Recursive Active Control for Controlling Chaos in Nonlinear Bloch Equations

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***Abstract*— The problem of chaos control in the nonlinear Bloch equations is considerd based on a new control technique. The new control technique combines a recursive approach and active control mechanism to design control functions that suppresses chaotic behaviours in nonlinear Bloch equations. The effeciency of the proposed Recursive Active Control (RAC) is demonstrated with numerical simulations.**

I. INTRODUCTION

Many physical systems can exhibit chaotic dynamics under certain conditions. Chaotic behaviours could be beneficial feature in some cases, however, it is undesirable in many engineering and other physical applications; and therefore it is often desired that chaos should be controlled, so as to improve the system performance. Thus, it is of considerable interest and potential utility, to devise control techniques capable of forcing a system to maintain a desired dynamical behaviour (the "goal" or "target") even when intrisically chaotic. The control of chaos and bifurcation is concerned with using some designed control input(s) to modify the characteristics of a parameterized nonlinear system. The control can be static or dynamic feedback control, or open-loop control. In most cases, the goal could be the stabilization and reduction of the amplitude of bifurcation orbital solutions, optimization of a performance index near bifurcation, reshapening of the bifurcation diagram or a combination of these [1], [2], [3].

For almost two decades, there has been intense research activities devoted to the design of effective control techniques. A large number of the proposed methods are based on the Ott, Grebogi and Yorke (OGY) closed-loop feedback method [4] and the Pyragas time-delayed auto-synchronization (TDAS) method [5]. In the recent times, numerous linear and nonlinear control methods have emerged. In particular, recursive backstepping nonlinear control scheme has been employed recently for controlling, tracking and synchronizing chaotic systems [6-12]. Recursive backstepping is a systematic design approach and consists in a recursive procedure that skillfully interlaces the choice of a Lyapunov function with the control.

In another development, Bai and Longrenn proposed an active control method for chaos synchronization [13]. The active control scheme has in the last one decade received considerable attention due to its simplicity and has been widely accepted as an effecient technique for synchronization of identical and non-identical chaotic systems (See for example Refs.[14-23] and refs. therein). Very recently, we reported the control of directed transports arising from co-existing attractors in ratchet motion using the active control mechanism [23].

It is known that chaos synchronization is closely related to observer problem in control theory [24]. Hence, it would be significant to develop a chaos control method for the active control scheme. To address this issue, we proposed very recently [25], a recursive active control (RAC) for controlling chaotic systems. In this paper, we extend our study on RAC, by setting up a recursive active control (RAC) scheme for controlling chaotic motion in the nonlinear Bloch equations (NBE). The method combines recursive approach with active control technique to design control functions that can suppress the chaotic behaviour in NBE.

II. THE MODEL

Motivated by the need to interpret various anomalies that had been observed in nuclear magnetic resonance (NMR) experiments, in terms of chaos theory, Abergel [26], recently examined the linear set of equations originally proposed by Bloch to describe the dynamics of an essemble of spins with minimal coupling. The model incorporates certain nonlinear effects arising from radiation damping based on feedback and consists of the three nonlinear modified Bloch equations (NBE) given in dimensionless units as

$$\begin{aligned} \dot{x} &= \delta y + \lambda z(x\sin\psi - y\cos\psi) - \frac{x}{\tau_2}, \\ \dot{y} &= -\delta x - z + \lambda z(x\cos\psi + y\sin\psi) - \frac{y}{\tau_2}, \\ \dot{z} &= y - \lambda\sin\psi(x^2+y^2) - \frac{z-1}{\tau_1}, \end{aligned} \tag{1}$$

where the dots denotes time derivatives, δ, λ, and ψ are the system parameters; and τ_1 and τ_2 are longitudinal time and transverse relaxation time respectively. The dynamics of system (1) has been extensively studied in Ref. [17], [26] for a fixed subset of the system parameters $(\delta, \lambda, \tau_1, \tau_2)$ and for a space area range of the radiation damping feedback ψ. The regions of ψ that would admit chaotic solutions were obtained. For instance, the NBE exhibits chaotic behaviour for $\delta = -0.4\pi$, $\lambda = 30, \psi = 0.173, \tau_1 = 5$ and $\tau_2 = 2.5$ as shown in Fig. 1 and Fig. 2.

Additionally, Moukam Kakmeni, Nguenang and Kofane [28], examined the dynamics of a variant NBE, extended to account for both the bi-axial property of the magnets to which the set of spins belongs and the presence of a back

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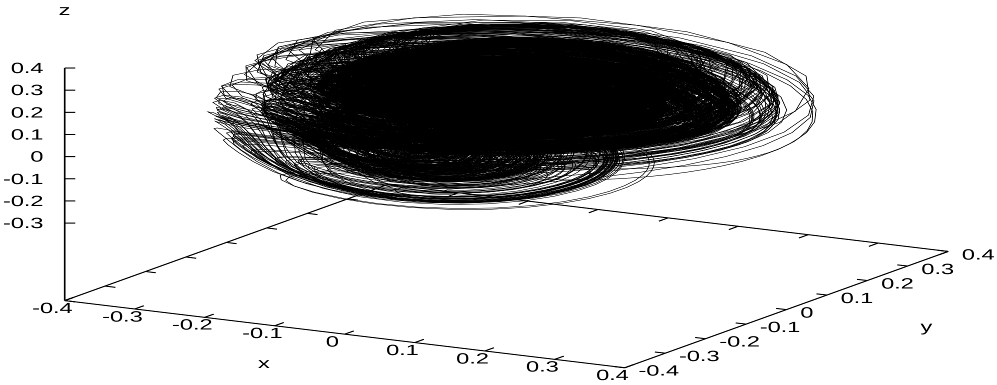

Fig. 1. Chaotic dynamics of the uncontrolled NBE. Parameters are: $\delta = -0.4\pi$, $\lambda = 30, \psi = 0.173, \tau_1 = 5$ and $\tau_2 = 2.5$

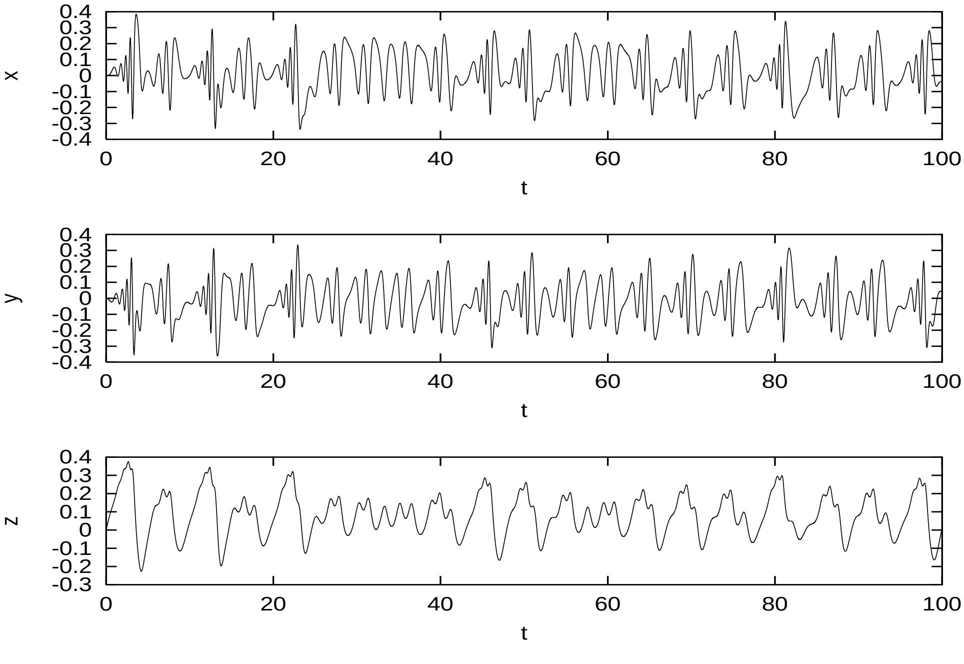

Fig. 2. Time history of the NBE system for the same parameter set as in Fig. 1

action from the probe. In [17], [27], [28], the synchronization behaviours of the NBE were also reported. Ucar et al. [17] studied the synchronization of drive-reponse system of the NBE with non-identical parameters using active control while in Ref. [27], Park studied the synchronization of the NBE with uncertain parameters. Moukam Kakmeni et. al. [28], considered the synchronization problem based on adaptive approach, using both linear and nonlinear feedback couplings. In all these reports, the control of the NBE chaotic behaviour to regular dynamics has not been addressed. In this paper, we set up a new chaos control scheme which we have recently developed for the NBE.

III. Design of RAC for NBE

To control the NBE chaotic attractor, we introduce the control functions $u_i (i = 1, 2, 3)$ as follows

$$\begin{aligned}
\dot{x} &= \delta y + \lambda z(x \sin\psi - y\cos\psi) - \frac{x}{\tau_2} + u_1, \\
\dot{y} &= -\delta x - z + \lambda z(x\cos\psi + y\sin\psi) - \frac{y}{\tau_2} + u_2, \quad (2) \\
\dot{z} &= y - \lambda \sin\psi(x^2 + y^2) - \frac{z-1}{\tau_1} + u_2,
\end{aligned}$$

and define the error dynamics as

$$\begin{aligned} e_x &= x - x_{1d}, \\ e_y &= y - x_{2d}, \\ e_z &= z - x_{3d}. \end{aligned} \tag{3}$$

For simplicity let

$$\begin{aligned} x_{1d} &= 0, \\ x_{2d} &= c_1 e_x, \\ x_{3d} &= c_2 e_x + c_3 e_y, \end{aligned} \tag{4}$$

where the $c_i's(i = 1, 2, 3)$ are arbitrary chosen control parameters; x_{1d} is the reference output; $x_{2d},\ x_{3d}$ are recursively introduced control inputs. Now, differentiating eq. (3) and (4); and substituting eqs. (2) into the resulting equations, we obtain the following error dynamics system:

$$\begin{aligned} \dot{e}_x &= \delta e_y + \lambda e_z(e_x \sin\psi - e_y \cos\psi) - \frac{e_x}{\tau_2} + u_1, \\ \dot{e}_y &= -\delta e_x - e_z + \lambda e_z(e_x \cos\psi + e_y \sin\psi) - \frac{e_y}{\tau_2} + u_2, \\ \dot{e}_z &= e_y - \lambda \sin\psi(e_x^2 + e_y^2) - \frac{e_z - 1}{\tau_1} + u_3. \end{aligned} \tag{5}$$

In (5), the $c_i's(i = 1, 2, 3)$ have be chosen so that the $\dot{e}_j(j = x, y, z)$ terms on the RHS varnishes. Here, $c_1 = c_2 = c_3 = 0$. In the absence of the control $u_i(i = 1, 2, 3)$, eg. (5) would have an equilibrium at $(0, 0, 0)$. If a $u_i(i = 1, 2, 3)$ is chosen such that the equilibrium remains unchanged, then the problem can be transformed to that of realizing asymptotic stabilization of system (5). Thus, the goal is to find the controls such that the system (5) is stabilized at the origin. Following the original method of active control, we re-define the control functions as follows

$$\begin{aligned} u_1 &= V_1 - \lambda e_z(e_x \sin\psi - e_y \cos\psi), \\ u_2 &= V_2 - \lambda e_z(e_x \cos\psi + e_y \sin\psi), \\ u_3 &= V_3 + \lambda \sin\psi(e_x^2 + e_y^2) - \frac{1}{\tau_1}. \end{aligned} \tag{6}$$

With (6), the error dynamics (5) becomes:

$$\begin{aligned} \dot{e}_x &= \delta e_y - \frac{e_x}{\tau_2} + V_1, \\ \dot{e}_y &= -\delta e_x - e_z - \frac{e_y}{\tau_2} + V_2, \\ \dot{e}_z &= e_y - \frac{e_z}{\tau_1} + V_3. \end{aligned} \tag{7}$$

We choose a feedback matrix **A** which will control the error dynamics (11) such that

$$\begin{pmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{pmatrix} = \mathbf{A} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \tag{8}$$

With

$$\mathbf{A} = \begin{pmatrix} k_1 + \frac{1}{\tau_2} & -\delta & 0 \\ \delta & k_2 + \frac{1}{\tau 2} & 1 \\ 0 & -1 & k_3 + \frac{1}{\tau_1} \end{pmatrix} \tag{9}$$

In (9) the three eigenvalues k_1, k_2 and k_3 are negative to ensure that a stable controlled state is achieved. Here, we fix $k_1 = k_2 = k_3 = -1$.

When the control is switched on, it is clear from the numerical simulation shown in Fig. 3 that the chaotic behaviour has been controlled as soon as the control is activated at $t = 100$.

IV. Conclusion

In this paper, we have combined a recursive approach with the active control technique to formulate a new control technique that elliminates the chaotic behaviour in nonlinear Bloch equations. The proposed Recursive Active Control (RAC) is simple to implement. Numerical simulations have been employed to confirm our results.

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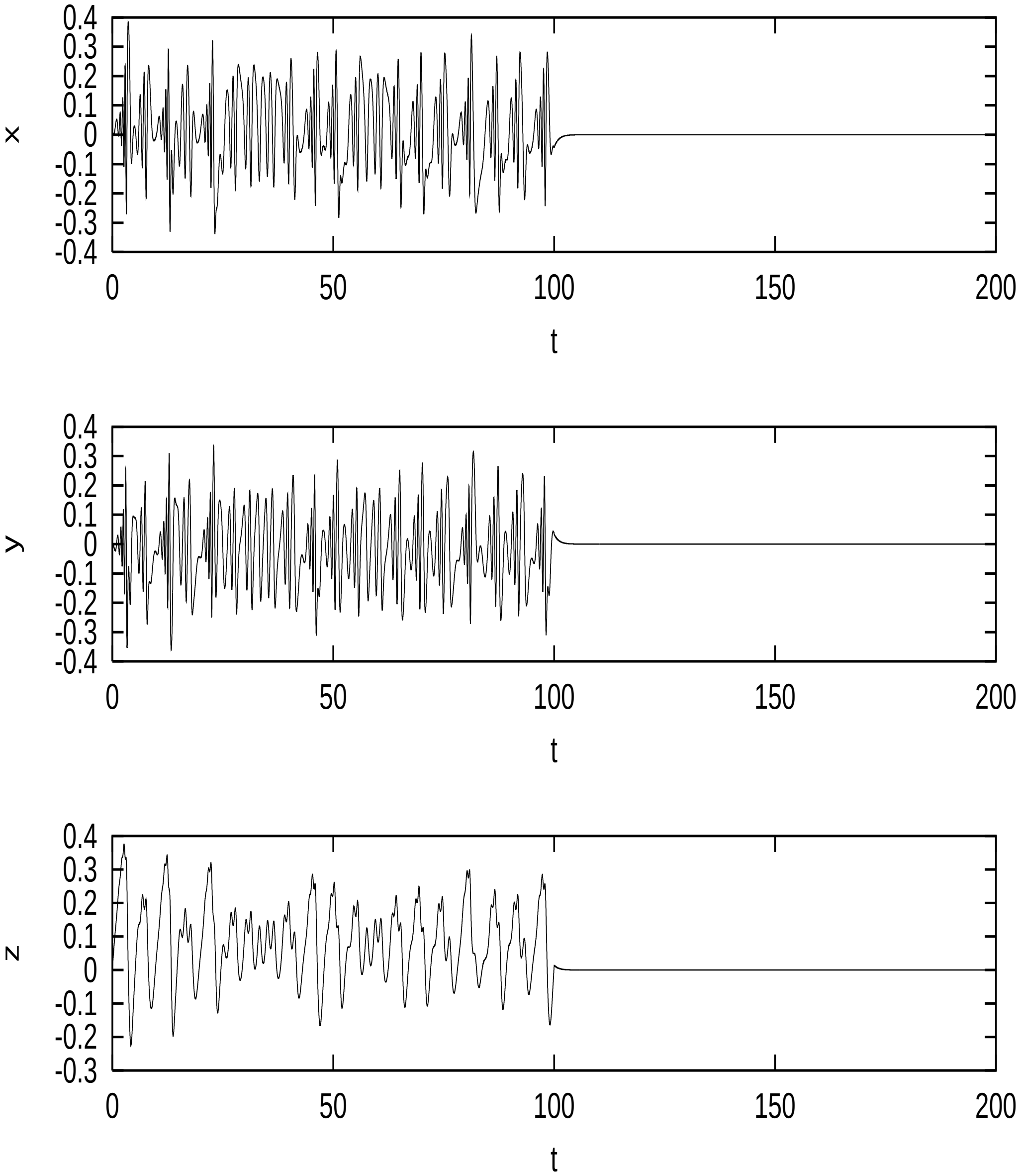

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