ON THE MOMENTUM OF ELASTIC WAVES 
AND ITS FORCE ON THE OBSTACLE

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Abstract
It’s illustrated with simple examples that for the 
calculation of the momentum of waves and the force 
exerted by the wave reflected from an obstacle it is 
necessary to consider nonlinear factor in motion 
equations of elastic systems and in boundary 
conditions. It is shown that the method using concepts 
of “wave momentum” and “wave pressure” for 
solution this problems is unreasonable.

Key words
elastic medium, momentum, pressure, wave

1. Interest to influence of waves on the reflecting 
obstacle has appeared long ago and is related with the 
assumption that the reflection of any physical wave 
exercises the nonzero pressure upon the reflector like 
the pressure of electromagnetic waves on their 
surface. There are many different points of view 
concerning the question about the influence of waves 
on the system boundary. The concept “wave 
momentum” [Vesnitsky, 2001; Vesnitsky, Kaplan and 
Utkin, 1983] is often used to answer this question. 
The change of this quantity caused by interaction of 
wave with obstacle explains the pressure existence on 
boundary which is called “wave pressure” as well.

Let 

\[ L = b \int_a^b \lambda dx \]

be a Lagrange functional of a one-
dimensional elastic system, where \( \lambda = \lambda(x,t,u(x,t),u_x(x,t),u_{xx}(x,t)) \) is the Lagrange 
function density and \( u(x,t) \) - the shift of system points. The density of wave momentum is defined as

\[ p^W(x,t) = -u_x \frac{\partial \lambda}{\partial u_t} \] (1).

The differential law of change \( p^W \) and \( T^W \) provided that outside forces are absent is given by

\[ \frac{\partial p^W}{\partial t} + \frac{\partial T^W}{\partial x} = \lambda_x \], \( T^W = \lambda - u_x \frac{\partial \lambda}{\partial u_x} \) - is so-
called the wave pressure force in arbitrary elastic 
system profile \( x \).

To all appearance in [Leech, 1961] wave 
characteristics concerned have been introduced for the 
first time. It’s emphasized the quantity defined by (1) 
differs from \( \frac{\partial \lambda}{\partial u_t} \) which is real medium momentum 
density. “It is new differential quantity”, which is 
proposed to term as “wave momentum density, 
because it’s not equal to zero only in wave motion 
when \( u_x \neq 0 \)”. At [Ostrovsky and Potapov, 2003] it’s 
stated that “wave momentum is one of the general 
physical characteristics of wave processes and is true 
for any type of waves”. The difference of generalized 
momentum and wave momentum is accentuated at that. 
“The first quantity is a linear function of variable \( u_t \) 
in distributed systems as well as in discrete systems. The second one is a new field characteristic 
of dynamical process and is reasonable only for 
distributed systems”.

For string waves the transport equation of “wave 
momentum” is a result of multiplication the equation 
of string vibrations \( \rho u_{tt} - N u_{xx} = 0 \) by \( u_t \):

\[ \frac{\partial}{\partial t} (-\rho u_t u_x) + \frac{\partial}{\partial x} \left[ \frac{1}{2} (\rho u_t^2 + N u_x^2) \right] = 0 \] (2)

where \( \rho \) - density on the unit of length, \( N \) - tightening 
force. Here \( p^W = -\rho u_t u_x \) is the “wave momentum” density 
on the unit of length, \( T^W = \frac{1}{2} (\rho u_t^2 + N u_x^2) \) - the force 
of “wave pressure”. In particular, for harmonic wave 
\( u = a \sin(\omega x - kx) \) we get

\[ p^W = \rho a^2 k \cos^2(\omega x - kx) \].

“Wave momentum” is proportional to the square of amplitude and has the same direction with wave 
motion. Therefore “wave pressure” is positive, that is 
an obstacle is pushed forward. The average value of mechanical momentum density for harmonic wave is 
zero: \( p = \rho u_t = 0 \), whereas the average “wave 
momentum” \( \rho \frac{1}{2} \rho u_t a^2 \neq 0 \).

Some questions occur about physical meaning of 
“wave momentum” and “wave pressure”, about 
necessity and reasonability of this quantities 
introduction for studying the dynamics of distributed 
systems, equations of which is following from 
Newton’s equations and for which classical motion 
characteristics - momentum and pressure - are 

enough.
The equation (1) looks like a conservation law for wave momentum \(- \rho u_x u_t\) and wave pressure
\[
\frac{1}{2}(\rho u_t^2 + Nu_x^2) = \frac{\partial a_1}{\partial t} + \frac{\partial c_1}{\partial x} = 0
\]  
It can be construed as the balance equation for the rate of change of the quantities of the first and second infinitesimal order:
\[
u = \nu_1 + \nu_2
\]
where \(\nu_1\) and \(\nu_2\) are the quantities of the first and second infinitesimal order.

The boundary condition at \(x=0\) is \(u(0,t) = 0\), \(u_t(0,t) = 0\).

Let's represent a solution as the sum of the quantities of the first and second infinitesimal order:
\[u = u_1 + u_2\]
In that case the task (4) is reduced to the next combined equations:
\[u_t - c_0^2 u_{xx} + \beta u_x = 0\]  
\[u_{2t} - c_0^2 u_{2xx} + \beta u_{1x} = 0\]  

Consider the wave traveling to the left (in boundary direction) and characterizing to a linear approximation by \(u_1(x,t) = f(x + c_0 t)\), a continuous function specified in the range \(x_0 - c_0 t < x < x_0\) correspond to the limit of integration.

Let's determine the pressure produced by the wave at the fixed end point of the rod. For that let’s find the solution of the equation at the second approximation
\[u_{2t} - c_0^2 u_{2xx} = - \frac{\beta}{2} f''(x + c_0 t)\]

By the d’Alembert’s formula, using zero initial conditions we obtain
\[u_2(x,t) = - \frac{\beta}{4c_0^2} \int_{x-c_0 t}^{x+c_0 t} \int_{-\infty}^{t} f''(\xi + c_0 t) d\xi d\xi = - \frac{\beta}{4c_0^2} \left[ f''(x - c_0 t) - f''(x + c_0 t) \right] t\]

The equation (4) can be written as
\[
\frac{\partial p}{\partial t} - \frac{\partial T}{\partial x} = 0
\]

where \(p = \rho u_t\) - generalized momentum density,
\[T = \rho \left( -\frac{\gamma}{2} u_x^2 + \frac{\beta}{2} u_x^2 \right)\]  
the force in profile \(x\).

The force on the boundary is defined by \(T = -R\). On the other side, the integration of the equation (6) over time at first and then over coordinate gives:
\[
\int \left[ T(x_2,t) - T(x_1,t) \right] dt = - \int_{x_1}^{x_2} \left[ P_{\text{amp}}(x,t_2) - P_{\text{max}}(x,t_1) \right] dx
\]

This implies that the total force on the rod of the distance \(x_2-x_1\) for the time \(t\) is equal to the difference between momentums of an incident wave and reflected from the limiter wave. Taking an interest in the force at the fixed end point set \(x_1=0\) and suppose \(x_2\) as such point which hasn’t been reached by disturbance yet, that is \(T(x_2,t) = 0\). Then the left side of the obtained rate is written as: \(\int_{x_0/c_0}^{x_2/c_0} T(0,t) dt\). The limits of integration are the time instants of the beginning and ending of wave interaction with the limiter.

The right side of the rate is equal to
\[
\int_{x_0-c_0 t_1}^{x_0-c_0 t_2} P_{\text{amp}}(x,t_2) dx - \int_{x_0-c_0 t_1}^{x_0+c_0 t_2} P_{\text{max}}(x,t_2) dx
\]

- time when wave hasn’t reached the limiter yet,
- time when the reflection has already completed.

Let's calculate momentum \(P\) and force on boundary \(R\) in linear model. The momentum of an incident wave \(P_{\text{max}}(t_1)\) is given by:
\[
P_{\text{max}}(t_1) = \int_{x_0-c_0 t_1}^{x_0+c_0 t_1} \rho u_t(x,t_1) dx = \rho \int_{x_0-c_0 t_1}^{x_0+c_0 t_1} f(x + c_0 t_1) dx = \rho \int_{x_0+c_0 t_1}^{x_0-c_0 t_1} f(x_0 + a) - f(0) = 0
\]

For the momentum evaluation of a reflected wave \(P_{\text{amp}}(t_2)\) we find its expression
\[
P_{\text{amp}}(t_2) = \int_{x_0-c_0 t_2}^{x_0+c_0 t_2} \rho u_t(x,t_2) dx
\]

Consider, following [Tikhonov and...
Integrating we obtain given by the expression

The same result for the force at the fixed end point is get:

traveling to the right.

The momentum of this wave

The disturbance of the first approximation doesn’t carry momentum under arbitrary function \( f \) which equals zero at the ends of interval. In that case the force on the boundary is equal to:

The same result for the force at the fixed end point is given by the expression \( \int_{0}^{x}(t_{0}) \text{d}x \), where

Thus, the problem solution of the force exerted by the wave on the elastic system boundary is different in the frame of linear and nonlinear model. The pressure appears only in the presence of nonlinearity and can be positive as well as negative according to the sign of the nonlinearity coefficient \( \beta \). The same is also concerned with the momentum of wave. The momentum isn’t connected with the wave traveling direction. The sign of momentum depends on the nonlinearity coefficient.

Notice that in linear model a momentum transfer is possible only if a longitudinal shifts function has a discontinuity at the disturbance area boundary. This condition is nonphysical. The analysis of wave motion should be specified by introduction nonlinearity to the model. In this case the local wave carries momentum on the condition of zero longitudinal displacements at the boundary too even if their first coordinate derivatives are zero.

Let’s consider what the solution of the task would be if we used the notions of “wave momentum” and “wave pressure”. Multiplying the first equation of (5) by \( \rho_{u} \) we get

\[
R_{\text{ex}} = \rho_{u}(x_{0}) \int_{x_{0}}^{\infty} f_{2}^{2}(x) \text{d}x .
\]

We get the same result in another way:

\[
R_{\text{ex}} = -\frac{\beta_{p}}{4c_{0}} \int_{x_{0}}^{\infty} f_{2}^{2}(x) \text{d}x .
\]

Thus, the problem solution of the force exerted by the wave on the elastic system boundary is different in the frame of linear and nonlinear model. The pressure appears only in the presence of nonlinearity and can be positive as well as negative according to the sign of the nonlinearity coefficient \( \beta \). The same is also concerned with the momentum of wave. The momentum isn’t connected with the wave traveling direction. The sign of momentum depends on the nonlinearity coefficient.

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\[
\frac{\partial f_{w}}{\partial x} + \frac{\partial f_{w}}{\partial x} = 0 , \quad \text{where} \quad p_{w} = -\rho_{u} \cdot u_{x} - \text{“wave momentum” } \text{density, } p_{w} = \frac{L}{2} \left[ u_{x}^{2} + c_{0}^{2} u_{x}^{2} \right] - \text{“wave momentum” } \text{in profile } x .
\]

“Wave momentum” of the incident wave \( P_{\text{inc}}(t_{1}) \) for disturbance \( u(x,t) = f(x+c_{0}t) \) is given by:

\[
P_{\text{inc}}(t_{1}) = -\rho_{u} \int_{x_{0}}^{\infty} c_{0} f_{2}^{2}(x+c_{0}t_{1}) \text{d}x = -\rho_{u} \int_{x_{0}}^{\infty} f_{2}^{2}(x) \text{d}x \]

“Wave momentum” of the reflected wave \( P_{\text{ref}}(t_{2}) \) is:
the boundary are analogous to the fixed end case. In
for finding the momentum of wave and its force on
pressure" on time:

\[ P^W(t_1) = \rho \int_{x_0}^{x_0 + a/c} f^2(x) dx \]

In that case the force on the boundary is equal to:

\[ \left. \begin{array}{l}
R_{\alpha = 0} = P_{\alpha \alpha}^W(t_1) - P_{\alpha}^W(t_2) = -2\rho c_0 \int_{x_0}^{x_0 + a/c} f^2(x) dx
\end{array} \right. \]

what coincides with the result of integration of "wave
momentum" on time:

\[ \left. \begin{array}{l}
R_{\alpha = 0} = -\int_{x_0}^{x_0 + a/c} T^W(0,t) \, dt
\end{array} \right. \]

Thus, “wave momentum” \( P^W \) is nonzero and has the
same direction with a traveling wave direction. The
change of “wave momentum” because of the
reflection of wave from the limiter coincides with the
positive “wave pressure” on the fixed end point of the
rod. This result differs from the solution obtained by
using classical concepts.

Consider the same task in the case of free end of the
rod, that is on condition \( u(x,0) = 0 \) at \( x = 0 \). Arguments
for finding the momentum of wave and its force on
the boundary are analogous to the fixed end case. In
the frame of a linear model the momentum \( P_{\alpha \alpha}(t_1) \) of
the incident wave \( u_1(x,t) = f(x + c_0 t) \) and the momentum
\( P_{\alpha \alpha}(t_2) \) of the reflected wave

\[ u_{\alpha \alpha}(x,t) = f(-x + c_0 t), \] \( t \geq \frac{x_0 + a}{c_0} \)

are equal to zero.

Let’s consider waves in the second approximation.

As the previous case the momentum \( P(t_1) \) at \( t_1 \leq \frac{x_0}{c_0} \) is
given by:

\[ P(t_1) = \int_{x_0}^{x_0 + a/c} \rho c_0 \left[ f^2(x + c_0 t_1) - f^2(x) \right] dx \]

Proceeding on the assumption that \( f_1(x_0) = f_2(x_0 + a) = 0 \) for momentum of the incident
wave we have

\[ P_{\alpha \alpha}(t_1) = -\frac{\beta \rho c_0}{8c_0} \left[ x_0 + a \right] f^2(x) dx \]

The momentum \( P(t_2) \) at the time \( t_2 \geq \frac{x_0 + a}{c_0} \) is:

\[ P(t_2) = -\frac{\beta \rho c_0}{8c_0} \left[ f^2(x + c_0 t_2) + 2c_0 f^2(2x - x_0 + c_0 t_2) \right] \]

After calculating we obtain

\[ P_{\alpha \alpha}(t_2) = -\frac{\beta \rho c_0}{8c_0} \left[ x_0 + a \right] f^2(x) dx = P_{\alpha \alpha}(t_1) \]

Thus, the momentum of wave doesn’t change its
direction after the wave reflection from the free end of
the rod and the total force on the boundary is equal to
zero in that case:

\[ \left. \begin{array}{l}
R\mid_{\alpha = 0} = P_{\alpha \alpha}(t_1) - P_{\alpha}(t_2) = 0
\end{array} \right. \]

However the solution of this task with the usage of
wave characteristics gives (7)-(9) as before. So the
“wave” method reduces to the false result and also
doesn’t make difference between cases of free end of
the rod and fixed end point.

So the wave momentum property to change or
conserv its direction with respect to wave
propagation direction under the type of boundary
condition exists. Note that it obtains in frame of linear
model too though it can be found the assertion about
the momentum transfer in the direction of wave
propagation.

Let \( f(x_0) \neq f(x_0 + a) \neq 0 \). The momentum of the
incident wave \( u_1(x,t) = f(x + c_0 t) \) is equal to

\[ P_{\alpha \alpha}(t_1) = \left[ \rho \mu u_1(x,t) \right] dx = \rho c_0 \left[ f(x_0 + a) - f(x_0) \right] \]

The momentum of the reflected wave

\[ u_{\alpha \alpha}(x,t) = f(-x + c_0 t), t \geq \frac{x_0 + a}{c_0} \]

under free boundary condition \( u_0(0,t) = 0 \) is given by

\[ P_{\alpha \alpha}(t_2) = \left[ \rho \mu u_{\alpha \alpha}(x,t) \right] dx = \rho c_0 \left[ f(x_0 + a) - f(x_0) \right] \]

that is the momentum has the same value as well as
direction though the wave propagation direction
changes by the opposite one as the interaction result
with rod boundary.

It’s easy to check up that the wave momentum
direction coincides with the wave propagation
direction after reflection from the boundary under
boundary condition \( u(0,t) = 0 \).

3. The investigation of the transverse motions of
elastic systems has another specialty which should be
considered for finding the force exerted on system. That is the necessity to take into
account the nonlinear connection of transverse
and longitudinal motions in motion equations as well as in
boundary conditions.

In case of the string transverse vibrations the Lagrange function density is

\[ \lambda = \frac{1}{2} \rho_0 \left( u_x^2 + v^2 \right) - \frac{1}{2} T_0 \left( u_x^2 + v^2 \right) \]

where the longitudinal and transverse shifts of string
points are denoted by \( u(x,t) \) and \( v(x,t) \) respectively, \( \rho_0 \) –
unperturbed density value, \( T_0 \) – tightening force, \( B = \frac{\rho_0}{\rho_x} \) -
the initial constant tension string.

The motion equations in the second approximation of
the perturbation theory are given by:

\[ u_{tt} - a^2 u_{xx} = u_t u_{tt} + 2 u_t v_t + v_{tt} + a^2 (1 - \gamma^2) v_{xx} \]

\[ v_{tt} - \gamma^2 a^2 v_{xx} = (u_t v_x) + a^2 (1 - \gamma^2) v_{xx} u_x \]

where \( a^2 = \frac{T_0}{\rho_0}, \gamma^2 = \frac{B}{1 + B} \).

Representing a solution as the sum of two quantities
of the first and second infinitesimal order
\[ u = u_1 + u_2, \quad v = v_1 + v_2, \] in the first approximation we get the independent equations of the string transverse and longitudinal motions:

\[
\begin{align*}
&u_{1t} - a^2 u_{1xx} = 0 \\
&v_{1t} - \gamma^2 a^2 v_{1xx} = 0
\end{align*}
\]

The shifts \( u_1 \) and \( v_1 \) are determined by the solution of the next system of equations:

\[
\begin{align*}
&u_{1t} - a^2 u_{2xx} = u_1 u_{1t} + 2u_{1x}u_{1y} + u_{1x}u_{1x} + a^2 (1-\gamma^2) u_{1x}u_{1x} \\
&v_{1t} - \gamma^2 a^2 v_{2xx} = (u_1 u_{1t})_y + a^2 (1-\gamma^2) (u_1 u_{1t})_y
\end{align*}
\]

The transverse waves excitation in the first approximation leads to the longitudinal waves generation in the second one. However, usually “wave pressure” is calculated to get the solution of linear problem. The inaccuracy of this method has been illustrated with the example of the longitudinal motions in a one-dimensional solid system. Here it will be shown that the solution of the question concerned with the force exerted by the wave on an obstacle essentially depends on the obstacle type as well as on its nonlinear factors. We’ll consider some static problems to show this and also to represent the obvious example.

Let’s consider the string of length \( l_0 \) in undistorted state. At initial instant of time the string is tighten between two fixed points which are positioned on the distance \( 2l_x \) from each other. The force of the constant string tension is equal to \( T_0 = k(2l_x - l_0) \), where \( k \) is the elasticity modulus. At middle point the constant force \( F \) acts on the string (fig.1). The angle of the string deviation from horizontal line is \( \alpha \) (\( \alpha << 1 \)). At that the connection of \( u_x \) and \( v_x \) with a follows from rates \( du = dl \cos \alpha, \quad dv = dl \sin \alpha \),

\[
dl = \sqrt{(1 + B + u_x)^2 + v_x^2} \quad dx - \text{the length of the string element after deformation. In static case it is } u_x = 0, v_x = 0. \quad \text{So the calculation of the string force on each fixed end point can be carried out from the expression } \]

\[
R = \frac{\partial L}{\partial u_x} = 0, \quad \text{as it was in the second part but it is possible to use the statics rates.}
\]

Let’s denote by \( R_0 \) the \( j \)-th component of the force exerted by the string on the boundary with number \( i \) (\( i = 1, 2; \ j = x, y \)). At the left fixed point the next relations are valid:

\[
R_{1x} = T \cos \alpha, \quad R_{1y} = T \sin \alpha ,
\]

where \( T = k \left( \frac{2l_x}{\cos \alpha} - l_0 \right) \) - tightening force.

From this, considering the members of the first and second infinitesimal order we obtain:

\[
R_{1x} \approx T_0 + k l_0 \frac{\alpha^2}{2}, \quad R_{1y} \approx T_0 \alpha .
\]

In that case the string deviation leads to additional longitudinal force which is in the opposite direction from the boundary because \( k l_0 \frac{\alpha^2}{2} > 0 \).

Similarly we get the next force components for the right fixed point:

\[
R_{2x} \approx -T_0 - k l_0 \frac{\alpha^2}{2}, \quad R_{2y} \approx T_0 \alpha .
\]

The additional longitudinal force is directed to the left tending to move the boundary to the left that is this force exerts the additional negative “pressure”.

Thus, the string forces on boundaries in case of fixed end points are equal and opposite directed. After the angle \( \alpha \) increasing, the force grows tending to join boundaries. Notice that in linear model the forces \( R_{1x}, R_{2x} \) are caused by the initial string tension:

\[
R_{1x} = R_{2x} = T_0 .
\]

Let’s consider the case of a fixed ended string as before but at point 1 and 2 there is a ring limiter that is the string is limited only in vertical displacement at these points (fig.2). In this example the initial tightening force is \( T_0 = k(2l_1 + 2l_x - l_0) \).

The force exerted by the string on the limiter at point 1 has the next components:

\[
R_{1x} = T \cos \alpha - T, \quad R_{1y} = T \sin \alpha ,
\]

and at point 2 the force components are:

\[
R_{2x} = -T \cos \alpha + T, \quad R_{2y} = T \sin \alpha .
\]

Here the tightening force of the string is

\[
T = k \left( \frac{2l_1}{\cos \alpha} + 2l_x - l_0 \right) .
\]

By the expansion procedure of the given expressions for force components in the second approximation we get:

\[
R_{1x} \approx -T_0 - \frac{\alpha^2}{2}, \quad R_{1y} \approx T_0 \alpha \quad \text{and } R_{2x} \approx T_0 + \frac{\alpha^2}{2}, \quad R_{2y} \approx T_0 \alpha .
\]

The force longitudinal component on the fixed point limiting only transverse shifts is nonlinear quantity about \( \alpha \). The force occurs in the presence of deviations only and is directed to limiters that is “presses down” on its. This essentially differs from the first case when at \( \alpha \neq 0 \) there is additional negative pressure.

Thus, the solution of the question about influence of elastic system on a boundary depends on the type of boundary conditions. The difference of force value as well as its sign becomes apparent only in the frame of nonlinear model.

In spite of the fact that “wave pressure” is proportional to the square of deformation amplitude it is usually calculated in linear models. In the static
problems under consideration “wave” method gives equal force at fixed points without dependence on the type of limiter.

The Lagrange function density corresponding to linear transverse vibrations of the string follows from (10) and is given by $\lambda = \frac{1}{2} \rho_0 \nu_0^2 - \frac{1}{2} T_0 \nu_0^2$, where $\rho_0$ – density on the unit of string length, $\nu(x,t)$ - the transverse shift of string points, $T_0$ – tightening force.

In static case there is $\nu_t = 0$, $\gamma_\alpha = \nu_x$. “Wave pressure” $T^W = \lambda - \frac{\partial \lambda}{\partial \nu_x} \nu_x$ is equal to $\frac{1}{2} T_0 \nu_x^2$ at that.

The horizontal force at fixed point $x=x_1$ is determined by $R_x = -T^W \bigg|_{x=x_1}$. From this it follows that the force on the boundary at point 1 is equal to $R_{1x} = -\frac{1}{2} T_0 \nu_1^2 \bigg|_{x=0}$, at point 2 - $R_{2x} = -\frac{1}{2} T_0 \nu_2^2 \bigg|_{x=2l}$. At the small angle $\alpha$ of deviation from horizontal line there is $R_{1x} = -\frac{1}{2} T_0 \alpha^2$, $R_{2x} = \frac{1}{2} T_0 \alpha^2$.

In the frame of “wave” method this result is valid for the both considered types of fixed points 1 and 2 (fig.1, fig.2). Presented solution coincides with the previous result obtained by statics method in the case of the ring limiters at point 1 and 2 only. These results are different at fixed end points.

Thus, in the case of elastic solid wave motion the formally introduced concepts of “wave momentum” and “wave pressure” which are interpreted from doubtful analogies and dimensions as the specific characteristics of wave motion can lead to wrong results.

The problems of wave influence on obstacles require detailed analysis in every particular case with consideration of different nonlinear factors. First of all it is concerned those situations when the wave doesn’t have momentum in linear model. The usage of “wave momentum” and “wave pressure” for the simplified solutions of questions about the dynamics of elastic systems is unreasonable.

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References


