PFC DESIGN REALIZING OUTPUT FEEDBACK EXPONENTIAL PASSIVITY FOR EXPONENTIALLY STABLE NON-LINEAR SYSTEMS

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Abstract: This paper deals with a design problem of a parallel feedforward compensator (PFC) which realizes the output feedback exponential passive (OFEP) system. One can easily design an adaptive output feedback control system for such nonlinear systems satisfying the OFEP conditions. We propose a design scheme of a PFC for making a non-OFEP nonlinear system OFEP. The nonlinear system considered here is exponentially stable but not restrict to minimum phase.

Keywords: adaptive control, parallel feedforward compensator, Output feedback

1. INTRODUCTION

The adaptive control of nonlinear systems has attracted a great deal of attention. As a result, many sorts of adaptive strategies based on the above mentioned geometric theory for nonlinear controlled systems were proposed in the early 90's (Kanellakopoulos et al., 1991a; Campion and Bastin, 1990; Sastry and Isidori, 1989; Pomet and Praly, 1992; Seron et al., 1995). Most of them, unfortunately, had been based on the state feedback strategies. Concerning output feedback based adaptive controls for nonlinear systems, adaptive output feedback controller designs based on backstepping strategy (Kanellakopoulos et al., 1991b; Krstic et al., 1995) have been widely developed (Marino and Tomei, 1993; Mizumoto et al., 2003). These adaptive output feedback methods essentially utilize an output feedback exponential passive (OFEP) property (Fradkov and Hill, 1998) of the controlled system in order to verify the stability of the control system. A nonlinear system is said to be OFEP (output feedback exponentially passive) if there exists an output feedback such that the resulting closed loop system is exponentially passive (Fradkov and Hill, 1998). The sufficient conditions for a nonlinear system to be OFEP have been provided (Fradkov and Hill, 1998) such as (1) the system has a relative degree of one, (2) the system be globally exponential minimum-phase and (3) the nonlinearities of the system satisfy the Lipschitz condition. It has been shown that, under these conditions, one can easily stabilize uncertain nonlinear systems with a simple high-gain output feedback based adaptive controller (Allgower et al., 1997; Fradkov, 1996; Fradkov et al., 1999). It has been shown that the methods have a strong robustness with respect to bounded disturbances in spite of its simple structure. Therefore the control methods based on the OFEP property of the controlled system are considered one of the powerful control tools for uncertain nonlinear systems. Unfortunately however, since most practical systems do not satisfy the OFEP conditions mentioned above, the OFEP conditions have imposed very severe restrictions to practical application of OFEP based adaptive output feedback controls. The backstepping strategy is adopted to realize a virtual OFEP system. However, using the backstepping strategy in controller designs, the structure of the controller might become complex for a system with a higher order relative degree because the number of steps in the recursive design of the controller through backstepping depends on the order of the relative degree of the controlled system. As an alternative method to realize an OFEP controlled system, the introduction of a parallel feedforward compensator (PFC) in parallel with the controlled non-OFEP system has been developed (Fradkov, 1996; Fradkov et al., 1999). This is a simple and innovative method to alleviate the restrictions imposed by the relative degree and/or the minimum-phase conditions on the OFEP property. However, the provided design methods of the PFC were only for minimum-phase systems with higher relative degrees and required *a priori* knowledge of nonlinearities in the control input term.

In this report, we propose a PFC design method which realizes a nonlinear system with OFEP property. The nonlinear systems dealt with here are exponentially stable but not restricted to minimum-phase and the existance of uncertain nonlinearities in the control input term. Firstly, it is shown that the exponentially stable nonlinear system can be rendered exponentially minimum-phase by introducing a PFC irrespective of its minimum-phase property. After that, the realization of the OFEP nonlinear system with the PFC will be shown.

2. PRELIMINARIES

Consider SISO affine nonlinear systems:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u(t),\ y(t) = h(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x}(t) \in R^n$ is a state vector, $u(t) \in R$ is a control input, $y(t) \in R$ is an output, and, $\boldsymbol{f}(\boldsymbol{x}), \boldsymbol{g}(\boldsymbol{x}) : R^n \to R^n$, $h(\boldsymbol{x}) : R^n \to R$ are sufficiently smooth (e.g. of class C^∞) functions such that $\boldsymbol{f}(\boldsymbol{0}) = \boldsymbol{0}, h(\boldsymbol{0}) = 0$. We assume that the system (1) has a relative degree of γ in R^n.

It is well known (Isidori, 1995) that if the system has relative degree of γ in R^n, then there exists a smooth nonsingular change of coordinate: $\boldsymbol{z}(t) = [z_1, z_2, \cdots, z_n]^T = \Phi(\boldsymbol{x})$, $\forall \boldsymbol{x} \in R^n$ such that the system (1) can be transformed into the following normal form:

$$\begin{aligned} \dot{z}_i(t) &= z_{i+1}(t) \quad (i = 1, \cdots, \gamma - 1) \\ \dot{z}_\gamma(t) &= a(\boldsymbol{z}_\xi, \boldsymbol{\eta}) + b_0 b(\boldsymbol{z}_\xi, \boldsymbol{\eta})u(t) \\ \dot{\boldsymbol{\eta}}(t) &= \boldsymbol{q}(\boldsymbol{z}_\xi, \boldsymbol{\eta}) \\ y(t) &= z_1(t) \end{aligned} \tag{2}$$

where

$$\boldsymbol{z}_\xi(t) = [z_1(t), \cdots, z_\gamma(t)]^T \tag{3}$$

$$\boldsymbol{\eta}(t) = [z_{\gamma+1}(t), \cdots, z_n(t)]^T \tag{4}$$

and

$$\begin{aligned} a(\boldsymbol{0}, \boldsymbol{0}) &= L_f^\gamma h(\boldsymbol{0}) = 0, \boldsymbol{q}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0} \\ b(\boldsymbol{z}_\xi, \boldsymbol{\eta}) &= L_g L_f^{\gamma-1} h(\boldsymbol{x}) \neq 0 \quad \forall \boldsymbol{x} \in R^n \end{aligned} \tag{5}$$

Definition 1:(output feedback exponentially passive: OFEP) (Fradkov and Hill, 1998) The system 1 is called OFEP, if there exists an output feedback:

$$u = \alpha(y) + \beta(y)v \tag{6}$$

such that the resulting closed loop system from v to y is exponentially passive, that is for the closed loop system, the following DI is satisfied

$$\dot{V}(\boldsymbol{x}) = \frac{\partial V(\boldsymbol{x})}{\partial \boldsymbol{x}}(f(\boldsymbol{x}) + g(\boldsymbol{x})v) \leq yv \tag{7}$$

with V which is a (C^2) positive definite function having the following properties:

$$\begin{aligned} &\delta_1 ||\boldsymbol{x}||^2 \leq V(\boldsymbol{x}) \leq \delta_2 ||\boldsymbol{x}||^2 \\ &\delta_3 ||\boldsymbol{x}||^2 \leq S(\boldsymbol{x}) \end{aligned} \tag{8}$$

where $\delta_1 \sim \delta_3$ are positive constants and $S(\boldsymbol{x})$ is a positive definite function.

3. PROBLEM STATEMENT

Let's consider an exponentially stable nonlinear system (2) with a relative degree of $\gamma (\gamma \geq 2)$. Now, express the system (2) as follows:

$$\begin{aligned} \dot{\boldsymbol{z}} &= \boldsymbol{f}_z(\boldsymbol{z}) + \boldsymbol{b}_0 b(\boldsymbol{z}, t)u \\ y &= [1, 0, \cdots, 0]\, \boldsymbol{z} = z_1 \end{aligned} \tag{9}$$

where

$$\boldsymbol{z} = \left[\boldsymbol{z}_\xi^T, \boldsymbol{\eta}^T\right]^T \tag{10}$$

$$\boldsymbol{f}_z(\boldsymbol{z}) = [z_2, \cdots, z_\gamma, a(\boldsymbol{z}), \boldsymbol{q}(\boldsymbol{z})]^T \tag{11}$$

$$\boldsymbol{b}_0 = [\underbrace{0, \cdots, 0, 1}_{\gamma}, \underbrace{0, \cdots, 0}_{n-\gamma}]^T \tag{12}$$

We impose the following assumptions on the system (9).

Assumptions:

(A-1) the system (9) is exponentially stable.

(A-2) there exist positive constants L_1, L_2 and functions g_a, g_q such that $a(\boldsymbol{z}_\xi, \boldsymbol{\eta})$ and $\boldsymbol{q}(\boldsymbol{z}_\xi, \boldsymbol{\eta})$ can be evaluated as

$$\begin{aligned} &|a(\boldsymbol{z}_{\xi 1}, \boldsymbol{\eta}) - a(\boldsymbol{z}_{\xi 2}, \boldsymbol{\eta})| \\ &\leq L_1 |z_{\xi 1\gamma} - z_{\xi 2\gamma}| + g_a(\overline{\boldsymbol{z}}_{\xi 1}, z_{\xi 1\gamma}, \overline{\boldsymbol{z}}_{\xi 2}, z_{\xi 2\gamma}, \boldsymbol{\eta}) \end{aligned} \tag{13}$$

$$\begin{aligned} &\|\boldsymbol{q}(\boldsymbol{z}_{\xi 1}, \boldsymbol{\eta}) - \boldsymbol{q}(\boldsymbol{z}_{\xi 2}, \boldsymbol{\eta})\| \\ &\leq L_2 |z_{\xi 1\gamma} - z_{\xi 2\gamma}| + g_q(\overline{\boldsymbol{z}}_{\xi 1}, z_{\xi 1\gamma}, \overline{\boldsymbol{z}}_{\xi 2}, z_{\xi 2\gamma}, \boldsymbol{\eta}) \end{aligned} \tag{14}$$

where g_a, g_q are any functions such as

$$g_a(\overline{\boldsymbol{z}}_\xi, z_{\xi 1\gamma}, \overline{\boldsymbol{z}}_\xi, z_{\xi 2\gamma}, \boldsymbol{\eta}) = 0 \tag{15}$$

$$g_q(\overline{\boldsymbol{z}}_\xi, z_{\xi 1\gamma}, \overline{\boldsymbol{z}}_\xi, z_{\xi 2\gamma}, \boldsymbol{\eta}) = 0 \tag{16}$$

(A-3) there exist positive constants $b_m \leq b_M$ such that $0 < b_m \leq b(\boldsymbol{z}, t) \leq b_M$.

(A-4) $\left|\frac{d}{dt}b(\boldsymbol{z},t)\right| \le \rho$, $\rho > 0$.
(A-5) $b(\boldsymbol{z},t)$ can be expressed as

$$b(\boldsymbol{z},t) = b(z_1, \cdots, z_{\gamma-1}, \boldsymbol{\eta}, t) \tag{17}$$

In this report, we propose a design scheme of a PFC which makes the resulting augmented system with the PFC exponentially minimum-phase and show that one can realize the OFEP system by designing the PFC with a relative degree of 1.

4. PFC DESIGN FOR REALIZATION OF OFEP SYSTEM

4.1 Design of a PFC

We first derive design conditions of a PFC which realizes an exponentially minimum-phase system.

Let's introduce the following PFC with a relative degree of $\gamma_f (\gamma_f < \gamma)$ for the system (9).

$$\begin{aligned} \dot{\boldsymbol{z}}_f &= A_f \boldsymbol{z}_f + \boldsymbol{b}_f u \\ y_f &= z_{f1} = [1, 0, \cdots, 0]\boldsymbol{z}_f \end{aligned} \tag{18}$$

where

$$\boldsymbol{z}_f = \begin{bmatrix} z_{f1} \\ \vdots \\ z_{f\gamma_f} \end{bmatrix}, A_f = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ f_1 & \cdots & \cdots & \cdots & f_{\gamma_f} \end{bmatrix}, \boldsymbol{b}_f = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ k_f \end{bmatrix} \tag{19}$$

The resulting augmented system with the PFC (18) can be represented by

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{z}} \\ \dot{\boldsymbol{z}}_f \end{bmatrix} &= \begin{bmatrix} \boldsymbol{f}_z(\boldsymbol{z}) \\ A_f \boldsymbol{z}_f \end{bmatrix} + \begin{bmatrix} \boldsymbol{b}_0 b(\boldsymbol{z}) \\ \boldsymbol{b}_f \end{bmatrix} u \\ y_a &= y + y_f \end{aligned} \tag{20}$$

Here, we will show design conditions of the PFC (18) with which the zero dynamics of the augmented system (20) are exponentially stable.

Now, consider deriving zero dynamics of the augmented system (20). Using the following nonsingular change of coordinate:

$$[\boldsymbol{z}_{a\xi}^T, \boldsymbol{\eta}_{a1}^T, \boldsymbol{\eta}_{a2}^T]^T = \Phi_z(\boldsymbol{z}_\xi, \boldsymbol{\eta}, \boldsymbol{z}_f)$$

with

$$\boldsymbol{z}_{a\xi} = \left[z_{a1}, \cdots, z_{a\gamma_f}\right]^T \tag{21}$$

$$\boldsymbol{\eta}_{a1} = \left[z_{a(\gamma_f+1)}, \cdots, z_{a(\gamma_f+\gamma)}\right]^T \tag{22}$$

$$\boldsymbol{\eta}_{a2} = \left[z_{a(\gamma_f+\gamma+1)}, \cdots, z_{a(\gamma_f+n)}\right]^T \tag{23}$$

and

$$\begin{aligned} z_{ai} &= z_i + z_{fi} & (i = 1, \cdots, \gamma_f) \\ z_{a(\gamma_f+j)} &= z_j & (j = 1, \cdots, \gamma - 1) \\ z_{a(\gamma_f+\gamma)} &= z_\gamma - \frac{1}{k_f} b(\boldsymbol{z},t) z_{f\gamma_f} & \\ z_{a(\gamma_f+k)} &= z_k & (k = \gamma + 1, \cdots, n), \end{aligned} \tag{24}$$

the transformed system can be represented as

$$\dot{\boldsymbol{z}}_{a\xi} = \begin{bmatrix} \dot{z}_1 + \dot{z}_{f1} \\ \vdots \\ \dot{z}_{\gamma_f} + \dot{z}_{f\gamma_f} \end{bmatrix} = \begin{bmatrix} z_{a2} \\ \vdots \\ z_{a\gamma_f} \\ z_{\gamma_f+1} + \boldsymbol{f}^T \boldsymbol{z}_f + k_f u \end{bmatrix} \tag{25}$$

$$\dot{\boldsymbol{\eta}}_{a1} = \begin{bmatrix} z_{a(\gamma_f+2)} \\ \vdots \\ z_{a(\gamma_f+\gamma-1)} \\ z_{a(\gamma_f+\gamma)} + \frac{1}{k_f} b(\boldsymbol{z},t) z_{f\gamma_f} \\ a(\boldsymbol{z}_\xi, \boldsymbol{\eta}) - \frac{1}{k_f}\dot{b}(\boldsymbol{z},t) z_{f\gamma_f} \\ \qquad - \frac{1}{k_f} b(\boldsymbol{z},t) \boldsymbol{f}^T \boldsymbol{z}_f \end{bmatrix} \tag{26}$$

$$\dot{\boldsymbol{\eta}}_{a2} = \begin{bmatrix} \dot{z}_{\gamma+1} \\ \vdots \\ \dot{z}_n \end{bmatrix} = \dot{\boldsymbol{\eta}} = \boldsymbol{q}(\boldsymbol{z}_\xi, \boldsymbol{\eta}), \tag{27}$$

where $\boldsymbol{f} = \left[f_1, \cdots, f_{\gamma_f}\right]^T$. It is apparent that the augmented system has a relative degree of γ_f from (25).

Defining

$$\boldsymbol{p}_1 = [0, \cdots, 0, 1]^T \in R^{\gamma_f} \tag{28}$$

$$\boldsymbol{p}_2 = [\underbrace{0, \cdots, 0, -1}_{\gamma_f}, \underbrace{0, \cdots, 0}_{\gamma-\gamma_f}]^T \in R^\gamma \tag{29}$$

$$P_3 = \left[I_{\gamma_f}, \boldsymbol{0}\right] \in R^{\gamma_f \times \gamma} \tag{30}$$

$$\boldsymbol{p}_4 = [0, \cdots, 0, 1]^T \in R^\gamma \tag{31}$$

$$P_5 = \boldsymbol{p}_4 \boldsymbol{p}_1^T \in R^{\gamma \times \gamma_f} \tag{32}$$

$$P_6 = \boldsymbol{p}_4 \boldsymbol{p}_2^T \in R^{\gamma \times \gamma}, \tag{33}$$

$z_{f\gamma_f}$, $\boldsymbol{z}_f$ and $\boldsymbol{z}_\xi$ can be represented by

$$z_{f\gamma_f} = \boldsymbol{p}_1^T \boldsymbol{z}_{a\xi} + \boldsymbol{p}_2^T \boldsymbol{\eta}_{a1} \tag{34}$$

$$\boldsymbol{z}_f = \boldsymbol{z}_{a\xi} - P_3 \boldsymbol{\eta}_{a1} \tag{35}$$

$$\boldsymbol{z}_\xi = \boldsymbol{\eta}_{a1} + \frac{1}{k_f} b(\boldsymbol{z},t) \left(P_5 \boldsymbol{z}_{a\xi} + P_6 \boldsymbol{\eta}_{a1}\right). \tag{36}$$

We have from (26),(34),(35) that

$$\begin{aligned} \dot{z}_{a(\gamma_f+\gamma-1)} &= z_{a(\gamma_f+\gamma)} \\ &\quad + \frac{1}{k_f} b(\boldsymbol{z},t) \big(\boldsymbol{p}_1^T \boldsymbol{z}_{a\xi} + \boldsymbol{p}_2^T \boldsymbol{\eta}_{a1}\big) \end{aligned} \tag{37}$$

$$\begin{aligned} \dot{z}_{a(\gamma_f+\gamma)} &= a(\boldsymbol{\eta}_{a1}, \boldsymbol{\eta}_{a2}) \\ &\quad + \{a(\boldsymbol{z}_\xi, \boldsymbol{\eta}_{a2}) - a(\boldsymbol{\eta}_{a1}, \boldsymbol{\eta}_{a2})\} \\ &\quad - \frac{1}{k_f}\dot{b}(\boldsymbol{z},t) \big(\boldsymbol{p}_1^T \boldsymbol{z}_{a\xi} + \boldsymbol{p}_2^T \boldsymbol{\eta}_{a1}\big) \\ &\quad + \frac{1}{k_f} b(\boldsymbol{z},t) \big(\boldsymbol{p}_7^T \boldsymbol{z}_{a\xi} + \boldsymbol{p}_8^T \boldsymbol{\eta}_{a1}\big) \end{aligned} \tag{38}$$

where

$$\boldsymbol{p}_7 = -\boldsymbol{f} \tag{39}$$

$$\boldsymbol{p}_8^T = \boldsymbol{f}^T P_3 = [f_1, \cdots, f_{\gamma_f}, \underbrace{0, \cdots, 0}_{\gamma-\gamma_f}]. \tag{40}$$

Thus $\dot{\boldsymbol{\eta}}_{a1}$ can be expressed as

$$\dot{\boldsymbol{\eta}}_{a1}=\begin{bmatrix} z_{a(\gamma_f+2)} \\ \vdots \\ z_{a(\gamma_f+\gamma)} \\ a(\boldsymbol{\eta}_{a1},\boldsymbol{\eta}_{a2}) \end{bmatrix}+\boldsymbol{p}_4\left\{a(\boldsymbol{z}_\xi,\boldsymbol{\eta}_{a2})-a(\boldsymbol{\eta}_{a1},\boldsymbol{\eta}_{a2})\right\}$$
$$+\frac{1}{k_f}b(\boldsymbol{z},t)\left(P_9\boldsymbol{z}_{a\xi}+P_{10}\boldsymbol{\eta}_{a1}\right)$$
$$-\frac{1}{k_f}\boldsymbol{p}_4\dot{b}(\boldsymbol{z},t)\left(\boldsymbol{p}_1^T\boldsymbol{z}_{a\xi}+\boldsymbol{p}_2^T\boldsymbol{\eta}_{a1}\right), \quad (41)$$

where

$$P_9=[\mathbf{0},\cdots,\mathbf{0},\boldsymbol{p}_1,\boldsymbol{p}_7]^T\in R^{\gamma\times\gamma_f} \quad (42)$$
$$P_{10}=[\mathbf{0},\cdots,\mathbf{0},\boldsymbol{p}_2,\boldsymbol{p}_8]^T\in R^{\gamma\times\gamma}. \quad (43)$$

Further, since $\dot{\boldsymbol{\eta}}_{a2}$ can be represented by

$$\dot{\boldsymbol{\eta}}_{a2}=\boldsymbol{q}(\boldsymbol{z}_\xi,\boldsymbol{\eta})$$
$$=\boldsymbol{q}(\boldsymbol{\eta}_{a1},\boldsymbol{\eta}_{a2})+\left\{\boldsymbol{q}(\boldsymbol{z}_\xi,\boldsymbol{\eta}_{a2})-\boldsymbol{q}(\boldsymbol{\eta}_{a1},\boldsymbol{\eta}_{a2})\right\}, \quad (44)$$

defining $\boldsymbol{\eta}_a=\left[\boldsymbol{\eta}_{a1}^T,\boldsymbol{\eta}_{a2}^T\right]^T$, the augmented system is expressed as

$$\dot{\boldsymbol{z}}_{a\xi}=\begin{bmatrix} z_{a2} \\ \vdots \\ z_{a\gamma_f} \\ z_{\gamma_f+1}+\boldsymbol{f}^T\boldsymbol{z}_f \end{bmatrix}+\begin{bmatrix} 0 \\ \vdots \\ 0 \\ k_f \end{bmatrix}u \quad (45)$$
$$\dot{\boldsymbol{\eta}}_a=\boldsymbol{q}_a(\boldsymbol{z}_{a\xi},\boldsymbol{\eta}_a),$$

where

$$\boldsymbol{q}_a(\boldsymbol{z}_{a\xi},\boldsymbol{\eta}_a)$$
$$=\boldsymbol{f}_z(\boldsymbol{\eta}_a)+\boldsymbol{p}_{11}\left\{a(\boldsymbol{z}_\xi,\boldsymbol{\eta}_{a2})-a(\boldsymbol{\eta}_{a1},\boldsymbol{\eta}_{a2})\right\}$$
$$+\frac{1}{k_f}b(\boldsymbol{z},t)\begin{bmatrix} I_\gamma \\ \mathbf{0} \end{bmatrix}\left(P_9\boldsymbol{z}_{a\xi}+P_{10}\boldsymbol{\eta}_{a1}\right)$$
$$-\frac{1}{k_f}\boldsymbol{p}_{11}\dot{b}(\boldsymbol{z},t)\left(\boldsymbol{p}_1^T\boldsymbol{z}_{a\xi}+\boldsymbol{p}_2^T\boldsymbol{\eta}_{a1}\right)$$
$$+\begin{bmatrix} \mathbf{0} \\ I_{n-\gamma} \end{bmatrix}\left\{\boldsymbol{q}(\boldsymbol{z}_\xi,\boldsymbol{\eta}_{a2})-\boldsymbol{q}(\boldsymbol{\eta}_{a1},\boldsymbol{\eta}_{a2})\right\} \quad (46)$$

and

$$\boldsymbol{p}_{11}=\left[\boldsymbol{p}_4^T,\mathbf{0}^T\right]^T\in R^n. \quad (47)$$

Thus the zero dynamics $\dot{\boldsymbol{\eta}}_a^*=\boldsymbol{q}_a(\mathbf{0},\boldsymbol{\eta}_a^*)$ of the augmented system is obtained as follows:

$$\dot{\boldsymbol{\eta}}_a^*=\boldsymbol{f}_z(\boldsymbol{\eta}_a^*)+\boldsymbol{p}_{11}\left\{a(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*)-a(\boldsymbol{\eta}_{a1}^*,\boldsymbol{\eta}_{a2}^*)\right\}$$
$$+\frac{1}{k_f}b(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*,t)\begin{bmatrix} I_\gamma \\ \mathbf{0} \end{bmatrix}P_{10}\boldsymbol{\eta}_{a1}^*$$
$$-\frac{1}{k_f}\boldsymbol{p}_{11}\dot{b}(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*,t)\boldsymbol{p}_2^T\boldsymbol{\eta}_{a1}^*$$
$$+\begin{bmatrix} \mathbf{0} \\ I_{n-\gamma} \end{bmatrix}\left\{\boldsymbol{q}(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*)-\boldsymbol{q}(\boldsymbol{\eta}_{a1}^*,\boldsymbol{\eta}_{a2}^*)\right\}. \quad (48)$$

Where $\boldsymbol{\phi}$ denotes $\boldsymbol{z}_\xi$ at $\boldsymbol{z}_{a\xi}=\mathbf{0}$, and from (36), it can be expressed by

$$\boldsymbol{\phi}=\boldsymbol{\eta}_{a1}^*+\frac{1}{k_f}b(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*,t)P_6\boldsymbol{\eta}_{a1}^*. \quad (49)$$

It should be noted that since the system (9) is exponentially stable from assumption (A-1), there exist a positive definite function $V_1(\boldsymbol{\eta}_a^*)$ and positive constants $\alpha_1\sim\alpha_4$ such that (Sastry and Isidori, 1989; Khalil, 1996)

$$\frac{\partial V_1(\boldsymbol{\eta}_a^*)}{\partial\boldsymbol{\eta}_a^*}\boldsymbol{f}_z(\boldsymbol{\eta}_a^*)\le-\alpha_1\left\|\boldsymbol{\eta}_a^*\right\|^2 \quad (50)$$
$$\left\|\frac{\partial V_1(\boldsymbol{\eta}_a^*)}{\partial\boldsymbol{\eta}_a^*}\right\|\le\alpha_2\left\|\boldsymbol{\eta}_a^*\right\| \quad (51)$$
$$\alpha_3\left\|\boldsymbol{\eta}_a^*\right\|^2\le V_1(\boldsymbol{\eta}_a^*)\le\alpha_4\left\|\boldsymbol{\eta}_a^*\right\|^2 \quad (52)$$

Theorem 1: For a system (9) which satisfies assumptions (A-1) to (A-5), consider an augmented system (20) with a PFC (18). Then the augmented system is exponentially minimum-phase provided that the PFC gain k_f is designed such that

$$k_f>\frac{\beta_1}{\alpha_1}, \quad (53)$$

where $\beta_1=\alpha_2\left\{b_M\left(L_1+L_2+\|P_{10}\|\right)+\rho\right\}$ and L_1,L_2 are positive constants satisfying (13) and (14) respectively. α_1,α_2 are positive constants in (50) and (51), and P_{10} has been defined in (43).

Proof: The time derivative of $V_1(\boldsymbol{\eta}_a^*)$ is obtained by

$$\dot{V}_1(\boldsymbol{\eta}_a^*)=\frac{\partial V_1}{\partial\boldsymbol{\eta}_a^*}\boldsymbol{f}_z(\boldsymbol{\eta}_a^*)$$
$$+\frac{\partial V_1}{\partial\boldsymbol{\eta}_a^*}\Big[\boldsymbol{p}_{11}\{a(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*)-a(\boldsymbol{\eta}_{a1}^*,\boldsymbol{\eta}_{a2}^*)\}$$
$$+\frac{1}{k_f}b(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*,t)\begin{bmatrix} I_\gamma \\ \mathbf{0} \end{bmatrix}P_{10}\boldsymbol{\eta}_{a1}^*$$
$$-\frac{1}{k_f}\boldsymbol{p}_{11}\dot{b}(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*,t)\boldsymbol{p}_2^T\boldsymbol{\eta}_{a1}^*$$
$$+\begin{bmatrix} \mathbf{0} \\ I_{n-\gamma} \end{bmatrix}\left\{\boldsymbol{q}(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*)-\boldsymbol{q}(\boldsymbol{\eta}_{a1}^*,\boldsymbol{\eta}_{a2}^*)\right\}\Big] \quad (54)$$

Since we have from assumptions (A-2), (a-3) that

$$\left|a(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*)-a(\boldsymbol{\eta}_{a1}^*,\boldsymbol{\eta}_{a2}^*)\right|\le\frac{1}{k_f}L_1b_M\left\|\boldsymbol{\eta}_a^*\right\| \quad (55)$$
$$\left\|\boldsymbol{q}(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*)-\boldsymbol{q}(\boldsymbol{\eta}_{a1}^*,\boldsymbol{\eta}_{a2}^*)\right\|\le\frac{1}{k_f}L_2b_M\left\|\boldsymbol{\eta}_a^*\right\| \quad (56)$$

and we have from assumption (A-4) that

$$\frac{1}{k_f}\left\|\boldsymbol{p}_{11}\dot{b}(\boldsymbol{\phi},\boldsymbol{\eta}_{a2}^*,t)\boldsymbol{p}_2^T\boldsymbol{\eta}_{a1}^*\right\|\le\frac{1}{k_f}\rho\left\|\boldsymbol{\eta}_a^*\right\|, \quad (57)$$

we obtain

$$\dot{V}_1(\boldsymbol{\eta}_a^*)\le\frac{\partial V_1}{\partial\boldsymbol{\eta}_a^*}\boldsymbol{f}_z(\boldsymbol{\eta}_a^*)+\frac{1}{k_f}\left\|\frac{\partial V_1}{\partial\boldsymbol{\eta}_a^*}\right\|(L_1b_M+L_2b_M$$
$$+\|P_{10}\|b_M+\rho)\left\|\boldsymbol{\eta}_a^*\right\|. \quad (58)$$

Finally, using (50),(51) and (52), $\dot{V}_1(\boldsymbol{\eta}_a^*)$ can be evaluated as

$$\dot{V}_1(\boldsymbol{\eta}_a^*) \leq -\alpha_1 \|\boldsymbol{\eta}_a^*\|^2 + \frac{1}{k_f}\alpha_2(L_1 b_M + L_2 b_M + \|P_{10}\| b_M + \rho) \|\boldsymbol{\eta}_a^*\|^2$$
$$= -\left(\alpha_1 - \frac{1}{k_f}\beta_1\right) \|\boldsymbol{\eta}_a^*\|^2$$
$$\leq -\left(\alpha_1 - \frac{1}{k_f}\beta_1\right) \frac{1}{\alpha_4} V_1(\boldsymbol{\eta}_a^*). \quad (59)$$

Thus, the zero dynamics of the augmented system (20) is exponentially stable provided that k_f is designed such that (53) is satisfied.

4.2 Realization of OFEP system

Introducing a PFC (18) with a relative degree of $\gamma_f = 1$ to the system (9), the resulting augmented system can be represented by

$$\begin{aligned} \dot{z}_{a\xi} &= f_a(z_{a\xi}, \boldsymbol{\eta}_a) + k_f u \\ \dot{\boldsymbol{\eta}}_a &= \boldsymbol{q}_a(z_{a\xi}, \boldsymbol{\eta}_a) \\ y_a &= z_{a\xi}. \end{aligned} \quad (60)$$

Here, from (25)

$$f_a(z_{a\xi},\boldsymbol{\eta}_a) = \begin{cases} \left\{\frac{1}{k_f} b(\boldsymbol{z},t) + f_1\right\}(z_{a1} - z_{a2}) + z_{a3} & \text{for } \gamma = 2 \\ f_1\,(z_{a1} - z_{a2}) + z_{a3} & \text{for } \gamma \geq 3 \end{cases} \quad (61)$$

and $f_a(z_{a\xi},\boldsymbol{\eta}_a)$ can be evaluated as

$$|f_a(z_{a\xi},\boldsymbol{\eta}_a)| \leq f_{a1}\,|z_{a\xi}| + f_{a2}\,\|\boldsymbol{\eta}_a\| \quad (62)$$

where

$$f_{a1} = \begin{cases} \frac{1}{k_f} b_M + f_1 & \text{for } \gamma = 2 \\ f_1 & \text{for } \gamma \geq 3 \end{cases} \quad (63)$$

$$f_{a2} = \begin{cases} \frac{1}{k_f} b_M + f_1 + 1 & \text{for } \gamma = 2 \\ f_1 + 1 & \text{for } \gamma \geq 3 \end{cases} \quad (64)$$

Further from the fact that

$$b(\boldsymbol{z}_\xi, \boldsymbol{\eta}, t) = b(\boldsymbol{\phi}, \boldsymbol{\eta}_{a2}, t) \quad (65)$$
$$\dot{b}(\boldsymbol{z}_\xi, \boldsymbol{\eta}, t) = \dot{b}(\boldsymbol{\phi}, \boldsymbol{\eta}_{a2}, t) \quad (66)$$

under assumption (A-5), we have

$$\|\boldsymbol{q}_a(y_a, \boldsymbol{\eta}_a) - \boldsymbol{q}_a(0, \boldsymbol{\eta}_a)\|$$
$$\leq \Bigg\| \boldsymbol{p}_{11}\left\{a(\boldsymbol{z}_\xi, \boldsymbol{\eta}_{a2}) - a(\boldsymbol{\phi}, \boldsymbol{\eta}_{a2})\right\} + \frac{1}{k_f} b(\boldsymbol{z}_\xi, \boldsymbol{\eta}, t) \begin{bmatrix} I_\gamma \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{p}_9 y_a + \frac{1}{k_f}\left\{b(\boldsymbol{z}_\xi, \boldsymbol{\eta}, t) - b(\boldsymbol{\phi}, \boldsymbol{\eta}, t)\right\} P_{10}\boldsymbol{\eta}_{a1} - \frac{1}{k_f}\boldsymbol{p}_{11}\dot{b}(\boldsymbol{z}_\xi, \boldsymbol{\eta}, t)\boldsymbol{p}_1^T y_a - \frac{1}{k_f}\boldsymbol{p}_{11}\left\{\dot{b}(\boldsymbol{z}_\xi, \boldsymbol{\eta}, t) - \dot{b}(\boldsymbol{\phi}, \boldsymbol{\eta}, t)\right\}\boldsymbol{p}_2^T \boldsymbol{\eta}_{a1} \begin{bmatrix} \boldsymbol{0} \\ I_{n-\gamma} \end{bmatrix} \left\{\boldsymbol{q}(\boldsymbol{z}_\xi, \boldsymbol{\eta}_{a2}) - \boldsymbol{q}(\boldsymbol{\phi}, \boldsymbol{\eta}_{a2})\right\} \Bigg\|$$

$$\leq |a(\boldsymbol{z}_\xi, \boldsymbol{\eta}_{a2}) - a(\boldsymbol{\phi}, \boldsymbol{\eta}_{a2})| + \frac{1}{k_f} b_M \|\boldsymbol{p}_9\|\,|y_a|$$
$$\frac{1}{k_f}\rho\,|y_a| + \|\boldsymbol{q}(\boldsymbol{z}_\xi, \boldsymbol{\eta}_{a2}) - \boldsymbol{q}(\boldsymbol{\phi}, \boldsymbol{\eta}_{a2})\|$$
$$\leq \frac{1}{k_f} K_1\,|y_a|, \quad (67)$$

where $K_1 = b_M\,(L_1 + L_2 + \|\boldsymbol{p}_9\|) + \rho$.

Then we have the following theorem.

Theorem 2: Consider an augmented system (60) with a PFC having a relative degree of $\gamma_f = 1$. Suppose that PFC gain k_f satisfies the condition (53). Then, with an input:

$$u = \frac{-K}{k_f} y_a + \frac{1}{k_f} v, \;\; K > 0, \quad (68)$$

the resulting closed loop system from v to y_a is exponentially passive for a sufficiently large.

Proof: The resulting closed loop system with the input (68) can be represented by

$$\dot{y}_a(t) = f_a(y_a, \boldsymbol{\eta}_a) - K y_a(t) + v(t) \quad (69)$$
$$\dot{\boldsymbol{\eta}}_a(t) = \boldsymbol{q}_a(y_a, \boldsymbol{\eta}_a). \quad (70)$$

Since the zero dynamics of the system (60):

$$\dot{\boldsymbol{\eta}}_a(t) = \boldsymbol{q}_a(0, \boldsymbol{\eta}_a) \quad (71)$$

is exponentially stable from Theorem 1, there exist a positive definite function $W(\boldsymbol{\eta}_a)$ which satisfies the following relations:

$$\frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\boldsymbol{q}_a(0, \boldsymbol{\eta}_a) \leq -\kappa_1 \|\boldsymbol{\eta}_a\|^2 \quad (72)$$
$$\left\|\frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\right\| \leq \kappa_2 \|\boldsymbol{\eta}_a\| \quad (73)$$
$$\kappa_3 \|\boldsymbol{\eta}_a\|^2 \leq W(\boldsymbol{\eta}_a) \leq \kappa_4 \|\boldsymbol{\eta}_a\|^2, \quad (74)$$

where $\kappa_1 \sim \kappa_4$ are appropriate positive constants.

Now consider the following positive definite function:

$$V(y_a, \boldsymbol{\eta}_a) = W(\boldsymbol{\eta}_a) + \frac{1}{2} y_a^2(t). \quad (75)$$

The time derivative of $V(y_a, \boldsymbol{\eta}_a)$ is obtained from (60) as

$$\dot{V}(y_a, \boldsymbol{\eta}_a) = \frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\dot{\boldsymbol{\eta}}_a(t) + y_a(t)\dot{y}_a(t)$$
$$= \frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\boldsymbol{q}_a(y_a, \boldsymbol{\eta}_a) + y_a(t) f_a(y_a, \boldsymbol{\eta}_a) - K y_a^2(t) + y_a(t) v(t). \quad (76)$$

Since we have from (72), (73) and (67) that

$$
\begin{aligned}
&\frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\boldsymbol{q}_a(y_a,\boldsymbol{\eta}_a)\\
&=\frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\boldsymbol{q}_a(0,\boldsymbol{\eta}_a)\\
&\quad+\frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\left\{\boldsymbol{q}_a(y_a,\boldsymbol{\eta}_a)-\boldsymbol{q}_a(0,\boldsymbol{\eta}_a)\right\}\\
&\leq -\kappa_1\left\|\boldsymbol{\eta}_a\right\|^2\\
&\quad+\left\|\frac{\partial W(\boldsymbol{\eta}_a)}{\partial \boldsymbol{\eta}_a}\right\|\left\|\boldsymbol{q}_a(y_a,\boldsymbol{\eta}_a)-\boldsymbol{q}_a(0,\boldsymbol{\eta}_a)\right\|\\
&\leq -\kappa_1\left\|\boldsymbol{\eta}_a\right\|^2+\kappa_2\frac{1}{k_f}K_1\left|y_a\right|\left\|\boldsymbol{\eta}_a\right\|, \qquad (77)
\end{aligned}
$$

and we have from (62) that

$$
y_a f_a(y_a,\boldsymbol{\eta}_a)\leq f_{a1}y_a^2+f_{a2}\left|y_a\right|\left\|\boldsymbol{\eta}_a\right\|, \qquad (78)
$$

the time derivative of $V(y_a,\boldsymbol{\eta}_a)$ can finally be evaluated by

$$
\begin{aligned}
&\dot{V}(y_a,\boldsymbol{\eta}_a)\\
&\leq -\kappa_1\left\|\boldsymbol{\eta}_a\right\|^2+\left(\kappa_2\frac{1}{k_f}K_1+f_{a2}\right)\left|y_a\right|\left\|\boldsymbol{\eta}_a\right\|\\
&\quad-\left(K-f_{a1}\right)y_a^2+y_a v\\
&\leq -\left(\kappa_1-\mu\right)\left\|\boldsymbol{\eta}_a\right\|^2\\
&\quad-\left\{K-f_{a1}-\frac{\left(\kappa_2\frac{1}{k_f}K_1+f_{a2}\right)^2}{4\mu}\right\}\left|y_a\right|^2+y_a v
\end{aligned}
\qquad (79)
$$

with any positive constant μ.

Setting $0<\mu<\kappa_1$, for a K such as

$$
K>K_0=f_{a1}+\frac{\left(\kappa_2\frac{1}{k_f}K_1+f_{a2}\right)^2}{4\mu}, \qquad (80)
$$

we obtain

$$
\dot{V}(y_a,\boldsymbol{\eta}_a)\leq -c_1\left\|\boldsymbol{\eta}_a\right\|^2-c_2\left|y_a\right|^2+y_a v \qquad (81)
$$

where c_1, c_2 are positive constants such that

$$
c_1=\kappa_1-\mu>0 \qquad (82)
$$

$$
c_2=K-K_0>0,\ \ K_0=f_{a1}+\frac{\left(\kappa_2\frac{1}{k_f}K_1+f_{a2}\right)^2}{4\mu} \qquad (83)
$$

Thus, for a sufficiently large K such as $K>K_0$, the closed loop system from v to y_a is exponentially passive (Fradkov and Hill, 1998) and we can conclude that the system is OFEP.

Note that for the OFEP augmented system, one can easily design an adaptive output feedback controller of the form:

$$
u=-ky_a,\ \ \dot{k}=-\gamma_I y_a^2,\ \gamma_I>0 \qquad (84)
$$

5. CONCLUSIONS

In this report, we proposed a PFC design method, which realizes a nonlinear system with OFEP property, for exponentially stable systems. The nonlinear systems dealt with here were exponentially stable but the proposed method can design PFCs irrespective of its minimum-phase property and the existence of uncertain nonlinearities in the control input term.

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