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A DELTA MODULATOR TECHNIQUE TO DRIVE THE CHAOTIC DUFFING OSCILLATOR

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Abstract
This paper presents a method to manipulate the chaotic behavior of the Duffing oscillator by employing a modified Delta modulator based on hysteresis modeling. Numerical experiments are given to support our design. Chaos is evidenced by utilizing Poincaré maps.

Key words
Duffing oscillator, chaos, Delta modulator, hysteresis.

1 Introduction

The Duffing system represents a single-mode approximation to the magnetically buckled beam governed by a periodically excitation force [Thomsen, 2003], [Khalil, 2003]. This oscillator is one of the most studied nonlinear dynamical systems, which serves as a model for various physical and engineering problems, such as particle in a forced double well, particle in a plasma, a defect in solids [Sharma, et al. 2012]. Moreover, diverse Duffing oscillators have been analyzed to clarify a wide range of physical applications in the real-world [Liu and Suh, 2012], such as mechanical structures, electric circuits, and biological systems [Danca and Lung, 2002], among others. On the other hand, one essential feature of the Duffing oscillator is its chaotic behavior when the Duffing system parameters are set to specific data.

Additionally, the digital Delta modulation is a simple and robust method of analog to digital conversion technique useful in systems requiring serial digital communications of analog signals [Donald, 1996]. It consists of a comparator in the forward path and an integrator in the feedback path of a simple control loop [Donald, 1996]. Furthermore, the Delta modulator has been used to translate continuous (on an *average sense*) feedback control signal into an implementable switch one with practically the same closed-loop performance than the continuous case [Morales et al., 2013]. Here, we propose a dynamic model of the digital Delta modulator useful to the objective of this paper. This example is based on hysteresis modeling presented in [Acho and Vidal, 2011] and [Acho, 2013].

The objective of this paper is to show how the Poincaré map of the chaotic Duffing oscillator can be altered by employing our proposed Delta modulator (to translate the Delta modulator from its digital version to an analog one). According to numerical experiments, the Duffing attractor and its Poincaré map can be altered preserving chaos.

2 Delta Modulator Modeling

The basic of the digital Delta modulator is shown in Fig. 1. The digital output, $u_{pw}(t)$, is either high or low at any given time. If $u(t)$ is grater than the integrator output, $u_{pw}(t)$ will be high and then the integrator output will be ramping up. Conversely, if $u(t)$ is lower than the integrator output, $u_{pw}(t)$ will be low and then the integrator output will be ramping down. The comparison is done at each clock period [Donald, 1996].

To realize an analog version of the digital Delta modulator, the scheme shown in Fig. 2 is proposed. The hysteresis block is employed as an alternative to the digital comparator. In this way, a kind of pause is acquired in the comparator process. A dynamic model of the hysteresis is suggested in [Acho, 2013], and [Acho and Vidal, 2011]:

$$\dot{z} = c[-z + b\ \mathrm{sign}(x + a\ \mathrm{sign}(z))], \qquad (1)$$

where the parameters a, b are described in Fig. 4. The parameter c regulates the dynamic time-transition from b to $-b$ (and viceversa) of $z(t)$. For instance, if

$$x(t) = u(t) = \sin(t), \quad (2)$$

$$u_{pw}(t) = z(t), \quad (3)$$

and $a = 0.1$, $b = 2$, and $c = 10$, Fig.5 shows the simulation result of the hysteresis behavior stated in (1). According to [Donald, 1996], integrating $u_{pwm}(t)$ (see Fig. 3), an estimation of $u(t)$ is obtained (see Fig. 6, $\hat{u}(t) = u_g(t)$ is the estimation of $u(t)$). In the numerical simulation experiments, we use the Euler method with an integration step-size of 0.01 seconds.

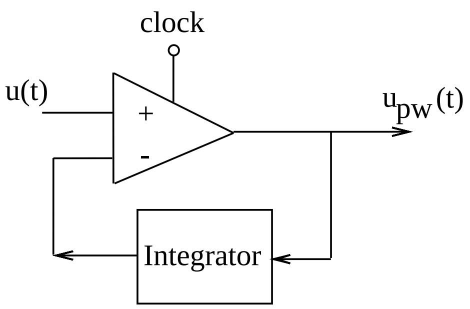
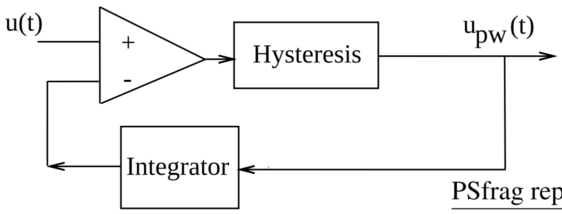


Figure 1. Block diagram of the delta modulator.



PSfrag replacements
$\hat{u}(t)$

Figure 2. Analog Delta modulator model.

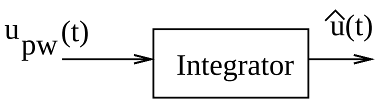


Figure 3. An integrator to estimate $u(t)$.

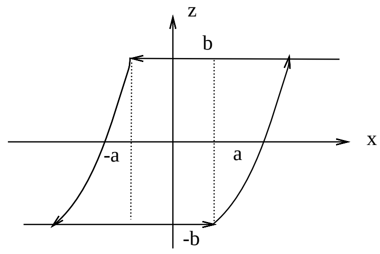


Figure 4. Hysteresis.

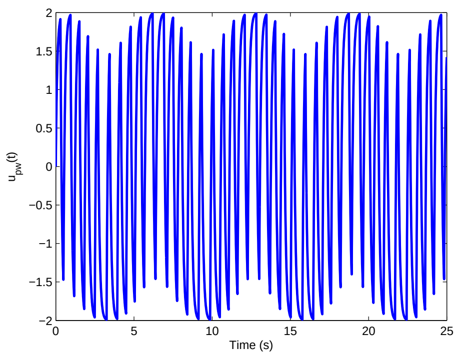


Figure 5. The output of the Delta modulator $u_{pw}(t)$ versus time.

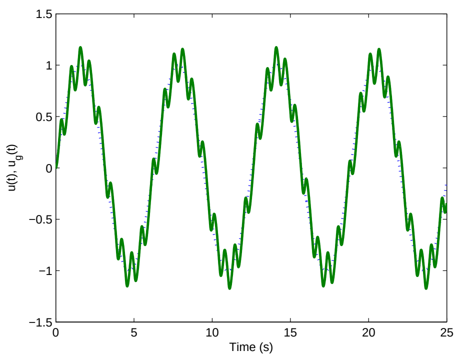


Figure 6. The estimation signal $u_g(t)$ (the continuous-green line) and the base signal $u(t)$ (the dotted-blue line), versus time.

3 Chaotic Duffing Oscillator Using the Delta Modulator

Given the chaotic Duffing oscillator:

$$\ddot{x} + 0.2\dot{x} - x + x^3 = 0.3f(t), \quad (4)$$

with $f(t) = \sin(t)$, we modify it as follows (see Fig. 7):

$$\ddot{x} + 0.2\dot{x} - x + x^3 = 0.3u_{pwm}(t), \quad (5)$$
$$\dot{z} = c[-z + b\ \text{sign}(u_e + a\ \text{sign}(z))], \quad (6)$$
$$u(t) = \sin(t), \quad (7)$$
$$\dot{y} = z(t), \quad (8)$$
$$u_e(t) = u(t) - y(t), \quad (9)$$
$$u_{pwm}(t) = z(t). \quad (10)$$

Using $a = 0.01$ and $c = 10$, simulation results for several values of b are shown in Figures 8-19 ($x(t) = x_1(t)$, and $\dot{x}(t) = x_2(t)$). From these figures, we can appreciate that the chaotic behavior of the Duffing oscillator can be altered using our design. In all the numerical experiments, the corresponding Poincaré maps were obtained by plotting the sequence of points $(x(t_k), \dot{x}(t_k))$, with $t_k = t_0 + 2\pi k$ [Thomsen, 2003]. According to [Thomsen, 2003], this tool is a valid tool to detect chaos. On the other hand, trying to find the Lyapunov exponents will be difficult due to the no-smooth conduct of the input signal to the Duffing system.

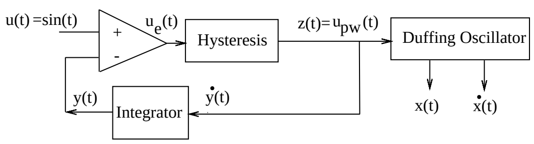


Figure 7. The Modified Duffing oscillator using the proposed Delta modulator.

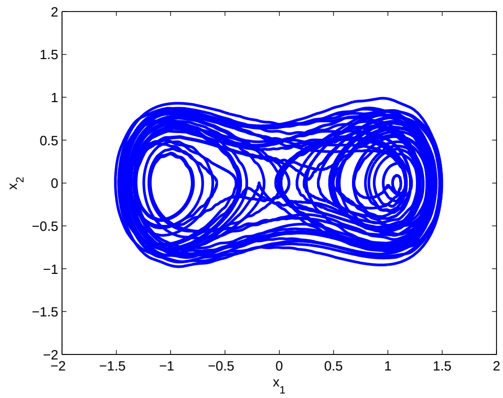


Figure 9. Chaotic attractor ($b = 1$).

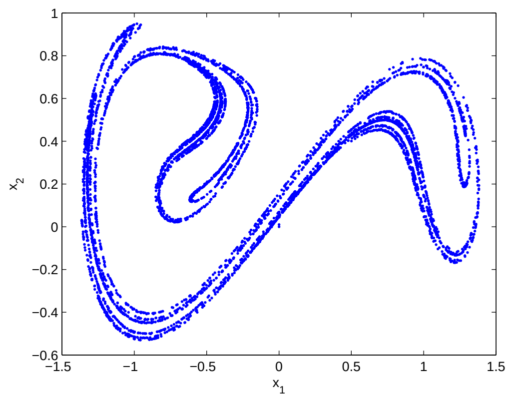


Figure 10. Poincaré map ($b = 1$).

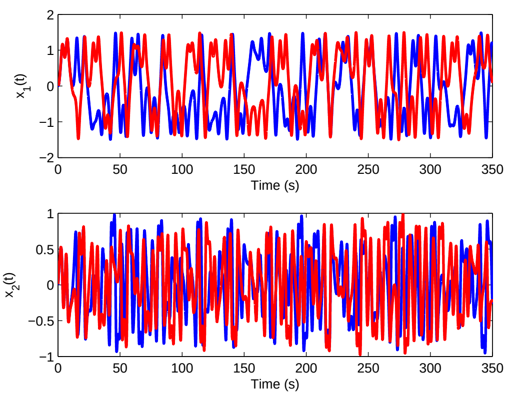


Figure 8. Sensibility condition test ($b = 1$).

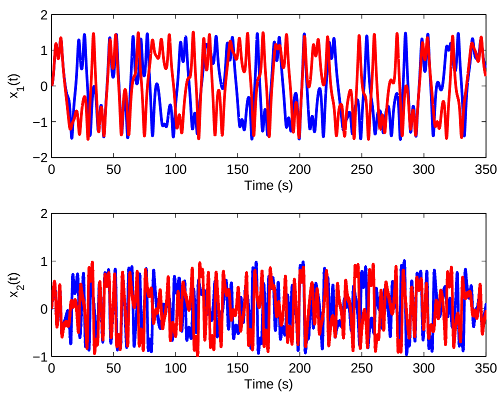


Figure 11. Sensibility condition test ($b = 10$).

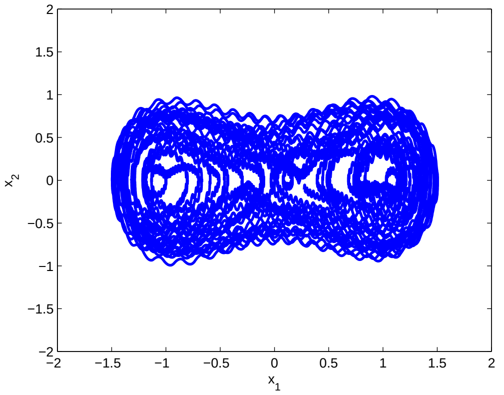


Figure 12. Chaotic attractor ($b = 10$).

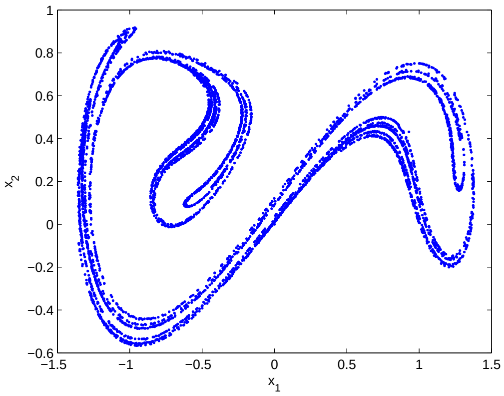


Figure 13. Poincaré map ($b = 10$).

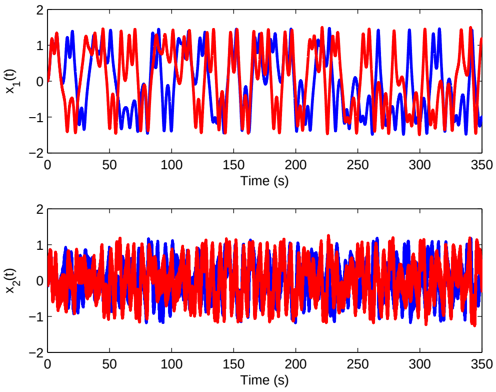


Figure 14. Sensibility condition test ($b = 100$).

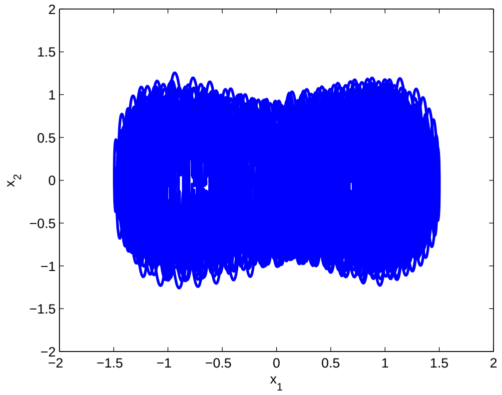


Figure 15. Chaotic attractor ($b = 100$).

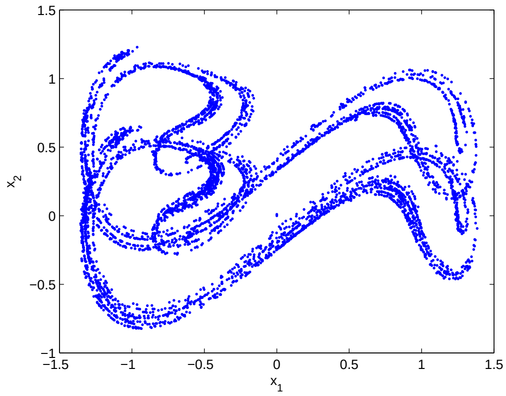


Figure 16. Poincaré map ($b = 100$).

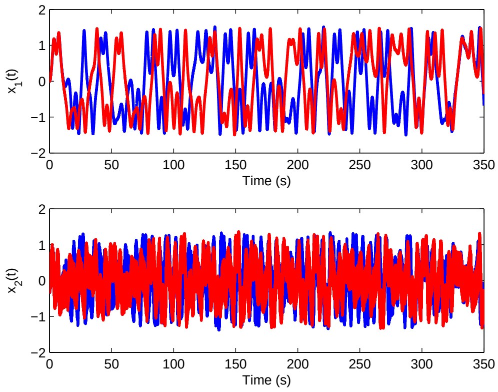


Figure 17. Sensibility condition test ($b = 150$).

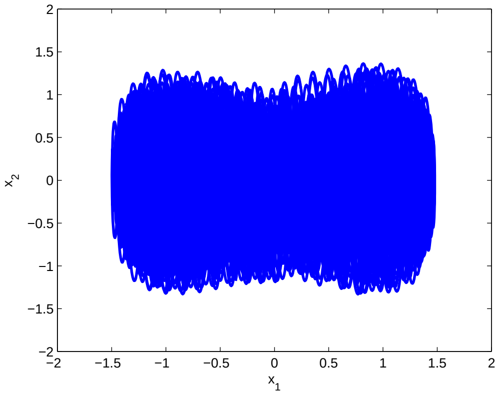


Figure 18. Chaotic attractor ($b = 150$).

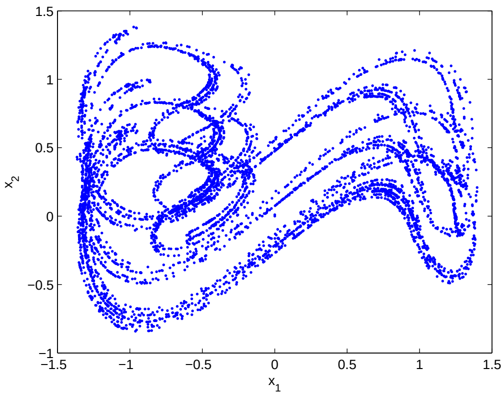


Figure 19. Poincaré map ($b = 150$).

4 Conclusions

A design based on a modified Delta modulator has been proposed to drive the Duffing oscillator as an option to manipulate its chaotic dynamics. Although there exist other ways to manipulate the Duffing system (see, for instance [Kakmeni et al., 2004]), our propose is obtained from an application of digital communication systems employing Delta modulators. Moreover, the obtained Duffing oscillator corresponds to a four-order dynamic system (it could be an hyper-chaotic system).

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