scheme could be effective for controlling dynamics of spatially extended chaotic system.

**Conclusion**

The method of controlling complex chaotic dynamics of the spatially distributed active medium "electron beam in Pierce diode" is discussed. The method is based on the ideas of controlling chaos in non-linear systems with few degree of freedom. The schemes of continuous delayed feedback, which is used for controlling, allow to stabilize the unstable equilibrium state of the distributed system and the unstable periodic spatio–temporal states analogous to the unstable periodic orbits of the chaotic attractor in the systems with few degree of freedom. The proposed methods can be used to achieve the desired regular dynamics of electron microwave systems.

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